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NUMERICAL SOLUTION OF A 2-D CONDUCTION-CONVECTION-RADIATION MODEL PROBLEM IN DIVERTOR GEOMETRY USING ADAPTIVE FINITE ELEMENTS

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ABSTRACT

Some of the essential features of plasma transport in the tokamak edge with realistic divertor geometry are simulated using adaptive finite elements to solve a time-dependent scalar anisotropic conduction-convection-radiation problem. Solutions are compared with those obtained on fixed finite element mesh and typically show comparable accuracy at a much lower cost or much better accuracy at comparable cost.

INTRODUCTION

A good understanding of the phenomena that take place in the scrape-off layer (SOL) region of tokamak plasmas is a critical issue for the success of the fusion program. From the modeling point of view, some of the most challenging difficulties arise from 1) the complex geometry of the tokamak SOL and 2) the strong anisotropy in the transport coefficients.

The first issue, in particular, led us in the past to develop the finite element (FE) code FELS for 2-fluid plasma modeling of the tokamak edge [1]. A detailed comparison [2] with the 5-point finite volume (FV) code B2 was performed on realistic divertor equilibrium with straight target quasi-orthogonal to the poloidal magnetic field. We showed that the FE code converged to the correct solution but, being the target geometry optimal for a 5-point FV computational molecule, conservative FV proved to be more robust than non conservative FE, on coarse meshes.

From the point of view of the comparison between FE and FV methods for the tokamak plasma edge, however, some aspects appear to deserve some further investigation. Here we shall concentrate on the following question: *Can an <u>adaptive</u> mesh, which is relatively easy to incorporate in an unstructured FE approach but difficult to realize with structured FV, bring significant advantages in the solution of SOL problems?*

The need for adaptivity has indeed become particularly apparent recently, with hydrogen/impurity radiation being considered as a promising way to reduce the thermal load on the divertor plates. In the presence of this type of transient phenomena, with nonlinear radiation sinks strongly depending on the plasma temperature, the accuracy of the numerical simulations will depend heavily on the possibility to locate and resolve (i.e., capture) moving structures (fronts) characterized by very strong gradients in the solution. Probably the most

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efficient and accurate way to solve these problems at a reasonable computational cost is to use an adaptive grid.

In rectangular geometry, the possibility to use an adaptive FE scheme to accurately compute a steady state radiation front ("detached plasma") with anisotropic heat conduction has already been demonstrated [3]. In this paper we extend the work of [3] by including time dependency, convection, and treatment of realistic divertor geometry.

We follow the evolution of a scalar quantity (e.g., temperature) in the plasma using a fully implicit scheme in time and Galerkin (P1) method in space. The set of nonlinear equations arising at each time step from the discretization is linearized with a globally convergent version of Newton's method based on a line-search backtracking procedure [3] (outer iterations) and then solved using GMRES (inner iterations). The plasma solver is coupled with an automatic unstructured (triangular) mesh generator [4] and the new solution is checked at each time step so as to determine how and when to update the mesh. Results are presented showing the flexibility of this strategy for modeling some of the essential features of edge plasma transport in realistic geometry.

MODEL EQUATION

We consider the following anisotropic conduction-convection-radiation model problem in a 2-D axis-symmetric domain (portion of the outer leg of a single-null SOL)

$$\frac{\partial u}{\partial t} + \nabla \cdot (\boldsymbol{a} u + \boldsymbol{q}) = S(u)$$

$$\boldsymbol{a} = a_{\|}\hat{b} + a_{\psi}\hat{e}_{\psi} \quad \boldsymbol{q} = -\boldsymbol{\kappa} \cdot \nabla u \quad \boldsymbol{\kappa} = \kappa_{\|}(u)\hat{b}\hat{b} + \kappa_{\psi}\hat{e}_{\psi}\hat{e}_{\psi}$$

Convection *a* and conduction *q* are split in components *parallel* to the magnetic field (unit vector *b*) and perpendicular to the magnetic surfaces (unit vector e_{ψ} briefly referred to as *radial* in the following). The radiation sink *S* is in general a strongly nonlinear function of *u* [3]. Magnetic field quantities are taken from a Grad-Shafranov solver and refer to actual divertor equilibria of the Asdex-Upgrade tokamak [2].

Although the fluid SOL problem is obviously a vector one (continuity, momentum and energy balances) this scalar problem can be considered as a good representative of the difficulties met in solving the full system.

