*DEPARTMENT OF ENERGETICS*

POLITECNICO DI TORINO

# Radiation Heat Transfer: Basic Physics and Engineering Modeling

## Numerical Heat Transfer

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# Outline of this Section

- Surface–to–Surface Radiation → Net Exchange Formulation based on Configuration Factors
- Radiation in Participating Media → Total Radiation Transfer Equation (RTE) → Traditional Solution Procedure:
  - (Hottel's) Zone Method
  - Discrete–Ordinate Approximation ($S_n$)
  - Differential Approximation ($P_N$)
- Statistical Approach → Monte Carlo Ray–Trace Method (MCRT)

# Radiation Heat Transfer: Basic Features

Wavelength range for **thermal** radiation

$10^{-7}$ - $10^{-4}$ m ($10^{-1}$ - $10^{2}$ μm)

ultraviolet - visible - infrared

The visible range is from 0.4 to 0.7 μm.

- Thermal radiation is an electromagnetic phenomenon → electromagnetic waves are capable to of carrying energy from one location to another, even in vacuum (broadcast radio, microwaves, X–rays, cosmic rays, light,…)
- Thermal radiation is the electromagnetic radiation emitted by a material substance solely due to its temperature
- Thermal radiation (as any other electromagnetic radiation) is characterized by a dual wave–particle nature

# Physics of Emission and Absorption

- Emission and absorption of photons by individual atoms is described in terms of
  - electronic transitions between energy levels within the atom
  - or between the energy of a free electron and an atomic energy level
- The probability of such transition is favored by high temperatures in gasses but it is relatively small in solids → the effects in terms of heat transfer are negligible
- A more efficient mechanism in solids exists → Each of the atoms in crystal structure acts as an oscillator whose stored energy is determined by its state of thermal equilibrium:
  - an instantaneous distortion in the electronic cloud may lead to an electrical dipole moment (atomic oscillator);
  - the atomic oscillator can also be modeled as a dipole antenna.

# Useful Definitions

- (Directional, Spectral) Intensity of Radiation
- (Directional) Total Intensity of Radiation
- Hemispherical Spectral Emissive Power
- Hemispherical Total Emissive Power

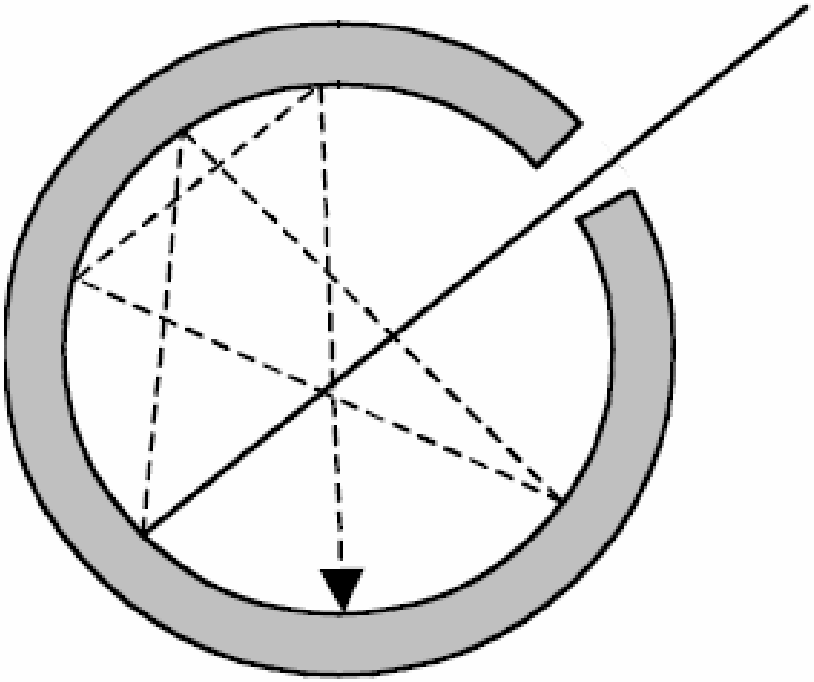
$$i_{\lambda\varphi} = \frac{\partial^3\phi}{\partial A_s \partial\lambda \partial\omega_s \cos\beta_s}$$

$$I_{\varphi} = \frac{\partial^2\phi}{\partial A_s \partial\omega_s \cos\beta_s}$$

$$e_{\lambda} = \frac{\partial^2\phi}{\partial A_s \partial\lambda}$$

$$E = \frac{\partial\phi}{\partial A_s}$$

# Black Body Cavity: Wave Density



$$\sqrt{k_1^2 + k_2^2 + k_3^2} = k = \frac{2\,L}{\lambda'} \leq \frac{2\,L}{\lambda}$$

$$N(\lambda' \geq \lambda) = \frac{2}{8}\,\frac{4\,\pi}{3}\left(\frac{2\,L}{\lambda}\right)^3 = \frac{8\,\pi\,L^3}{3\,\lambda^3}$$

$$\frac{\partial N}{\partial V} = \frac{8\,\pi}{3\,\lambda^3}$$

# Black Body: Quantum Mechanics (1900)

$$e_\lambda^n(T,\lambda) = \frac{\partial}{\partial\lambda}\left(\frac{c}{4}\,J_m\,\frac{\partial N}{\partial V}\right) = \frac{c}{4}\,J_m\,\frac{\partial^2 N}{\partial\lambda\partial V} = \frac{2\pi\,c\,J_m}{\lambda^4}$$

$$J_m^C = \frac{\int_0^\infty \exp(-\frac{\hbar f}{k_B T})\,\hbar f\,df}{\int_0^\infty \exp(-\frac{\hbar f}{k_B T})\,df} = k_B T$$

$$J_m^Q = \frac{\sum_{n=0}^\infty \exp(-\frac{n\hbar f}{k_B T})\,n\hbar f}{\sum_{n=0}^\infty \exp(-\frac{n\hbar f}{k_B T})} = \frac{\hbar f}{\exp(\frac{\hbar f}{k_B T}) - 1}$$

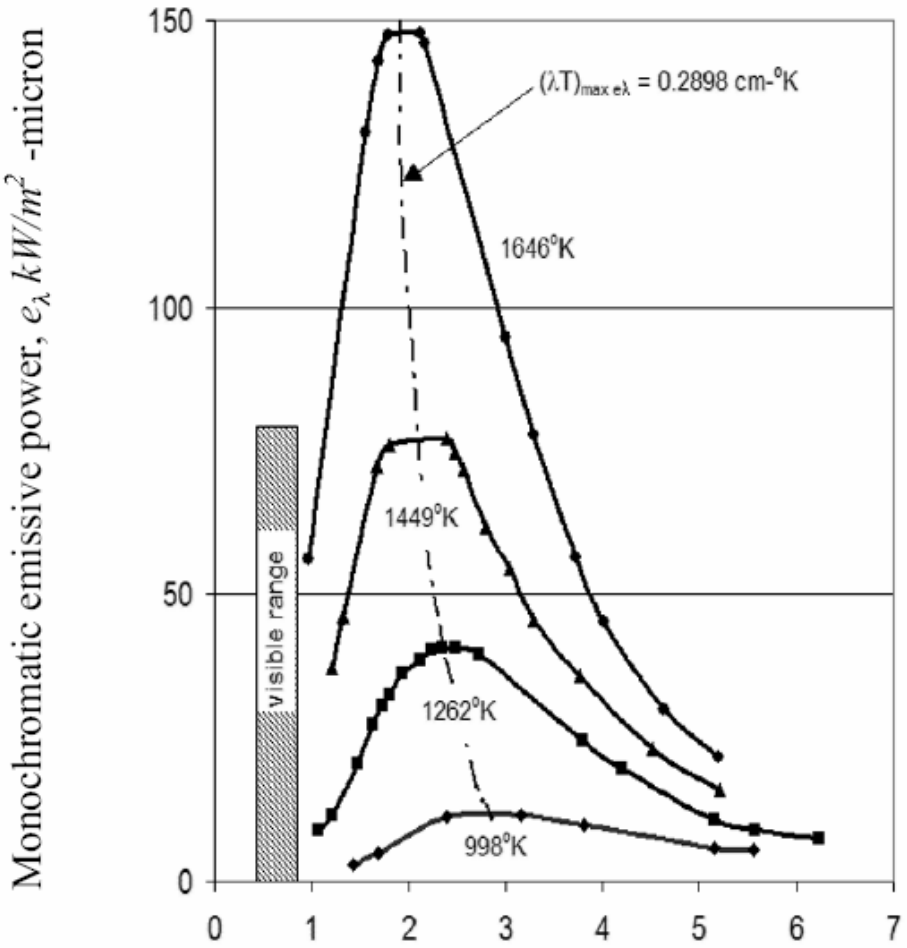
# Emissive Power of Back Body

$$e_\lambda^n(T,\lambda) = \frac{2\pi\, c^2\, \hbar}{\lambda^5 \left[\exp\left(\frac{\hbar\, c}{\lambda\, k_B\, T}\right) - 1\right]}$$

