Friction Factor Correlation with Application to the Central Cooling Channel of Cable-in-Conduit Super-Conductors for Fusion Magnets

Roberto Zanino, Piero Santagati and Laura Savoldi
Dipartimento di Energetica, Politecnico, Torino, Italy
Andre' Martinez and Sylvie Nicollet
CEA, Cadarache, France

Abstract – **A correlation has been developed for the turbulent friction factor f of a circular channel with a helical rib roughness of rectangular cross section, which is relevant to the central channel (hole) in two-channel cable-in-conduit conductors. The correlation is based on data we measured on a pipe with three different types of helix. It relates f with the Reynolds number Re and with suitable dimensionless combinations of all relevant geometrical parameters of the problem, i.e., hole diameter and helix gap and thickness. A limited comparison with actual (QUELL) conductor data shows good agreement.**

I. Introduction

The Central Solenoid and Toroidal Field Model Coils (CSMC and TFMC, respectively) of the International Thermonuclear Experimental Reactor are being built using super-conductors with a two-channel topology. The annular cable bundle region is separated from the central channel (hole) by a helical spring-like interface. The hole provides a low impedance parallel path for helium flow. Additionally, overpressure of the supercritical helium coolant, which can originate in the bundle as a consequence of thermal-hydraulic transients in the cable, can be relieved in the hole through the perforation of the spring.

The empirical correlation of Katheder [1] and/or extensions thereof accurately models the friction factor in the bundle in the proper range of Re. On the contrary, the friction factor f for the central channel has been traditionally modeled (e.g., for the QUench Experiment on Long Length – QUELL) starting from the classical smooth circular tube correlation $f = 0.046\,(Re)^{-0.2}$, and correcting it with an ad-hoc multiplier [2]. Very recently, however, experimental tests [3] have shown that geometrical details of the helix can significantly influence the overall pressure drop along the conductor, while analysis [4] indicated that a more accurate model of f is needed to reproduce the measured mass flow rate.

Although the situation we are analyzing is tightly related to problems of heat transfer enhancement in heat exchangers [5]-[8], the geometries previously considered in the literature were characterized to the best of our knowledge by somewhat different ranges of geometrical parameters. Here, a correlation for the Fanning friction factor is derived based on a set of experimental data we obtained in the OTHELLO facility at CEA Cadarache on a tube with a helical-rib-roughened inner wall. The correlation is then applied to and validated against actual QUELL conductor data.

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R. Zanino, +39 011 564 4490, fax +39 011 564 4499, zanino@polito.it, http://www.polito.it/~zanino.
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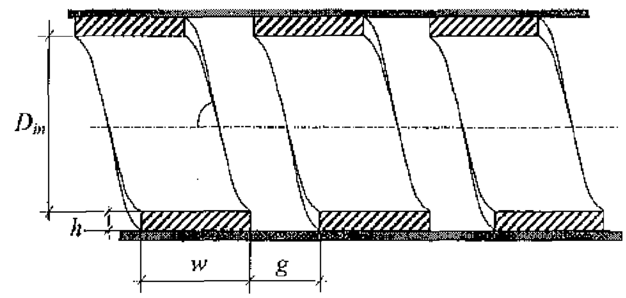

Fig. 1. Sketch of the tested pipe (longitudinal cross section)

II. Experimental Setup

Three different helices with rectangular cross section, used for different pancakes in the TFMC, were tested: CORTAILLOD (C), HITACHI (H) and SHOWA (S). A sketch of a helix is shown in Fig. 1, and its main features are summarized in Table I: the inner diameter D_{in}, the height (thickness) h, the width w, the gap length g. The angle formed by the helix with its axis is $\theta \sim \mathrm{atan}\,[\pi(D_{in}+h)/p]$, where $p = w + g$ is the pitch.

For the helices in Table I, θ varies from 71° (C) to 76° (S), i.e., these helices are very "closed". Notice also that only g varies significantly among the three. Both of these observations will be important in what follows.

The dependence $f\,(Re)$ is obtained from data of pressure drop Δp vs. mass flow rate m as follows: starting from the definitions of Re and of Δp

$$Re = \rho\, U\, D_h \,/\, \mu \quad (1)$$
$$\Delta p \,/\, L = 2\, f\, \rho\, U^2 \,/\, D_h \quad (2)$$

and eliminating the average (core) flow speed $U = m \,/\, (\rho A)$, where $A = \pi D_{in}^2/4$ is the minimum flow area, gives

$$Re = m\, D_h \,/\, (\mu A) \quad (3)$$
$$f = (1/2)\,(\Delta p/L)\, \rho\, D_h\, A^2 \,/\, m^2 \quad (4)$$

where L = 5 m is the length of the hydraulic path, and D_h is the hydraulic diameter assumed = D_{in} for the core flow. The dynamic viscosity μ and the density ρ are obtained from pressure and temperature measurements, using GASPAK.