ADAPTIVE STRATEGY

The adaptive strategy (see also [3]) relies on a local interpolation error control based on the Hessian matrix *H*, which is computed at each node of the mesh before creating the new one. The user has essentially 4 free input parameters available: maximum norm μ and minimum norm ν of the metric computed from *H* (see below), maximum r_M and minimum r_m aspect ratio of the triangles in the mesh.

To get the new mesh parameters, one must know the characteristic lengths along the directions parallel and perpendicular to the *poloidal* magnetic field, l_θ and l_ψ, which are computed as follows: first the eigenvalues λ_1, λ_2 of H and the corresponding characteristic lengths: $l_j = 1/\sqrt{|\lambda_j|}$ are determined. Then we construct the ellipse with axes l_1 and l_2 parallel to the Hessian eigenvectors, and take the intersections with the axes parallel and perpendicular to the *poloidal* magnetic field, respectively. At this point we construct the aspect ratio $r = (l_\theta/l_\psi)$, and chop it if $r>r_M$ or $r<r_m$.

In the reference frame with axes along the poloidal field and along e_ψ respectively the metric will be proportional to

$$\mathrm{M} = \begin{pmatrix} r^2 & 0 \\ 0 & 1 \end{pmatrix}.$$

To get the actual metric $\mathrm{M}_1=\gamma\, \mathrm{M}$ a suitable value for the scaling factor γ is needed. This is used to have *the norm* of the metric span a desired range, which then influences the refinement of the mesh to be generated [3]. If we define I_1=[min $\|H\|$, max $\|H\|$], and $I_2 = [\nu,\mu]$, a monotonically increasing mapping $I_1 \to I_2$ will provide a way to associate the metric norm to the given H, so that the scaling factor γ can be obtained. Until now we have used linear or square root mappings (the latter leading to larger refined zones). The set of local metrics is then provided to the BL2D code [4], which will generate an adapted mesh.

In conclusion, the influence of the input parameters on the mesh should be as follows:

- The metric norm controls the size of l_θ and l_ψ: the bigger is the norm, the shorter are the characteristic lengths, and the finer is the grid.
- The aspect ratio controls the alignment of the elements with the poloidal magnetic field: the bigger is the aspect ratio, the smaller is the misalignment.

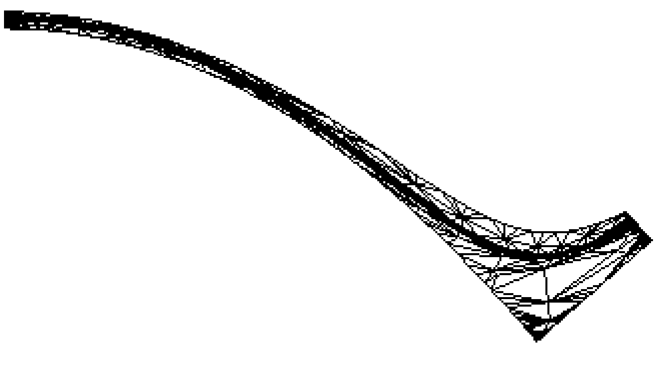

Figure 1 Final adaptive mesh for radial step with purely parallel conduction

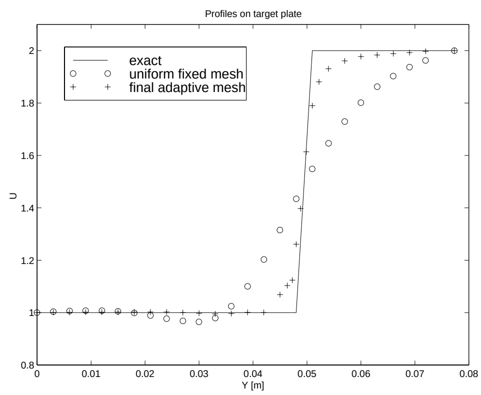

Figure 2 Target profiles for radial step with purely parallel conduction (fixed mesh with 6927 nodes, final adaptive mesh with 883 nodes)

RESULTS

<u>Effect of parallel conduction on radial layer broadening (pseudo-transient to steady state)</u>. It is well known (see, e.g., [5]) that insufficient alignment of the triangles in the FE mesh to the poloidal magnetic field will lead to large errors in the parallel fluxes and to spurious broadening of layers in the radial direction (computed SOL thickness larger than the actual one). When a fixed mesh is used this is indeed a problem and quickly overrides the advantage of using an unstructured mesh for complex geometry [2]. Here we want to check if an adaptive strategy can automatically provide enough alignment even with an unstructured mesh. Since the mentioned difficulties are caused essentially by the dominant $\kappa_{\|}$ we consider first a test problem where this is the only transport mechanism ($\kappa_{\|}=10^2 u^{5/2}$, $\kappa_{\perp}=0$). We follow the evolution to steady state of an initial profile of u with a radial step but uniform along a magnetic flux coordinate. Since this type of "contact discontinuity" should be maintained in the presence of purely parallel conduction, all broadening in the computed steady state shall be attributed to the effect of mesh misalignment.