- The emissive power of the black body shows a maximum

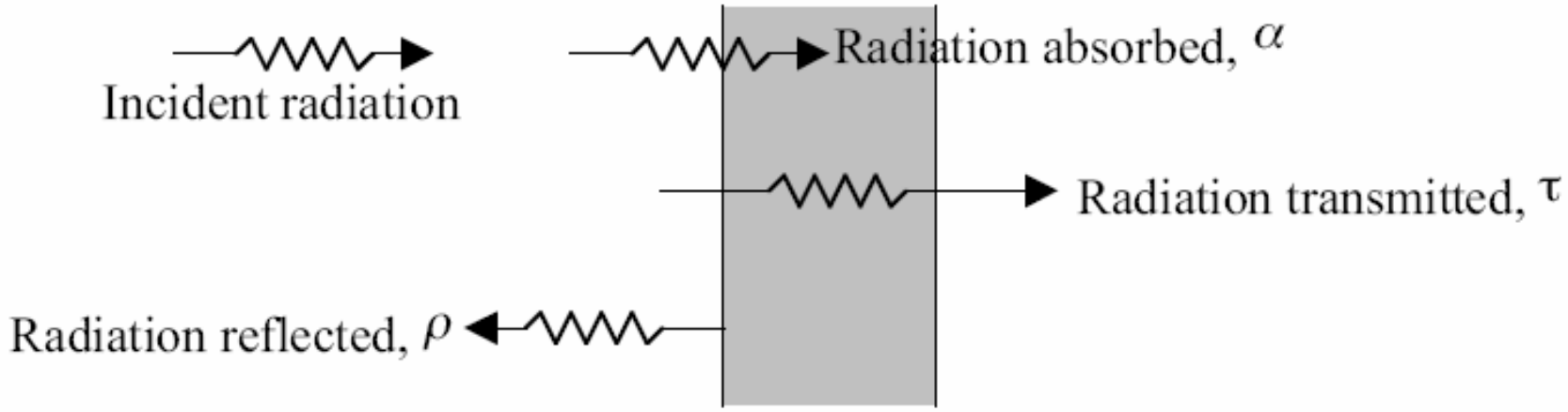
$$\lambda_M\, T = b \qquad b = 2898\,[\mu m\, K]$$

$$E^n(T) = \int e_\lambda^n(T,\lambda)\, d\lambda = \sigma\, T^4$$



# Interactions between Photons and Surfaces

$\alpha$ = absorptance - fraction of incident radiation absorbed
$\rho$ = reflectance - fraction of incident radiation reflected
$\tau$ = transmittance – fraction of incident radiation transmitted





# Kirchhoff's Law

Emittance: $\varepsilon = \dfrac{E}{E_b}$ ← radiation from real body at $T$ / ← radiation from black body at $T$

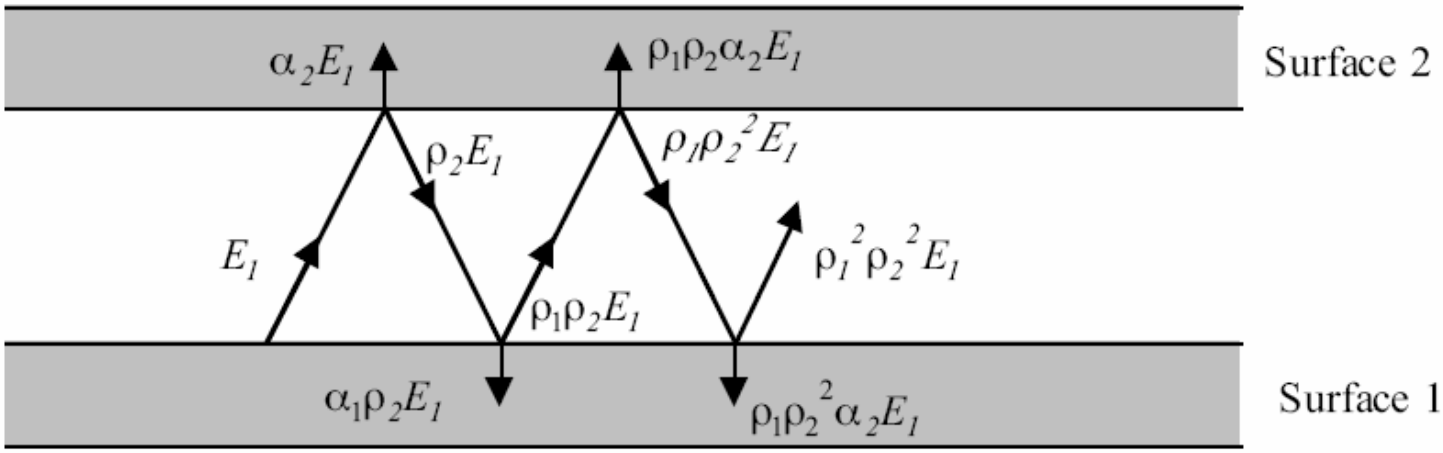
Values of emittance vary greatly for different materials. They are near unity for rough surfaces such as ceramics or oxidized metals, and roughly 0.02 for polished metals or silvered reflectors.

The level of the emittance can be related to the absorptance using the following arguments. Suppose we have a small non-black body in the cavity. The power absorbed per unit area is equal to $\alpha H$. The power emitted is equal to $E$. An energy balance gives $E = E_b\varepsilon = \alpha H = \alpha E_b$. Thus

$$\frac{E}{E_b} = \alpha = \varepsilon \tag{9.3}$$

Equation (9.3), the relation $\alpha = \varepsilon$, is known as Kirchhoff's Law. It implies that good radiators are good absorbers. It was derived for the case when $T_{body} = T_{surroundings}$ (cavity) and is not strictly true for all circumstances when the temperature of the body and the cavity are different, but it is true if $\alpha_\lambda = \alpha, \varepsilon_\lambda = \varepsilon$, so the absorptance and emittance are not functions of $\lambda$. This situation describes a "gray body". Also, since $\alpha_\lambda$, $\varepsilon_\lambda$ are properties of the surface, $\alpha_\lambda = \varepsilon_\lambda$.

# Radiation involving Gray Surfaces




Consider the two infinite gray surfaces shown in Figure . We suppose that the surfaces are thick enough so that, $\alpha + \rho = 1$ (no radiation transmitted so transmittance = 0). Consider a photon emitted from Surface 1 (remembering that the reflectance $\rho = 1 - \alpha$):

# Recursive Interaction Chain

| | |
|---|---|
| Surface 1 emits | $E_1$ |
| Surface 2 absorbs | $E_1\alpha_2$ |
| Surface 2 reflects | $E_1(1-\alpha_2)$ |
| Surface 1 absorbs | $E_1(1-\alpha_2)\alpha_1$ |
| Surface 1 reflects | $E_1(1-\alpha_2)(1-\alpha_1)$ |
| Surface 2 absorbs | $E_1(1-\alpha_2)(1-\alpha_1)\alpha_2$ |
| Surface 2 reflects | $E_1(1-\alpha_2)(1-\alpha_1)(1-\alpha_2)$ |
| Surface 1 absorbs | $E_1(1-\alpha_2)(1-\alpha_1)(1-\alpha_2)\alpha_1$ |

We can add up all the energy $E_1$ absorbed in 1 and all the energy $E_2$ absorbed in 2. In doing the bookkeeping, it is helpful to define $\beta = (1-\alpha_1)(1-\alpha_2)$. The energy $E_1$ absorbed in 1 is

$$E_1(1-\alpha_2)\alpha_1 + E_1(1-\alpha_2)\alpha_1(1-\alpha_2)(1-\alpha_1)+\ldots$$

This is equal to

$$E_1(1-\alpha_2)\alpha_1\left(1+\beta+\beta^2+\ldots\right) = \frac{E_1(1-\alpha_2)\alpha_1}{1-\beta}$$

# Simplified Formula

$$E_2 - \left( \frac{E_2\left(1-\alpha_1\right)\alpha_2}{1-\beta} \right) = \frac{E_2\alpha_1}{1-\beta}$$

is absorbed by 1. The net heat flux from 1 to 2 is

$$\dot{q}_{net\ 1\ \ to\ 2} = E_1 - \frac{E_1\left(1-\alpha_2\right)\alpha_1}{1-\beta} - \frac{E_2\alpha_1}{1-\beta} = \frac{E_1\alpha_2 - E_2\alpha_1}{\alpha_1 + \alpha_2 - \alpha_1\alpha_2}.$$

If $T_1 = T_2$, we would have $\dot{q} = 0$, so from Equation (9.4)

$$\frac{E_1}{\alpha_1} = \frac{E_2}{\alpha_2} = f(T).$$ If body 2 is black, $\alpha_2 = 1$, and $E_2 = \sigma T^4$.

or, as the final expression for heat transfer between gray, planar, surfaces:

$$\dot{q}_{\text{net 1 to 2}} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}.$$