Typically, N2 enters the test section at T ~ 235 K and exits at T ~ 295 K, the temperature increase being due mainly to heat exchange with the surrounding environment at room temperature. Different series of tests were performed with different N2 pressures, in the range between 5 bar and 40 bar.

Concerning the H(itachi) data it should be noticed finally that only a subset of the measured data was used for the correlation, i.e., those showing a clearly decreasing trend of

TABLE I
GEOMETRICAL PARAMETERS OF THE TESTED HELICES

Type	D_{in} [mm]	h [mm]	w [mm]	g [mm]
C	10.1	1	6.5	5.3
H	9.65	1	6.2	3.0
S	9.9	1	6.25	2.4

f with Re. In the literature, increasing f (Re) is found for smooth pipes in the laminar-to-turbulent transition region, which however occurs well below $Re \sim$ 1e4. For rough pipes it is found in the smooth-to-rough transition region, which occurs at decreasing Re for increasing (h/D), e.g., at $Re \sim$ 1e4 for $(h/D) \sim 0.01$ [8]. Some of our data series, showing an increasing trend of f with Re for $Re >$ 1e5, have been therefore discarded.

III. QUALITATIVE DISCUSSION

The experimental results (see, e.g., Fig.3 below) show that, at given Re, an increase of (g/h) of up to ~ 100%, see Table I, all other parameters being essentially unchanged, leads to an increase of f of up to 50%. Therefore it can be considered important experimental evidence that *the gap is playing the major role in determining the friction factor.*

The helix can be considered as a roughness of average height h on the underlying pipe of diameter $D_{out} \equiv D_{in}+2h$ (alternately, one could think that a helical gap has been *grooved* in a pipe of diameter D_{in}). Geometries similar to that of Fig. 1 have been often considered in studies of turbulence promoters for heat exchangers. The obvious trade-off is that also friction increases with respect to the case of a smooth pipe, so that it needs to be simultaneously taken into account.

Friction on the flowing fluid in the geometry of Fig. 1 is extremely complex because of the 3-D nature of the flow field (indeed, not even numerical solutions appear to exist to date for this geometry). The analysis can be however simplified by the observation made above that the helices we are considering are very closed, i.e., they may be approximated to some extent by a 2-D model with a series of rings separated by gaps g.

Conceptually, we may further split the force exerted by the wall on the fluid in two contributions: that from the helix width w, and that from the helix gap g. On the width a laminar sub-layer attempts to build up, but it quickly separates at the gap. Since w is so short, no significant shear can build up there, and this contribution to friction can be considered negligible (of course, in the limit of vanishing (g/h) this contribution cannot be neglected). For this reason (and also considering that w is essentially unchanged in the three helices tested) the helix width will not enter the correlation for f.

On the contrary, in the gap a separated re-circulating vortex appears. This is due to the fact that the layer is not expected to reattach at the gap bottom unless $(g/h) > 6$-8 [5], whereas for us (g/h) is always < 6, see Table I. Therefore, we can assume that the major contribution to friction comes from the interaction of the core flow in the channel with the salient edges of the gap and with the re-circulating vortex, and the parameter (g/h) will play an essential role in the correlation, which we are going to derive.

IV. FRICTION MODELS

The conventional approach to the problem of friction in internally rough pipes is based on the study of the interaction of the boundary layer at the pipe wall with obstacles of given height and shape. Although strictly speaking justified only in this case, the same approach has been often *empirically* applied also to situations where, as seen above for our case, formation of the boundary layer was not guaranteed. We shall also follow this line eventually, for the derivation of our correlation, in the absence of more rigorous approaches.