We impose as boundary conditions a radial step at the mid-plane boundary, fixed u at the outer wall and at the upper and lower separatrix, whereas u is left free at the target (i.e., vanishing normal gradient is imposed). During this transient the dimensionless time-step Δt automatically increases from 1e-6 to 1e-2 (the maximum allowed value). The initial uniform mesh has ~ 7000 nodes. It is then automatically adapted 3 times and the final mesh, shown in Fig.1, has ~ 1000 nodes. The final mesh is significantly refined near the step and well aligned to it. The adaptive mesh fairly accurately captures the discontinuity, whereas if the mesh is kept fixed a much more significant numerical broadening of the layer results (Fig.2).

<u>Effect of parallel convection on MARFE-like structure (transient)</u>. We consider here the parallel convection towards the X-point of a Gaussian shaped hole in u, initially located near the outer target (see Fig.3), with $a_{\|}=10^4$, $a_{\perp}=0$. Initially, at the bottom of the Gaussian $u=u_{min}=1$ while a few "radii" away from its center $u=u_{max}=2$ everywhere. As opposed to the case of Cartesian geometry the hole should not exactly preserve its shape, because b is not divergence free. Therefore, a time convergence study was needed to determine the evolution of u_{min} as shown in Fig.3 (a space convergence study showed that 9936 nodes are enough to obtain a grid independent solution on a fixed grid). It is seen that an adaptive grid with typically ~ 1000 nodes (Fig.3) can give a solution of comparable accuracy than a fixed grid with many more nodes. This is true, however, only if the grid is not adapted too often. With Δt=5e-8, 57 adaptations over a total transient time of 5e-5 give a solution which is only as accurate as a solution with Δt=5e-7 on the fixed grid (Fig.3). On the contrary, if a criterion for adaptation is used that leads to only 20 adaptations during the transient, the solution is comparable to that obtained with Δt=1e-7 on the fixed grid (Fig.3). The observed "smoothing" effect is due to the averaging intrinsic in the interpolation used in the adaptation process. Macroscopically it results in some profile distortion and broadening as shown in Fig.3.

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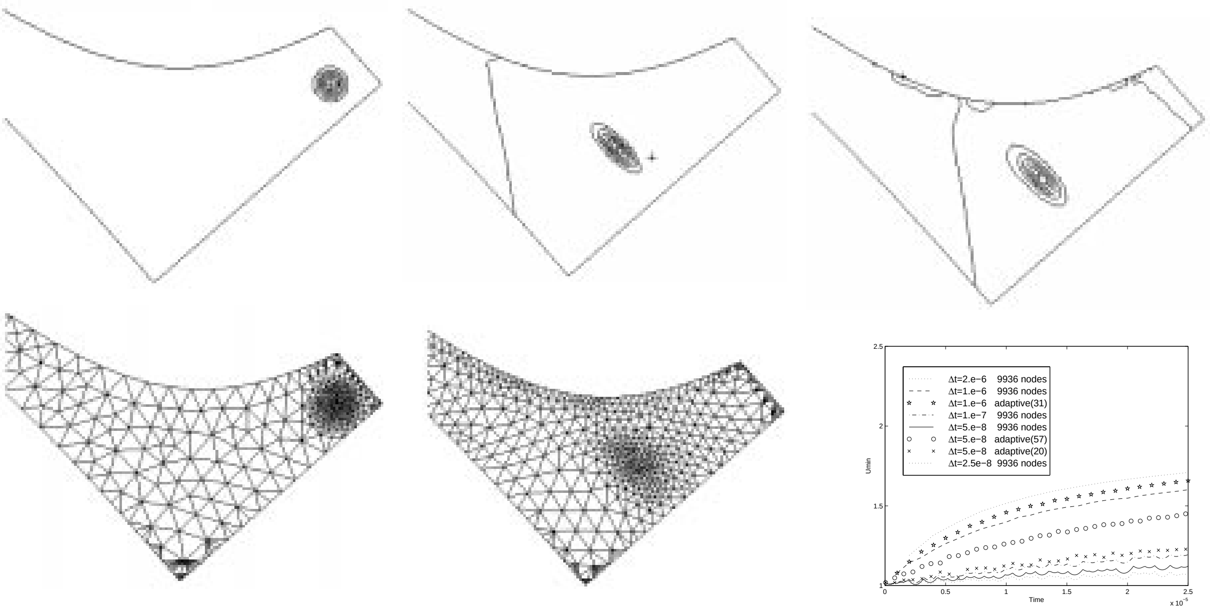