# Net Exchange Formulation

- Let us consider two gray surfaces realizing a radiation heat transfer by means of recursive emission and absorption phenomena
- Radiosity (R) is defined as the total radiation heat flux leaving a surface (it includes both emitted and reflected power), without regard to direction or wavelength
- Irradiance (I) is defined as the total radiation heat flux incident to the surface element from the hemispherical space above it

$$R = \epsilon E_r^n + (1 - a) I$$

$$I = (R - \epsilon E_r^n)/(1 - a)$$

# Equivalent Thermal Resistance

$$\phi = (R_1 - I_1)\, A_1 = \frac{\epsilon_1\, A_1}{1-\epsilon_1}(E_1^n - R_1)$$

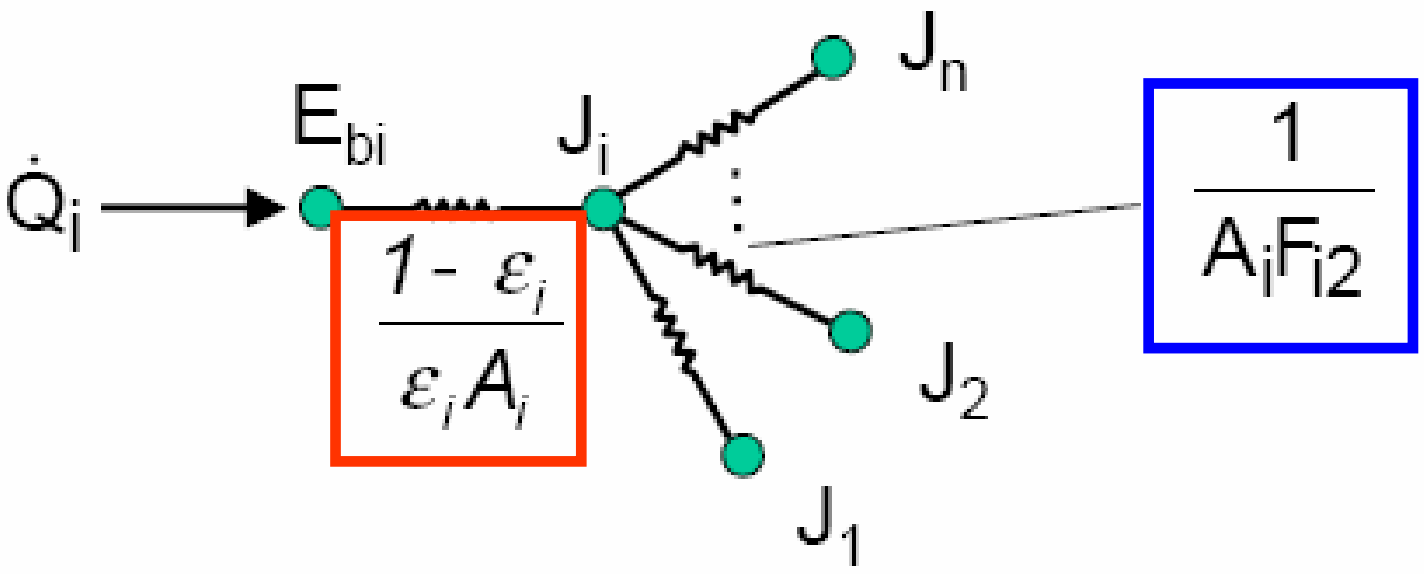
$$\phi = (I_2 - R_2)\, A_2 = \frac{\epsilon_2\, A_2}{1-\epsilon_2}(R_2 - E_2^n)$$

$$\phi = \pi\, (I_{R\varphi}^1 - I_{R\varphi}^2)\, A_1\, F_{12}$$

$$F_{12} = \frac{1}{\pi\, A_1} \int\int \frac{\cos\beta_1 \cos\beta_2 dA_1 dA_2}{r^2}$$

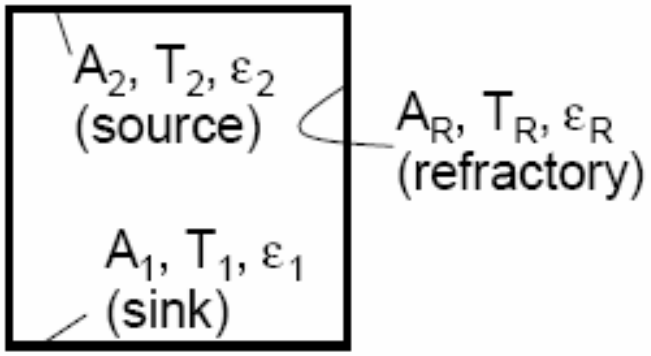
$$\phi = \frac{\sigma\,(T_1^4 - T_2^4)}{\frac{1-\epsilon_1}{A_1\,\epsilon_1} + \frac{1}{A_1\,F_{12}} + \frac{1-\epsilon_2}{A_2\,\epsilon_2}}$$

# Thermal Circuit for Gray Enclosure



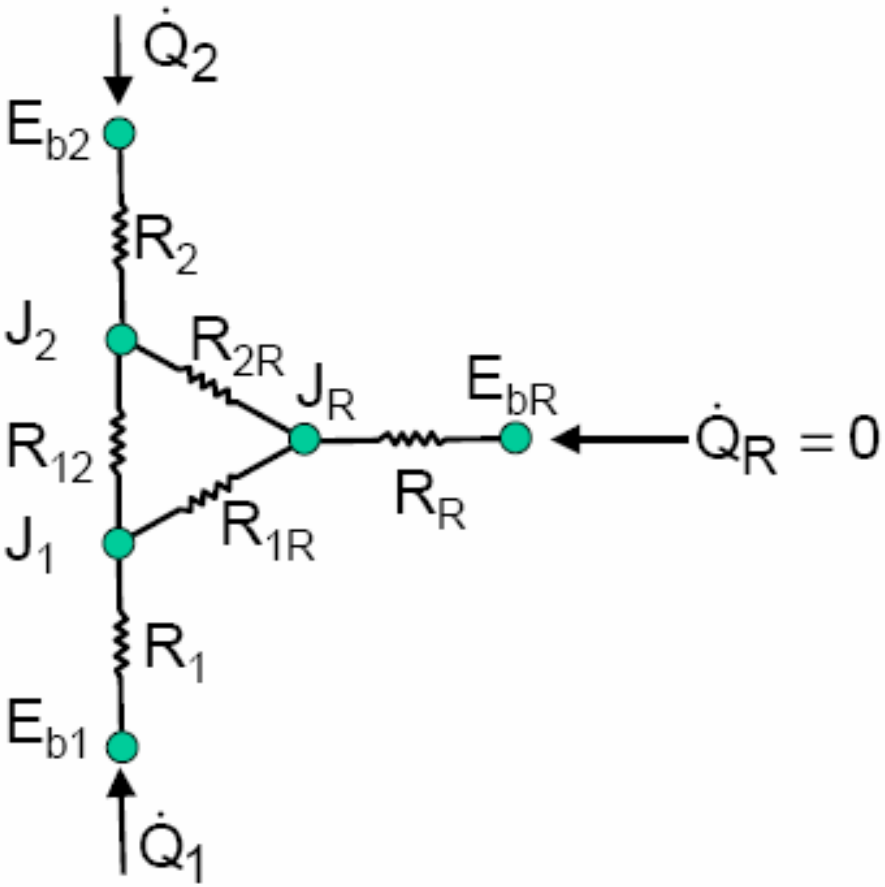
# Simple Thermal Network

A source, a sink, and a refractory




$$\dot{Q}_2 = -\dot{Q}_1 = \frac{E_{b2} - E_{b1}}{R_1 + R_\Delta + R_2}$$

$$\frac{1}{R_\Delta} = \frac{1}{R_{12}} + \frac{1}{R_{2R} + R_{1R}}$$



# Surface–to–surface Radiation Model (1)

- For example in Fluent®, the surface-to-surface radiation model (S2S) can be used to account for the radiation exchange in an enclosure of gray-diffuse surfaces.
- The energy exchange between two surfaces depends in part on their size, separation distance, and orientation. These parameters are accounted for by a geometric function called a “shape / view / configuration factor”, i.e. $F_{12}$ .
- The main assumption of the S2S model is that any absorption, emission, or scattering of radiation can be ignored; therefore, only “surface-to-surface” radiation need be considered for analysis.

# Clustering

- The surface–to–surface radiation model is computationally very expensive when there are a large number of radiating surfaces. To reduce the computational time as well as the storage requirement, the number of radiating surfaces is reduced by creating surface “clusters”. The surface clusters are made by starting from a face and adding its neighbors and their neighbors until a specified number of faces per surface cluster is collected.
- Since the radiation source terms are highly non-linear (proportional to the fourth power of temperature), the surface cluster temperature is obtained by area averaging the forth power of the single element temperature → in this way, the radiation flux is preserved as much as possible.



