DEPARTMENT OF ENERGETICS

POLITECNICO DI TORINO

Multiphase Flows: Basic Physics and Engineering Modeling

Numerical Heat Transfer

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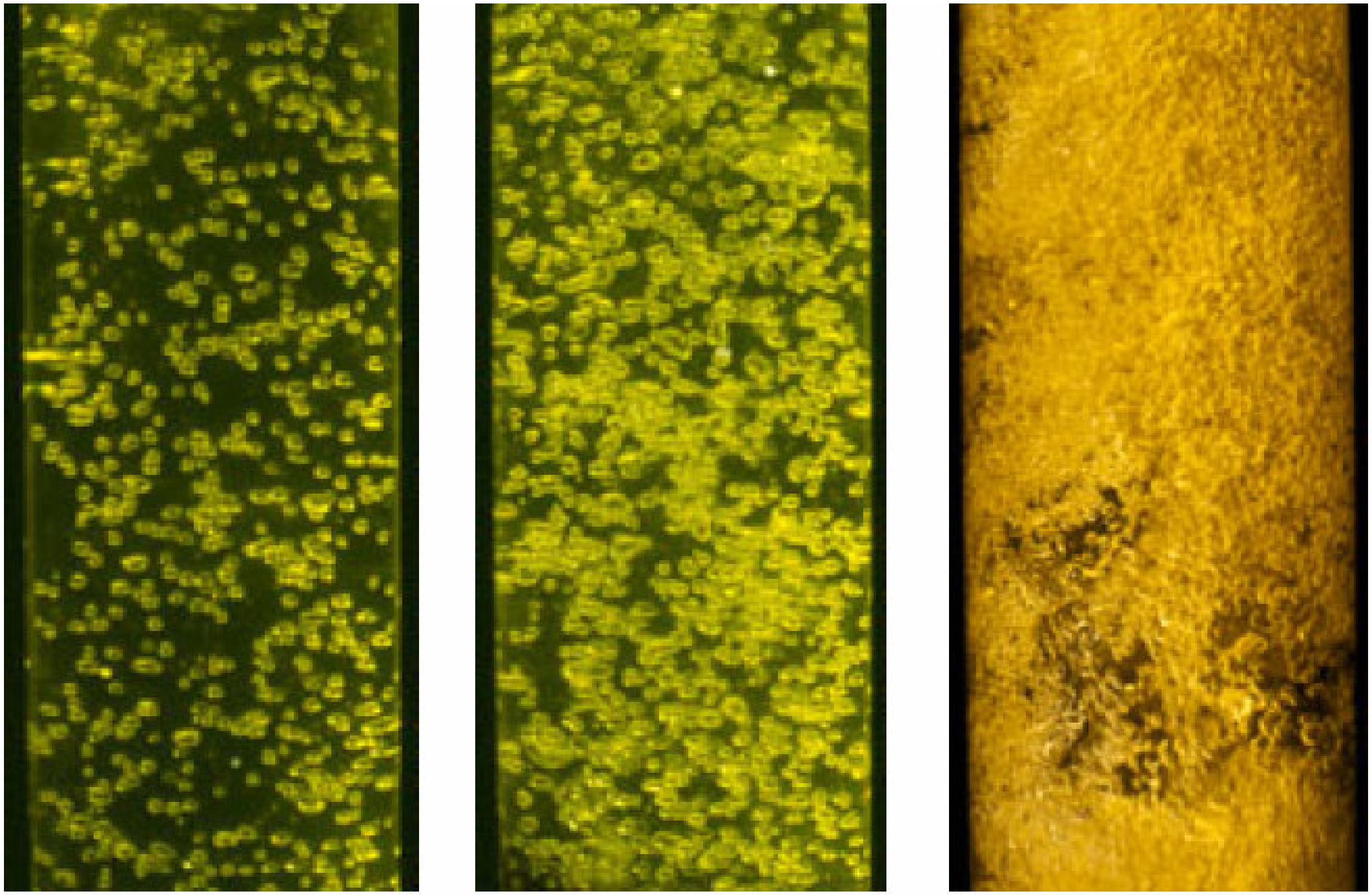
Spring 2007, TOP – UIC Program: The Master of Science Degree of the University of Illinois at the 'Politecnico di Torino'

Outline of this Section

- Basic physics of multiphase flows:
  - flow patterns;
  - flows around droplet and bubble;
  - evolution equation for bubble dynamics.
- Discrete phase models (DPM):
  - particle based (Lagrangian) formulation.
- Continuous phase models (CPM):
  - single–fluid approach (Mixture models, Volume–of–Fluid VOF, Cavitation models);
  - two–fluid approach (Eulerian – Eulerian models, and Eulerian – granular models).

Air / Water Mixtures in Vertical Pipe



Photographs of air/water flow in a 10.2cm diameter vertical pipe (Kytömaa 1987). Left: 1% air; middle: 4.5% air; right: > 15% air.

Flow Pattern Classification

- Disperse Flows:
  - **homogeneous flow**, it is the asymptotic limit of a disperse flow in which the disperse phase is distributed as an infinite number of infinitesimally small particles, bubbles, or drops → this limit implies zero relative motion between the phases;
  - **bubbly or mist flow**, it is a flow quite disperse in that the particle size is much smaller than the pipe dimensions but in which the relative motion between the phases is significant.
- Separated Flows:
  - **annular or film flow**, it is a flow where the droplets are an important feature and therefore it can only be regarded as partially separated;
  - **fully separated flow**, it consists of two single phase streams when low velocity flow of gas and liquid in a pipe.