If we consider the case of a smooth circular pipe, there are two fundamentally different regions in the boundary layer, which forms near the wall in turbulent flow: a laminar region next to the wall and a turbulent region next to the channel core [6]. In the laminar sub-layer the velocity distribution can be represented by a linear dependence of u^+ ($\equiv u/u^*$) on y^+ ($\equiv yu^*/\nu$). Here u is the time average (mean) of the turbulent flow speed along the channel, y is the radial coordinate measured from the wall, $u^* \equiv \sqrt{(\tau_0/\rho)}$, τ_0 is the viscous tangential stress at the wall ($y = 0$), and $\nu \equiv \mu/\rho$. In the outer (turbulent) boundary region it is easy to obtain $u^+ = 2.5 \ln (y^+) + \mathrm{R}$, where R is an integration constant to be determined from the experiment.

Although the boundary layer does not extend strictly speaking beyond $y \sim 0.15\ (D_h/2)$, it is common practice to use the expression of u^+ for the turbulent layer, up to the center of the channel. Thus, the u (y) profile is known, and it is easy to relate U with u^*, i.e., with τ_0. Using the previous formulas, an implicit expression for f (Re) can be obtained:

$$\sqrt{(2/f)} = 2.5 \ln [\sqrt{(f/2)}\, Re / 2] + \mathrm{R} - 3.75 \,. \qquad (5)$$

Notice that (5) can either be seen as an equation for f (Re), provided R is known from experiment, or as a way of determining R from f (Re) measurements.

Consider now the case of sand grains of diameter h homogeneously glued on the inner surface of a circular pipe. The major difference with respect to the above treatment of a smooth pipe is that R is now expected to depend on the roughness. More precisely one finds in this case $u^+ = 2.5 \ln (y/h) + \mathrm{R}\ (h^+)$, where

$$h^+ \equiv (h/D_h)\, Re \sqrt{(f/2)} = (u^* h/\nu) \qquad (6)$$

is a roughness Reynolds number. In the case of a rough tube, (5) generalizes therefore to

$$\boxed{\mathrm{R}\ (h^+) = \sqrt{(2/f)} + 2.5 \ln (2h/D_h) + 3.75 \,.} \qquad (7)$$

For homogeneous sand grain Nikuradse found that, for $h^+ > 70$, the so-called fully rough regime, R (h^+) ~ 8.48, a universal value. Notice finally that (7) stays valid for any geometrically similar type of roughness, i.e., provided only (h/D_h) is varied, and all the rest is kept fixed.

In order to understand how additional geometrical parameters, besides (h/D_h), are empirically brought into the model for geometrically more complex roughness, we shall consider now two relevant examples of two-parameter correlation.

In the first study, heat transfer and friction in tubes with repeated-rib roughness (annular obstacles of given periodicity p and height h, and negligible width) were considered [5], and (p/h) was used as second correlation parameter. In the second study, turbulent heat transfer and fluid friction in a helical-wire-coil-inserted tube were considered [7], and the helix angle θ was used as second correlation parameter. (Such a device was also considered at MIT [9]. However the parameters of the MIT helix (D_{in} = 9.1mm, $h = w$ = 1.55mm, g = 1.6mm, giving $\theta \sim 85^\circ$), fall outside the domain of validity of [7].)

In both cases just discussed, the correlation formula has the following structure

$$\mathrm{R}\,(h^+) = \alpha (h^+)^{\beta}\,[parameters]^{z} \tag{8}$$

which will be used as a guideline in the development of our correlation. It should be stressed, however, that this procedure is rather empirical since, e.g., (p/h) is *not* a geometrically similar roughness parameter.

V. Correlation for R (h^+)

Starting from $f\,(Re)$ experimental data, it is possible to construct the R (h^+) diagrams using (7). The result, shown in Fig.2a, indicates that, as expected, R (h^+) needs to be properly corrected as in (8), taking into account the additional relevant geometrical parameters of the helix, if we want the correlation to be representative of all data.

The helices we are considering are defined by four independent geometrical parameters (see Table I). However, in view of the discussion of Section III, we can restrict our choice to the following two dimensionless combinations:

Rib-height on hydraulic diameter = h/D_h,

Gap length on rib-height = g/h.

We then look for the friction function R in the form

$$\mathrm{R}\,(h^+) = \alpha\,(h^+)^{\beta}\,(g/h)^{\gamma} \tag{9}$$

and multivariate least squares regression (IMSL routine RLSE) is used to find α, β, γ (notice that h/D_h is hidden in h^+).