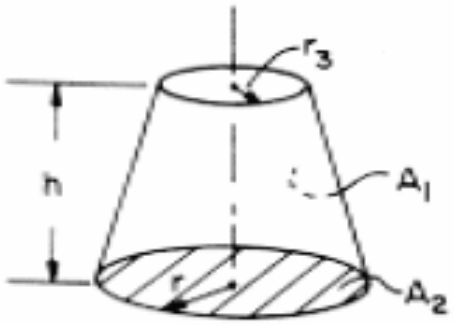
# Catalog of Configuration Factors

www.me.utexas.edu/~howell/index.html

John R. Howell
University of Texas at Austin

## TABLE OF CONTENTS

# Right Circular Cone to Base




Definitions:

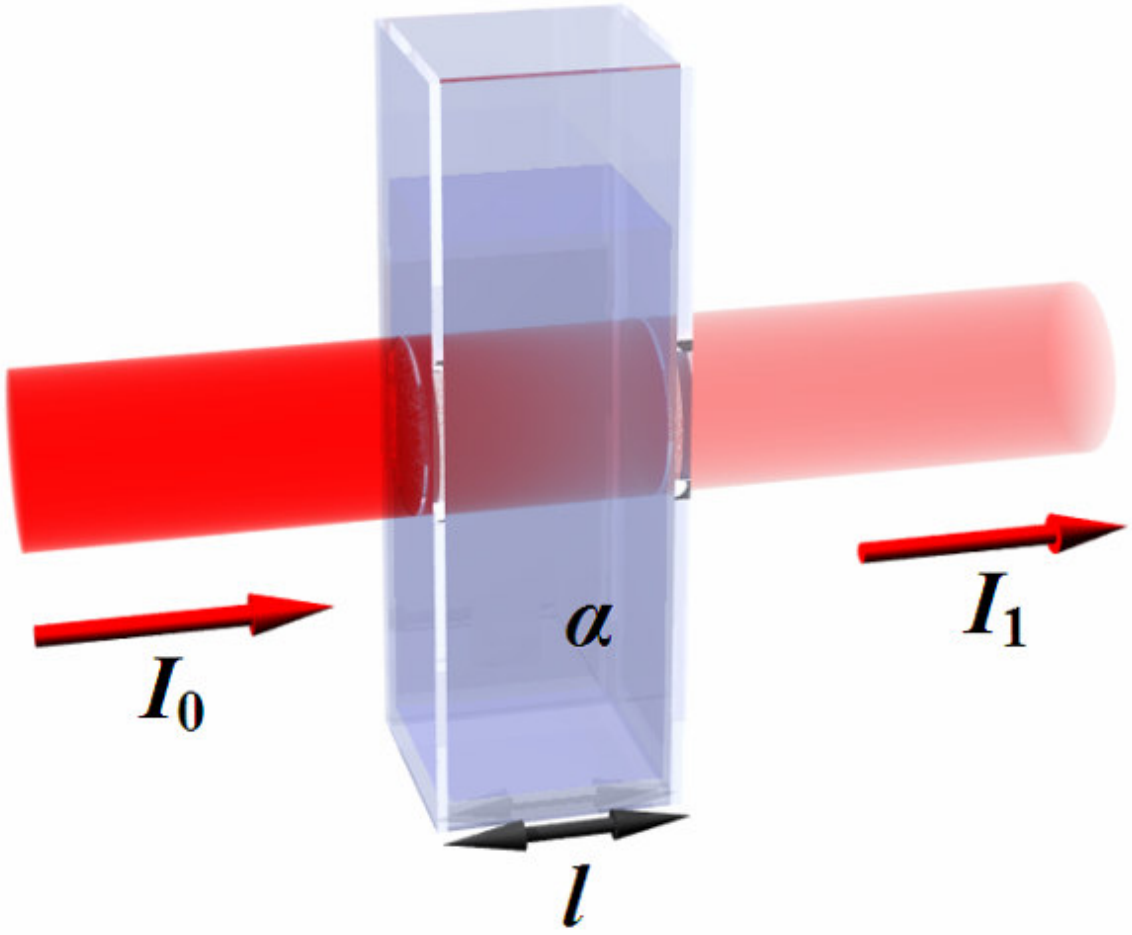
$H = h/r_3$; $R = r/r_3$;

$X = (1 + R^2 + H^2)$

Governing Equation:

$$F_{1-2} = \frac{2R^2 - X + \left(X^2 - 4R^2\right)^{1/2}}{2(X - 2R)^{1/2} \cdot (1 + R)}$$

# Beer's Law




$$A = \beta l$$

$$\frac{I_1}{I_0} = 10^{-A} = 10^{-\beta l}$$

$$A = \log_{10}\left(\frac{I_0}{I_1}\right)$$

# Participating Media: Optical Regimes

- If $\beta L << 1$, then a medium is said to be optically thin → radiation emitted within an optically thin medium travels to the bounding walls of the enclosure → the radiation problem is highly simplified
- If $\beta L \sim 1$, then the medium is neither optically thin nor optically think → the governing equations of transfer are integro–differential equations
- If $\beta L >> 1$, then a medium is said to be optically thick → radiation exchange occurs only among neighboring volume elements → this is diffusion limit in which the governing radiative transport equations are differential equations

# Emission and Absorption Mechanisms (gas)

- A gas molecule stores energy in one of the four modes (with regards to radiation): electronic (E), vibrational (V), rotational (R), translational (T) →
  - (E) The electronic storage mode leads to both line and continuum absorption and emission
  - (R and V) Infrared region of the spectrum is dominated by rotational and vibrational bands associated with water vapor and carbon dioxide
  - (T) Translational degree of freedom does not in itself lead to emission, but it translational energy can be converted into any the previous forms by collisions.

# Emission and Absorption

Consider an isothermal enclosure at equilibrium which is filled with an homogeneous participating media. Any gain in I must be exactly offset by a decrease in I.

$$\frac{dI}{ds} = 0 = -k_a I + k_a I_b \quad \text{where} \quad I = I_b$$

In general, assuming local equilibrium,

$$\boxed{\frac{dI}{ds} = -k_a I + k_a I_b \quad (\text{no scattering})}$$

(IV.F.3) can also be written on a spectral basis as

$$\frac{dI_\lambda}{ds} = -k_{a\lambda} I_\lambda + k_{a\lambda} I_{b\lambda} \quad (\text{no scattering})$$

# Hottel's Zone Method

- The previous equations can be quite easily solved (pure differential equations) → it is possible to find the conditions leading to a uniform temperature distribution within an absorbing emitting medium bounded by infinite parallel black walls
- Non–trivial cases are those where the effect is due to the perfect balance with other modes of heat transfer
- Then it is always possible to subdivide the generic domain into isothermal zones of appropriate shape such that the constant stepwise profile approximates the actual solution → the analysis (Hottel's method) allows to recover the consistent thermal fluxes at the boundaries of the arbitrary zones

# Analytical Solution for a Ray

$$\frac{dI}{ds} + aI = \frac{a\sigma T^4}{\pi}$$

where $a$ = gas absorption coefficient

$I$ = intensity

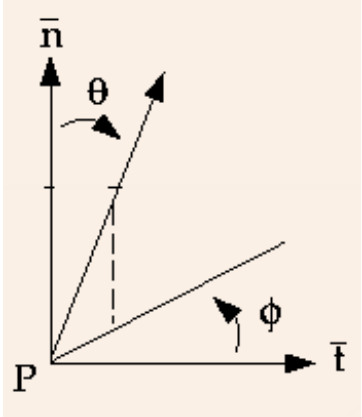
$T$ = gas local temperature

$\sigma$ = Stefan-Boltzmann constant ($5.672 \times 10^{-8}$ W/m$^2$-K$^4$)

$$I(s) = \frac{\sigma T^4}{\pi}(1 - e^{-as}) + I_0 e^{-as}$$

# Discrete Transfer Radiation Model (2)

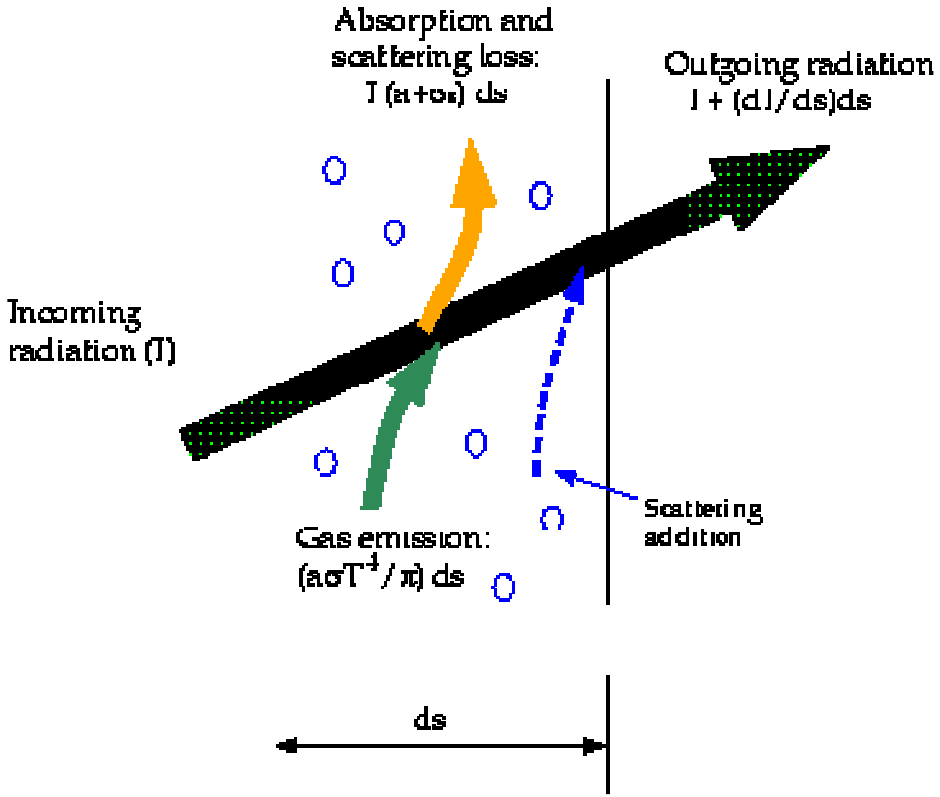
- For example in Fluent®, the DTRM integrates the previous equation along a series of rays emanating from boundary faces → The energy source in the fluid due to radiation is then computed by summing the change in intensity along the path of each ray that is traced through the fluid control volume
- The "ray tracing" technique used in the DTRM can provide an estimation of radiative heat transfer between surfaces without explicit view-factor calculations → At each radiating face, rays are fired at discrete values of the polar and azimuthal angles