<u>Figure 3</u> Parallel convection of MARFE-like structure towards X-point using constant Δt=5e-8. Zooms of the computational domain including X-point (lowermost point) region and straight target on the right. Solution @ t=0 (upper left), and @ t=5e-5 on fixed mesh with 9936 nodes (upper center) and on adaptive mesh with 828 nodes (upper right). Adaptive mesh with 518 nodes @ t=5e-8 (lower left), @ t=5e-5 (lower center) and time convergence study of u_{min} (lower right).

<u>Effect of nonlinear radiation sink (evolution to "detached" steady state)</u>. Here we reconsider, in actual divertor geometry with oblique plate and using a time dependent treatment, the problem of the balance between conduction ($\kappa_{\parallel}=10^2 u^{5/2}$, $\kappa_{\perp}=1$) and nonlinear (ionization/impurity) sink discussed in [3]. Boundary conditions are: u=50 at the core plasma, u=2 at target and first wall, vanishing flux at mid-plane and private flux region. The sink *S(u)* has a (slightly smoothed) square wave dependence on u and vanishes outside the u interval (5,10). In order to simulate a "detachment" transient the amplitude of *S(u)* is increased in time from 1e7 (attached solution) to 2e7 (detached solution) as shown in Fig.4. The final adaptive mesh (Fig.4) has about 12000 nodes, most of which concentrated in the strong curvature region 5<u<10 where the sink is active, and appears to properly follow the solution. We also attempted to solve this problem on fixed meshes: a structured one, quasi-orthogonal and flux-surface fitted with ~ 4000 nodes as used in [2], and a uniform unstructured one with ~ 15000 nodes. In both cases we were unable to properly reproduce the solution of Fig.4 and negative values of u appeared systematically during the transient.

CONCLUSIONS AND PERSPECTIVE

We have shown the potential of adaptive finite elements to accurately simulate, for a time-dependent scalar anisotropic conduction-convection-radiation problem, some of the essential features of plasma transport in the tokamak edge with realistic divertor geometry. The solution and adaptive strategies, however, still have to be fully optimized.

We plan to extend this treatment to the full vector problem of a 2-fluid plasma as modeled by the FELS code [1-2].

Figure 4 *"Detachment" transient simulated by increasing the amplitude S of the nonlinear radiation sink S(u). Zooms of the computational domain including X-point (lowermost point) region and oblique target on the right. Attached solution for S=1e7 (left) used as initial condition for evolution to detached steady state with S=2e7 (center). Final adaptive mesh with 12631 nodes (right).*

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Table 1 *Mesh characteristics. BW=maximum band width, $<\alpha>$=average misalignment [2], $<\alpha A>/<A>$=area averaged misalignment[2], m=type of metric mapping (L=linear, S=square root).*

Problem / Mesh type	Nodes	Elements	BW	$<\alpha>$ (^{o})	$<\alpha A>/<A>$ (^{o})	ν	μ	r_m	r_M	*m*
Layer / unif. fixed **(Fig.2)**	6927	13150	9	12.	12.	-	-	-	-	-
Layer / adapt. final **(Figs.1,2)**	883	1570	19	2.7	3.8	1.e-8	1.e8	5	50	L
Marfe / unif. fixed	9936	18971	10	15.	14.	-	-	-	-	-
Marfe / adaptive @ t=5e-8 **(Fig.3)**	518	840	19	13.	10.	4.e4	4.e6	1	1	S
Marfe / adaptive @ t=5e-5 **(Fig.3)**	828	1353	14	13.	13.	4.e4	4.e6	1	1	S
Sink / unif. unstruct. fixed	14947	29462	10	1.6	1.6	5.e7	5.e7	15	15	
Sink / unif. struct. fixed	3750	7192	7	0.24	0.31	-	-	-	-	-
Sink / adaptive final **(Fig.4)**	12631	24959	13	3.2	3.8	1.e5	7.e9	5	5	L