The result is given by the first row of Table II, i.e.,

$$\boxed{\mathrm{R}\,(h^+) = 11.88\,(h^+)^{0.039}\,(g/h)^{-0.299}} \tag{10}$$

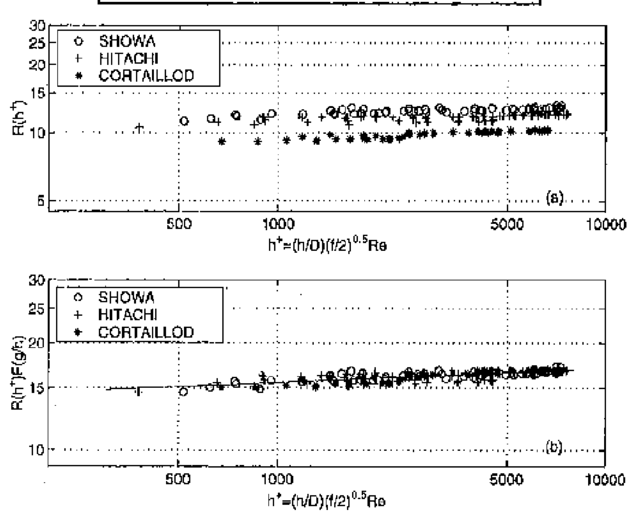

Fig. 2. (a) Experimental data for friction function R vs. h^+, for different helices. (b) Corrected friction function R $(g/h)^{-\gamma}$ vs. h^+; experimental data (symbols), computed from correlation (10) (solid line).

TABLE II

Coefficients in (9) and Accuracy of (7) + (10) in the Prediction of f

	α	β	γ	σ_S (%)	σ_C (%)	σ_H (%)
C+S+H	11.88	0.039	−0.299	3.4	3.6	4.8
C+S	11.37	0.045	−0.300	3.5	3.2	5.0

and shown in Fig.2b (where F ≡ $(g/h)^{-\gamma}$).

It may be noticed in (10) that the dependence on h^+ is very weak (but not negligible, see Fig.2a). This is typical of the high Re regimes considered here, and in qualitative agreement with results shown in [10] for a somewhat different geometry and parameter range.

The friction factor can now be computed *iteratively* combining (7) and (10). (Notice that, in a typical MITHRANDIR run, the relative cost of the evaluation of both (bundle + hole) friction factors increases from ~ 0.2% to ~ 0.5% when going from an explicit $f\,(Re)$ to the present, implicit relation, i.e., it remains absolutely negligible.) The result of our correlation is given by the solid lines in Fig.3, with an average error $\sigma \sim 4\%$ with respect to the experimental data (see Table II).

As a measure of the *predictive* capability of the correlation we have also repeated the exercise using only C and S data, which leads to the second row in Table II and to the dashed lines in Fig.3. Notice the good accuracy of the prediction for the H data, with average error ~ 5% (see Table II). The latter is significantly smaller than the average relative scattering of the data.

Probably the most striking feature of Fig.3, and indeed the major motivation behind this work, is the rather significant increase of f when (g/h) increases, for a given Re. This parametric dependence has been investigated also by other authors, who found increasing f for $2 < (g/h) < 8.5$ [11] or for $5 < (p/h) < 10$ [6], in somewhat different geometries. (In all instances a maximum is predicted when the reattachment of the boundary layer occurs just before the next obstacle – a fact which should not happen, as seen above, for our parameter range.) Qualitative justification of the dependence on (g/h) is not easy, in view of the extreme complexity of the flow pattern. However it can be observed that an increasing deviation of the streamlines could be expected downstream of the obstacle as the gap increases, and this should increase the form drag. Simulations (based on the k-ε turbulence model) and measurements, performed for a geometrically similar situation, appear to qualitatively confirm this picture [12].

Notice finally that both in the limit of vanishing and of infinite (g/h), one has to recover the smooth tube situation, i.e., $f \sim$ 3-4 times smaller than with our helices, see Fig.3.

VI. Application to Two-Channel CICC

The application of our correlation to an actual conductor is not entirely obvious since the helix is not attached to a solid wall, and one can now have flow through the gaps, which allow communication between bundle and hole regions of the two-channel CICC. In particular: 1) the flow pattern in the gaps can be significantly perturbed with respect to the experimental condition in OTHELLO; 2) the effective height of the obstacle could fall in principle anywhere between 0 and h. (For the latter, assuming that the helix is attached to a "wall of strands", we shall use the actual helix thickness.)