# Scattering

- Scattering in participating media is analogous to reflection in surface radiation → the main difference is that scattering occurs in the whole $4\pi$-space
  - In the Rayleigh scattering model, the scattered energy in any direction varies as the forth power of the frequency → this theory applies when the scattering particles are small compared to the wavelength of the scattered light
  - In the MIE scattering model, the scattering centers are described as semi–transparent bodies → it describes the interaction between particles and incident radiation when scattering particles are not small compared to the wavelength of the scattered light

# Total Radiation Transfer Equation (RTE)




$$\frac{dI(\vec{r},\vec{s})}{ds} + (a+\sigma_s)I(\vec{r},\vec{s}) = an^2\frac{\sigma T^4}{\pi} + \frac{\sigma_s}{4\pi}\int_0^{4\pi} I(\vec{r},\vec{s}\,')\,\Phi(\vec{s}\cdot\vec{s}\,')\,d\Omega'$$

# Total Radiation Transfer Equation (RTE)

$$\frac{dI(\vec{r},\vec{s})}{ds} + (a+\sigma_s)I(\vec{r},\vec{s}) = an^2\frac{\sigma T^4}{\pi} + \frac{\sigma_s}{4\pi}\int_0^{4\pi} I(\vec{r},\vec{s}\,')\,\Phi(\vec{s}\cdot\vec{s}\,')\,d\Omega'$$

where $\vec{r}$ = position vector

$\vec{s}$ = direction vector

$\vec{s}\,'$ = scattering direction vector

$n$ = refractive index

$\sigma_s$ = scattering coefficient

$\Phi$ = phase function

$\Omega'$ = solid angle

# Traditional Methods for RTE Analysis

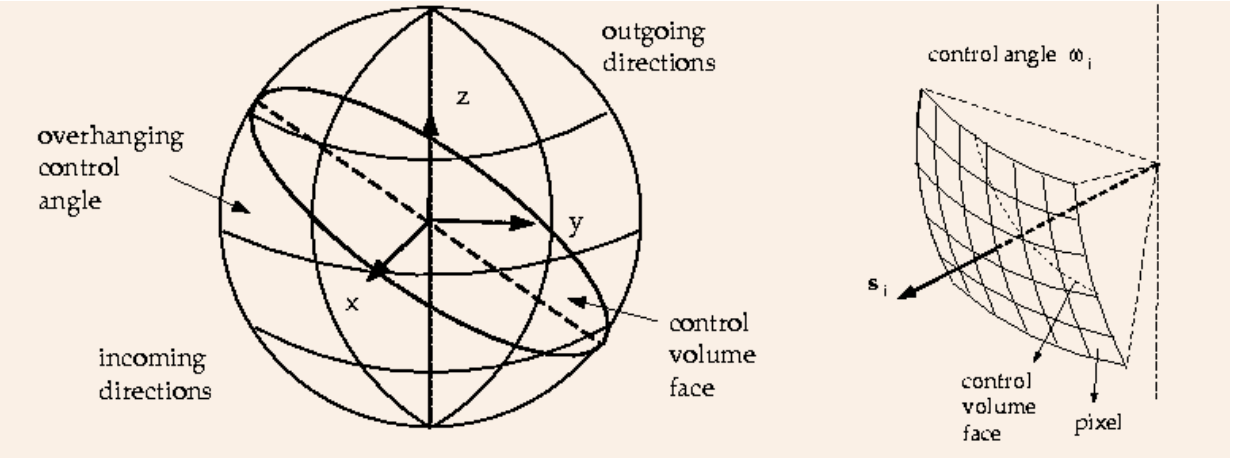
- In all those cases, where scattering is usually not isotropic, reflection is seldom diffuse and geometry is complicated, traditional methods to solve RTE are:
  - Discrete–Ordinate Approximation ($S_n$); the integrals taking into account the scattering are approximated as appropriately weighted sums;
  - Differential Approximation ($P_N$); the intensity is approximated as truncated series of appropriate transcendental functions.

# Discrete–Ordinate Approximation ($S_n$)

- The terms “ordinate” here refers to direction, or axis, in $4\pi$-space. For every ordinate passing in one direction a second ordinate passes in the opposite direction → therefore the ordinates appear in pairs ($S_2$ , $S_4$ , …)
- Each particular scheme is identified by the set of direction cosines and related weights → Gaussian sets are widely documented, but this is still an area of active research
- This scheme solves for as many transport equations as the considered directions → The numerical techniques are similar to those used in order to solve the other equations governing the fluid flow

# Discrete Ordinates Model (3)

- For example in Fluent®, each octant of the angular space at any spatial location is discretized into solid angles, called control angles.
- Each control angle is subdivided in smaller portions (pixelation) and the energy contained in each pixel is then treated as incoming or outgoing to the face (stepwise approximation)



# Differential Approximation ($P_N$)

- The basic idea is that the intensity in a participating medium can be represented as a rapidly converging series whose terms are based on orthogonal spherical harmonics → usually a small number is usually adequate ($P_1$ , $P_3$ , …) → for example in $P_1$ approximation only zeroth and first order moments of the intensity are considered
- Essentially the deviation of local intensity from its local means value is expressed in terms of the local gradients → this allows to derive a diffusion expression for the radiation flux and consequently to derive an advection–diffusion equation for the local mean intensity

# Approach to "Equilibrium"

$$\frac{dI(\vec{r},\vec{s})}{ds} = -a\left(I(\vec{r},\vec{s}) - I(\vec{r},\vec{s})^{b}\right)$$

- The RTE can be expressed in terms of approach to "equilibrium", in order to model the absorption and emission processes.
- On the other hand, the scattering term represents an additional forcing term, which substantially does not modify the structure of the equation (even though it is an integro-differential operator, leading to long-distance coupling)
- There are some similarities with some simplified kinetic equations (BGK approximation)

# P-1 Radiation Model (4)

- For example in Fluent®, the P-1 model solves an advection–diffusion equation for the quantity G, which is the mean local incident radiation
- Consequently the gradient of the radiation flux can be directly substituted into the energy equation to account for heat sources (or sinks) due to radiation

$$q_r = -\Gamma \nabla G$$

$$\nabla (\Gamma \nabla G) - aG + 4a\sigma T^4 = S_G$$

$$\nabla \cdot (\Gamma \nabla G) + 4\pi \left( a\frac{\sigma T^4}{\pi} + E_p \right) - (a + a_p)G = 0$$

# Rosseland Model (5)

- Moreover in Fluent®, when the medium is optically thick, it is possible to further simplify the numerical model by means of the Rosseland approximation
- Instead of solving an equation for G, it is possible to assume the theoretical value prescribed by the back body expression and then to group the consequent radiation terms in some modified transport coefficients for the energy equation (liminal transport coefficients)

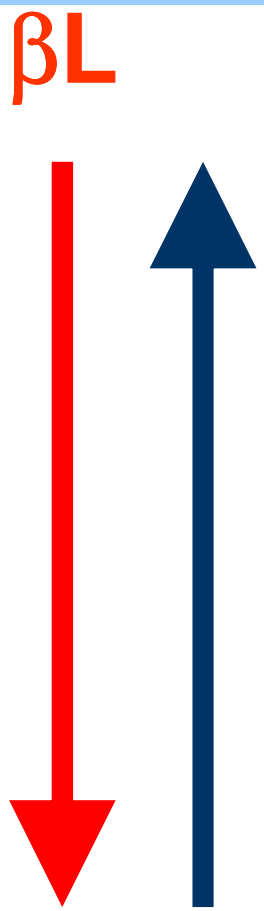
$$q_r = -\Gamma \nabla G$$

$$q_r = -16\sigma\Gamma T^3 \nabla T$$



# Overview of Radiation Models in Fluent®

| | |
|---|---|
| Surface-to-surface Model(1) | S2S |
| Discrete Transfer Radiation (2) | DTRM |
| Discrete-ordinate Model (3) | DO |
| P-1 Differential Approximation (4) | P-1 |
| Rosseland Model (5) | RM |

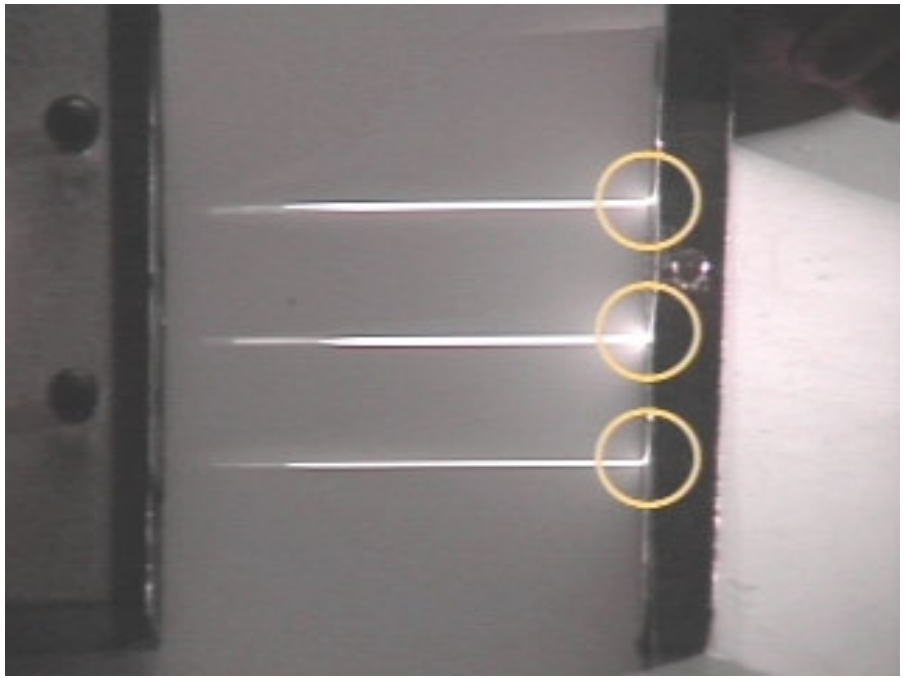


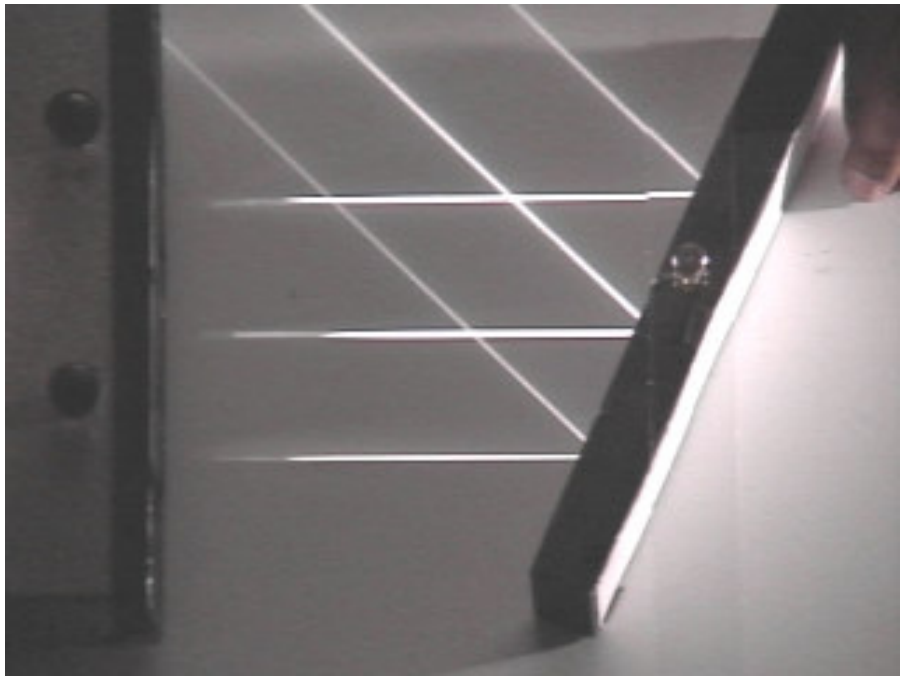

**Long-distance coupling**

# Critical Comparison

| | | |
|---|---|---|
| **S2S Model (1)** | Simple, good for enclosures, quite fast (without view factors) | No participating media, all surface are diffuse, gray radiation |
| **DTR Model (2)** | Simple, the accuracy depends on the number of rays | No scattering, all surfaces are diffuse, gray radiation |
| **DO Model (3)** | Complete, capable to deal with wide range of optical regimes | Anisotropic surfaces and non-gray radiation (gray-band) still under development |
| **P-1 Model (4)** | RTE becomes a simpler advection-diffusion equation | All surface are diffuse, gray radiation, it may over-predict radiation |
| **Rosseland Model (5)** | No extra equations: radiation moves directly in energy eq. | Optically thick media, somehow very rough |

# Diffuse vs. Specular Reflection





- Diffuse (isotropic), specular (anisotropic) or bidirectional reflection (which can be considered as the proper weighted sum of previous cases)

# Limits of Traditional Approaches

- The traditional approaches for gaseous radiation (absorption, emission and scattering) are not easily coupled with directionally dependent surface radiation models (anisotropic, non-diffuse surfaces)
- In some applications, the surfaces are engineered in such a way to show special features with regards to radiation (low–observable military aircrafts, vehicles with onboard radiation traps,…), where the directionality is the main goal of the design (highly bidirectional surfaces)
- Some applications exist (jet engine exhaust plume, infrared image processing, thermal radiometer,…) where higher accuracy is required

# Advantages of Statistical Approach

- It is easier to deal with surface radiation models directionally dependent surface radiation models (anisotropic, non-diffuse surfaces) because the actually solved analogous process is simpler than the original physical one (even though the calculations are statistically repeated a huge number of times in order to catch good accuracy)
- The desired accuracy can be tuned by means of the actual number of performed statistical experiments (unfortunately the convergence rate is quite slow) → There are no intrinsic limits reducing accuracy in principle, as far as the analogous process is a reasonable estimation of the original physical one

# Monte Carlo Methods

- Monte Carlo (MC) methods can be thought of as statistical simulation methods that utilize a sequences of (pseudo-) random numbers to perform the calculation
- The name "Monte Carlo" was coined by N.C. Metropolis (1915-1999) and inspired by S. Ulam (1909-1986), because of the similarity of statistical simulation to games of chance, and because Monte Carlo is a center for gambling
- In a typical process one compute the number of points in a set **A** that lies inside box **R**. The ratio of the number of points that fall inside **R** to the total number of points tried in **A** is equal to the ratio of the two areas
- The accuracy of the ratio depends on the number of points used, with more points leading to a more accurate value



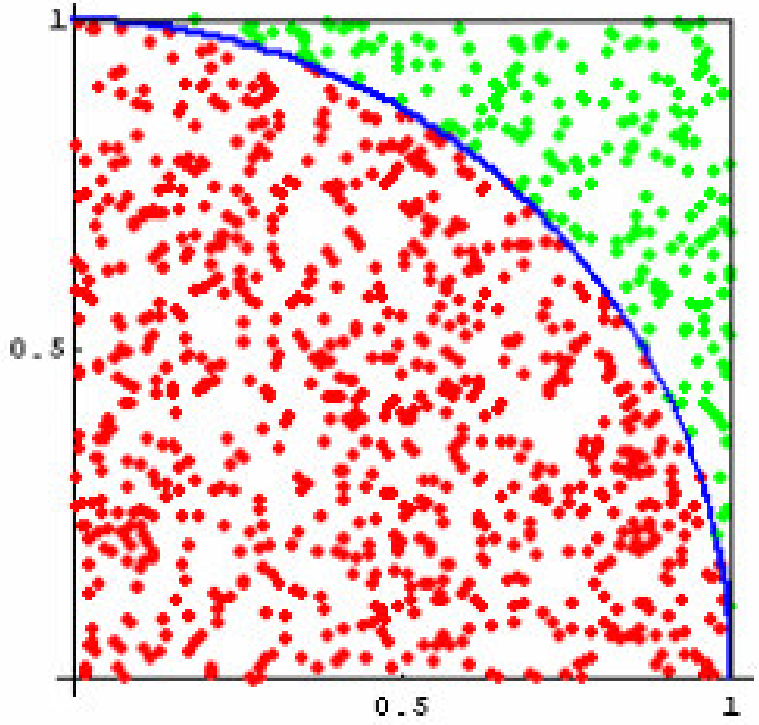
# Monte Carlo Methods: Details

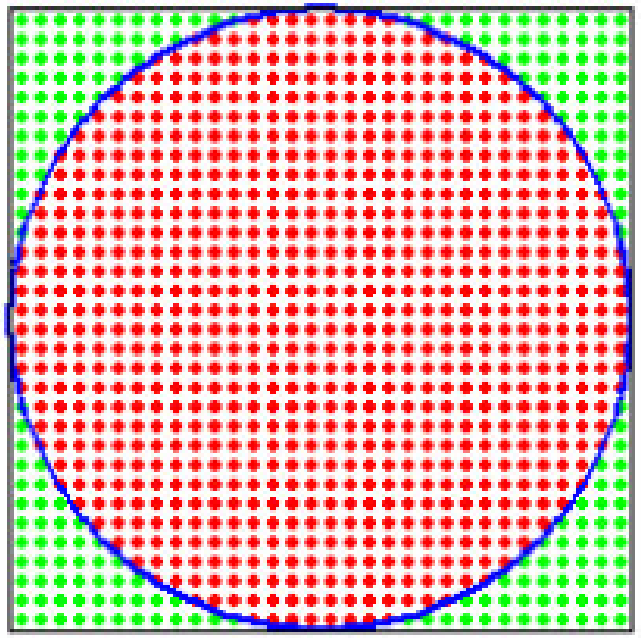
- The use of MC methods to model physical problems allows us to examine more complex systems than we otherwise can → With MC methods, a large system can be sampled in a number of random configurations, and that data can be used to describe the system as a whole
- As long as the function in question is reasonably well-behaved, it can be estimated by randomly selecting points and taking some kind of average of the function values → convergence ($1/\sqrt{N}$) (law of large numbers)
- Computer-generated numbers aren't really random, since computers are deterministic → Given a number to start with, generally called a random number seed, a number of mathematical operations can be performed on the seed so as to generate unrelated (pseudo-random) numbers