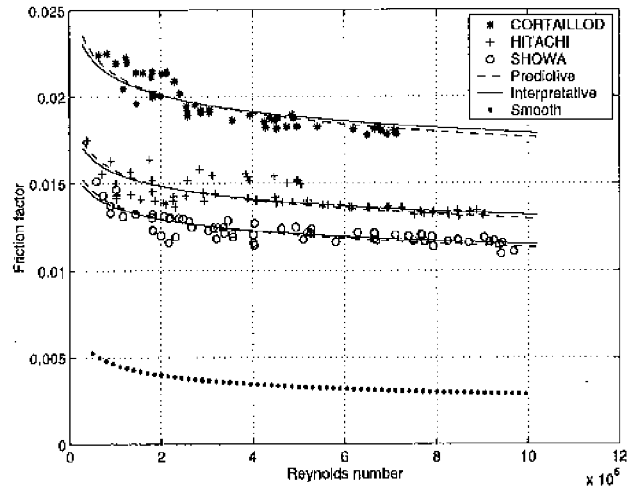

Fig. 3. Dependence of the friction factor f on the Reynolds number Re for different helices: experimental data (symbols), computed from correlation (7), (10) using whole data set (solid), computed from correlation (7), (10) using only C and S data set (dashed). Blasius correlation for smooth tubes (dotted) is also shown for comparison.

It should also be remarked that hydrodynamic similarity between the gaseous N2 flow in OTHELLO and the liquid He flow in an actual conductor would in principle require not only same Re but also same Mach number M (= U/C_S where C_S is the sound speed). However, while the He flow in an actual CICC is typically very much subsonic (e.g., $M \sim 0.01 \ll 1$ was typical in QUELL), N2 inlet $M \sim 0.2$-0.25 would be sufficient to reach $M \sim 0.3$-1.0 at the test section outlet in the case at hand. We checked therefore the outlet M in our cases and it turned out to be always < 0.1. Furthermore, in experiments by other authors [13] M does not appear to influence f significantly.

We now consider the application of the correlation just developed to data measured on QUELL [2]. (Notice owhever that the geometrical parameters of the QUELL helix, D_{in} = 5.9mm, h = 0.5mm, w = 5.9mm and g = 1mm, are a bit outside the range of Table I.) Starting from the raw Δp (m) data, where m is now the *total* (bundle + hole) mass flow rate, and following the strategy presented in [2], we have constructed a set of $f(Re)$ couples as given by the symbols in Fig.4. It should be observed also here the rather surprising occurrence of data series with increasing trend of f with Re. The prediction of our correlation is also shown in Fig.4, together with the correlation suggested in [2], i.e., Blasius corrected by a factor N = 2.5, as well as used, e.g., in [4]. One

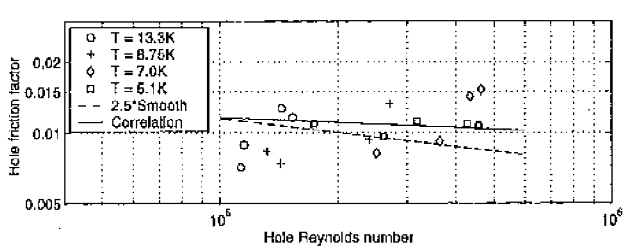

Fig. 4. Dependence of the hole friction factor f on the hole Reynolds number Re in QUELL: experimental data derived from Δp (m) measurements reported in [2] for different He temperatures (symbols), computed from correlation (7), (10) (solid). Blasius correlation for smooth tubes, corrected by a factor of 2.5 as suggested in [2], is also shown for comparison (dashed).

can see that, considering all uncertainties in the problem, the agreement between our correlation and the experimental data is good. Comparison with the correlation proposed in [2] appears difficult, because of the spread of the data.

VII. Conclusion and Perspective

The correlation developed in the present paper predicts within ~ 5% accuracy the friction factor f in a helical-rib-roughened tube similar to the central channel in two-channel CICC, for Reynolds number $5e4 < Re < 1e6$, and geometrical parameters of the helix in the range specified by Table I. Limited validation on actual (QUELL) conductor data also gives good agreement.

Observing the significant increase in pressure drop caused in all cases by the helix, with respect to the smooth tube, a reduction of the gap, with respect to the values of Table I, should give an improvement, at least from this point of view. Optimization of the helix geometry, however, should take into account also other aspects not considered here, e.g., heat exchange, mechanical issues, etc.

From an engineering point of view the accuracy of the results presented here can be considered satisfactory. However, the physics of friction in a geometry like that of Fig.1 is still not fully understood, and more work will be needed in the future.

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