# MC Calculation of π → Hit and Miss

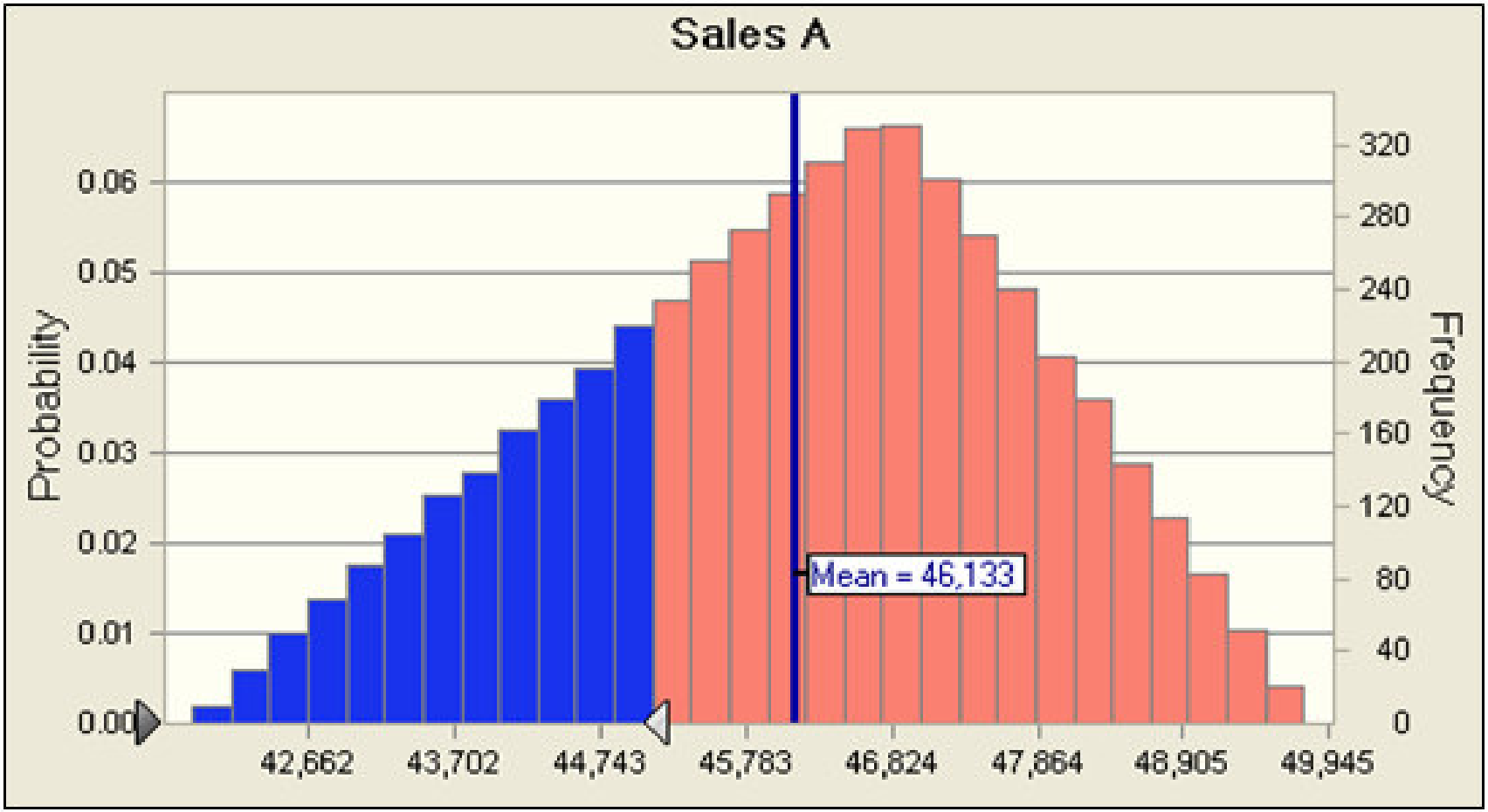
$$\rho = \frac{m}{n} = \frac{787}{1000} = 0.787$$

$$\rho = \frac{812}{812 + 212} = \frac{812}{1024} = 0.792$$

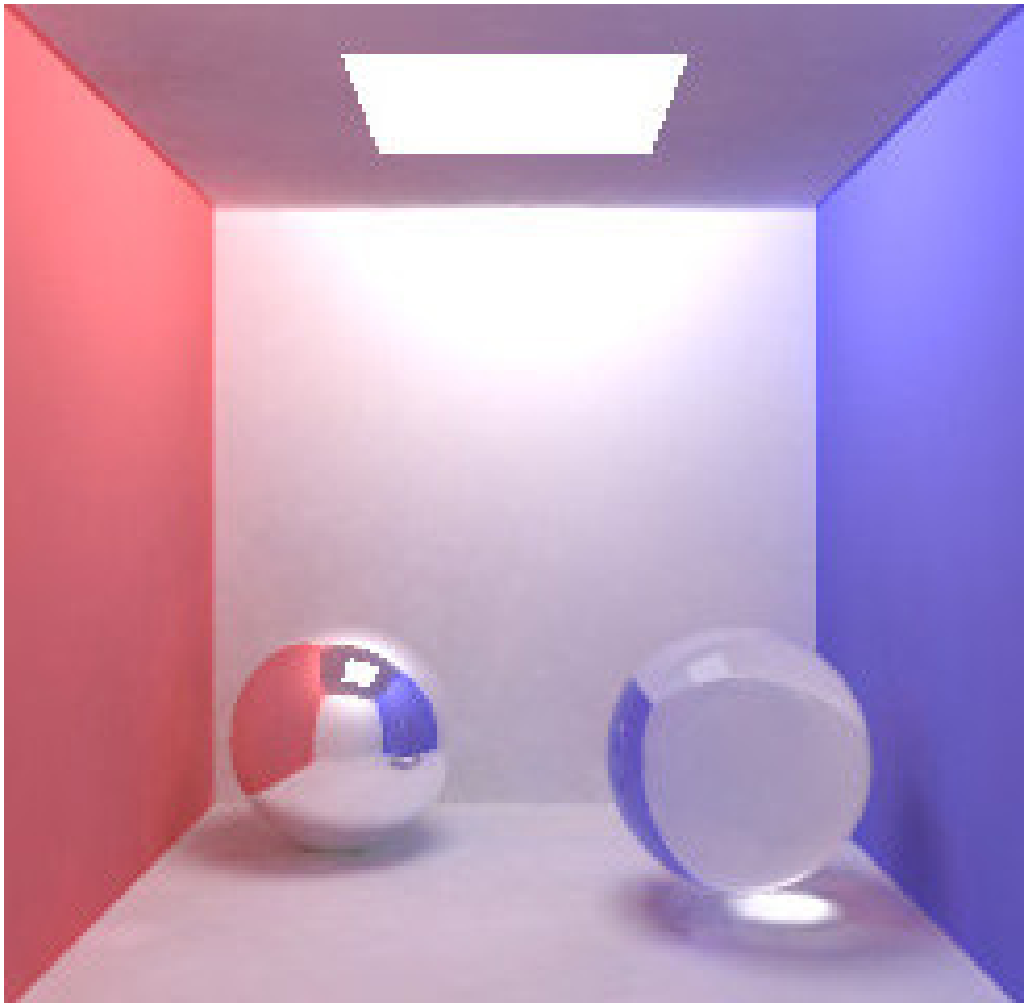


# MC Calculations for Risk Management



# MC Calculations for Light Tracing



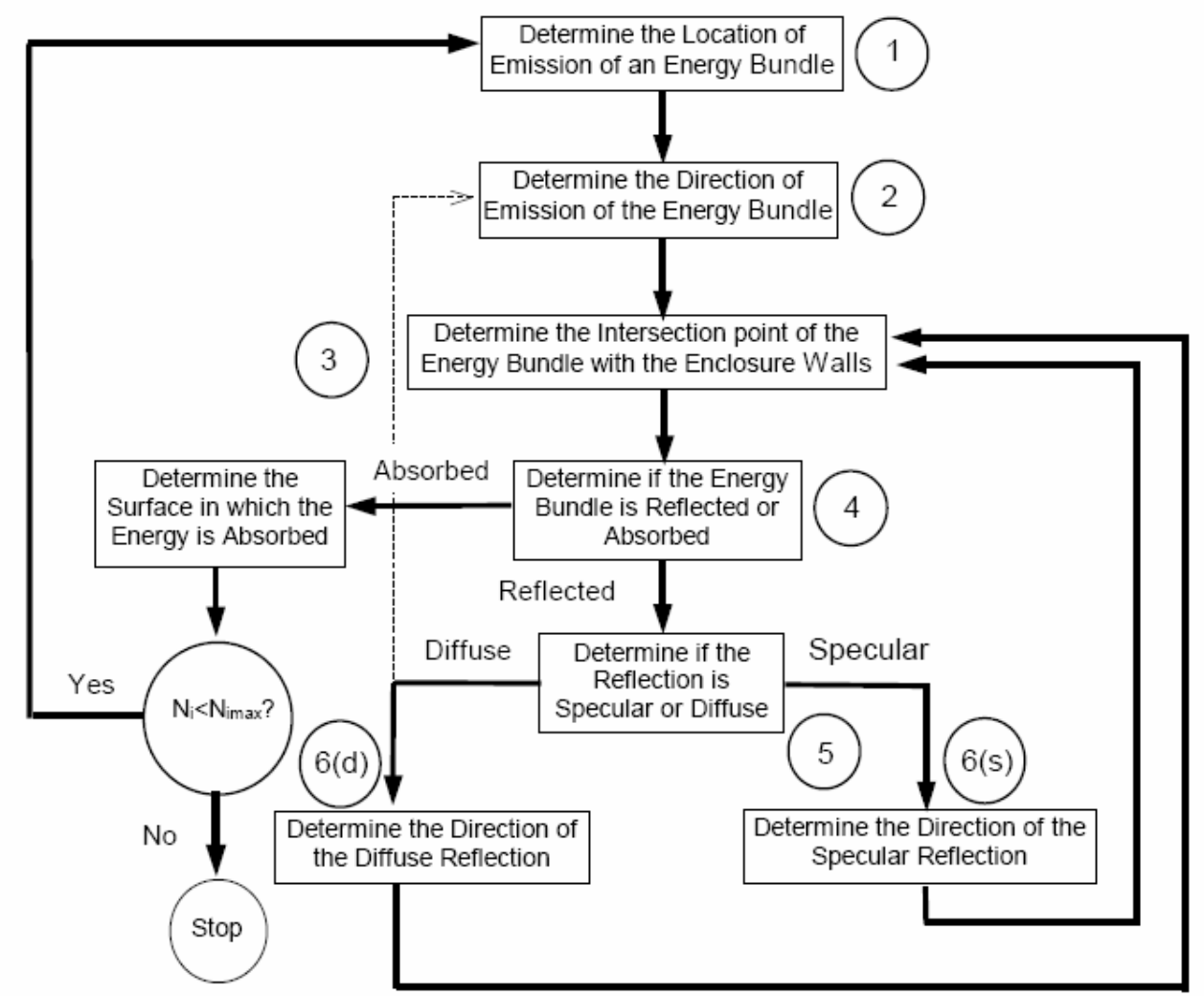
# Application to Radiation Heat Transfer

- For some physical processes, we have statistical models of behavior at a microscopic level from which we attempt to derive an analytic model of macroscopic behavior
- However, if we are not interested in forming a general model, but instead want to know about the behavior of a particular radiation surface in a particular environment, we can just numerically simulate the behavior of the radiation surface
- To do this we computationally "emit" photons from the radiation surface and keep track of where the photons go → This simple method is from MC methods and can be a very easy, though often slow, way to numerically solve physics problems

# Radiation Heat Transfer: Assumptions

- In practice, very few Monte Carlo simulations model the full physical process. Instead, an analog process is found that is easier to simulate, but retains all the important behavior of the original physical process
- An analog process that is almost always employed in radiation heat transfer is to replace photons with set wavelengths with power carrying beams that have values across the entire spectrum → energy bundle
- The properties of the enclosure and the laws of probability (for example, $P_\alpha=\alpha$) are used to determine the number of energy bundles absorbed by a given surface element
- Usually 10,000 energy bundles are emitted from each surface (anyway in high accuracy case up to $10^6$)

# MCRT Algorithm in Gray Enclosure



# Non-Gray Gasses (1)

- Non-gray gasses can be approximated by means of band-averaged (or gray-band) approximation, i.e. the spectrum is subdivided in a proper set of bands and the radiation properties are assumed constant in each of them (stepwise spectrum)
- In case of a complete spectral analysis, the quantity being traced must be a quantity of energy, or power, whose value is properly weighted with respect to the Plank radiation distribution function → It is better to refer to (band-averaged) energy bundles instead of rays
- It is essential that each emission event from a given surface involves the same amount of energy, otherwise the total thermal balance will be violated

# Non-Diffuse Surfaces (2)

- A band-averaged spectral surface scattering phase function (***SSF***) must be considered → it is the probability that an energy bundle in wavelength interval $\Delta\lambda$ incident from a specific direction (I) will be reflected in another given direction (R)
- This function traduces the spectral reflectivity properties of the surface in a statistical probability concerning the dynamics of the incoming energy bundle (it requires an internal computational "engine" for producing pseudo-random numbers) → The actual performance of this function can be compared with the complete original physical data (statistically on $10^6$ experiments)

# MCRT for Participating Medium (3)

- The enclosure volume (filled with a participating medium) is suitably subdivided into volumes elements and enclosure walls are subdivided into compatible surface elements
- We need to generalize the concept of radiation configuration (or coupling) factor $D_{12}$ between volume elements of a participating medium, and between a surface element and a volume element
- Radiative analysis is frequently performed on the basis of the results (segregated formulation) of an earlier CFD analysis used to establish the flow → it may be convenient to use the same mesh already defined

# Simplified Description of Scattering

- Bulk scattering in the participating medium is analyzed by means of an analogous process (from space to ensemble variability) → band-averaged spectral bulk scattering phase function (***BSF***) must be considered (based on Rayleigh model, MIE model or experimental data)
- This function traduces the spectral scattering properties of the participating medium in a statistical probability concerning the dynamics of the scattered energy bundle (clearly the main assumption is that the scattering is modeled by means of a rough change in the direction of the energy bundle) → The actual performance of this function can be compared with the physical data

# Characteristic Lengths

- Bulk Emission is randomly assumed in the participating medium (both the emission site and direction)
- Concerning the bulk Absorption, let us define
  - the absorption-free path length (***AFL***) as the free path of the energy bundle before it will suffer an absorption event
- Concerning the bulk Scattering, let us define
  - the scattering-free path length (***SFL***) as the free path of the energy bundle before it will suffer a scattering event
- Clearly the MC algorithm will take into account all the discrete combinations derived from the previous lengths

# Essential Steps of MCRT Algorithm

- ***AFL*** and ***SFL*** are computed by means of the physical properties of the participating medium → discrete cases
- The MC experiment is a time-marching process aiming to mimic the dynamics of an energy bundle based on a chain of discrete events, selected from a fixed (small) set:
  - self-absorption
  - direction change due to scattering
  - direction change due to crossing the interface between adjacent elements (Snell's law)
  - absorption by bulk fluid
  - absorption or reflection by bounding surface
- Clearly the (statistical) phase functions ***SSF*** and ***BSF*** are the essential conditions affecting the bundle dynamics

# MCRT Uncertainties

- MCRT method is based on a probabilistic interpretation of the radiative properties of surface and volume elements, and the radiation configuration factor is itself a (coupling) probability
- Moreover the actual process solved by MCRT is "analogous" in the sense that it mimics the original physics, in case of large ensemble of experiments
- Hence it is possible to use statistical inference to state, to a specific level of confidence, i.e. the uncertainty of a result obtained → Error bars and level of confidence can be used to decide (iteratively) the number of experiments to be performed (however actual reliability depends on how good is the analogous process)

# Summary

- MCRT method involves a drastic simplification in the description of the single phenomenon → However the actual success of this description is drastically improved by means of the (statistical) error compensation
- The essential idea behind this approach is that the space (directional) variability of surface and bulk scattering is reformulated in terms of ensemble variability (huge repetition of similar numerical experiments) → In this way, most of the geometrical complexity is gone and the problem becomes the increased computational demand required by running more experiments in order to get an acceptable convergence



# Further Readings

- R. Siegel, J.R. Howell, Thermal Radiation Heat Transfer, Taylor & Francis, 1992.
- J.R. Mahan, Radiation Heat Transfer: a Statistical Approach, John Wiley & Sons, Inc., 2002.
- Fluent® Manual version 5.5
- A catalog of radiation heat transfer configuration factors: http://www.me.utexas.edu/~howell/index.html

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