Multiphase Flows

- Multiphase flow is used to refer to any fluid flow consisting of more than one phase (*) (or component)
- We exclude those circumstances in which the components are well mixed above the molecular level → Consequently, the flows considered here have some level of phase or component separation at a scale well above the molecular level → This still leaves an enormous spectrum of different multiphase flows

(*) Phase: in thermodynamics, a chemically and physically uniform quantity of matter that can be separated mechanically from a non – homogeneous mixture. It may consist of a single substance or of a mixture of substances.

Disperse Flows: Trajectory Models

- Disperse flows are those consisting of finite particles, drops or bubbles (the disperse phase) distributed in a connected volume of the continuous phase
- In **trajectory models**, the details of the flow around each of the particles are subsumed into assumed drag, lift and moment forces acting on and altering the trajectory of those particles → The following considerations hold:
  - particle-particle interactions and the effects of the particle volume fraction on the gas phase are negligible (this issue implies that the discrete phase must be present at a fairly low volume fraction, usually less than 10 %);
  - the thermal history of the particles can also be tracked if it is appropriate to do so → in this case, different mass transfer phenomena are affecting the convective and radiative heat transfer.

Disperse Flows: Two–fluid Models

- In **two–fluid models**: the disperse phase is treated as a second continuous phase intermingled and interacting with the continuous phase → The following considerations hold
  - effective conservation equations (of mass, momentum and energy) are developed for the two fluid flows → these included interaction terms modeling the exchange of mass, momentum and energy between the two flows;
  - the two-fluid models neglect the discrete nature of the disperse phase and approximate its effects upon the continuous phase (this implies a fairly high volume fraction, usually larger than 10 %);
  - inherent in this approach, are averaging processes necessary to characterize the properties of the disperse phase → these involve significant difficulties (for example, appropriate boundary conditions).

Separated Flows

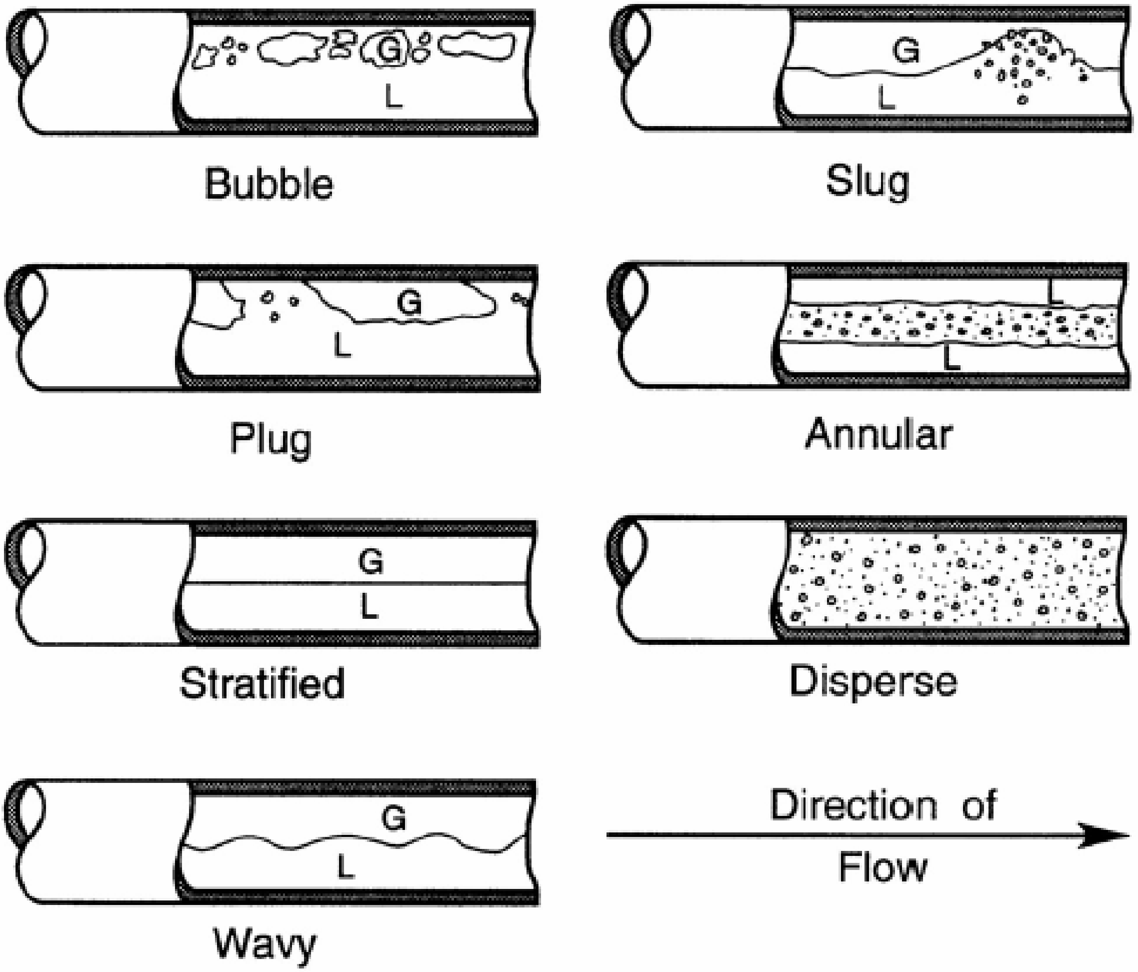
- Separated flows are those where two different streams can readily travel at different velocities → The essential features of this type of flows are:
  - the relative motion (and consequently, from the modeling point of view, the relative velocity);
  - the separating interfaces (which implies that they must be tracked in the computational domain).
- These flows can be modeled by means of
  - **single–fluid (or mixture) models**, where the main stream (?!) is modeled first and then the relative drifts;
  - **two–fluid models**, where the single phase fluid flow equations is applied in the two streams, coupling them through appropriate kinematic and dynamic conditions at the interface.

Flow Patterns

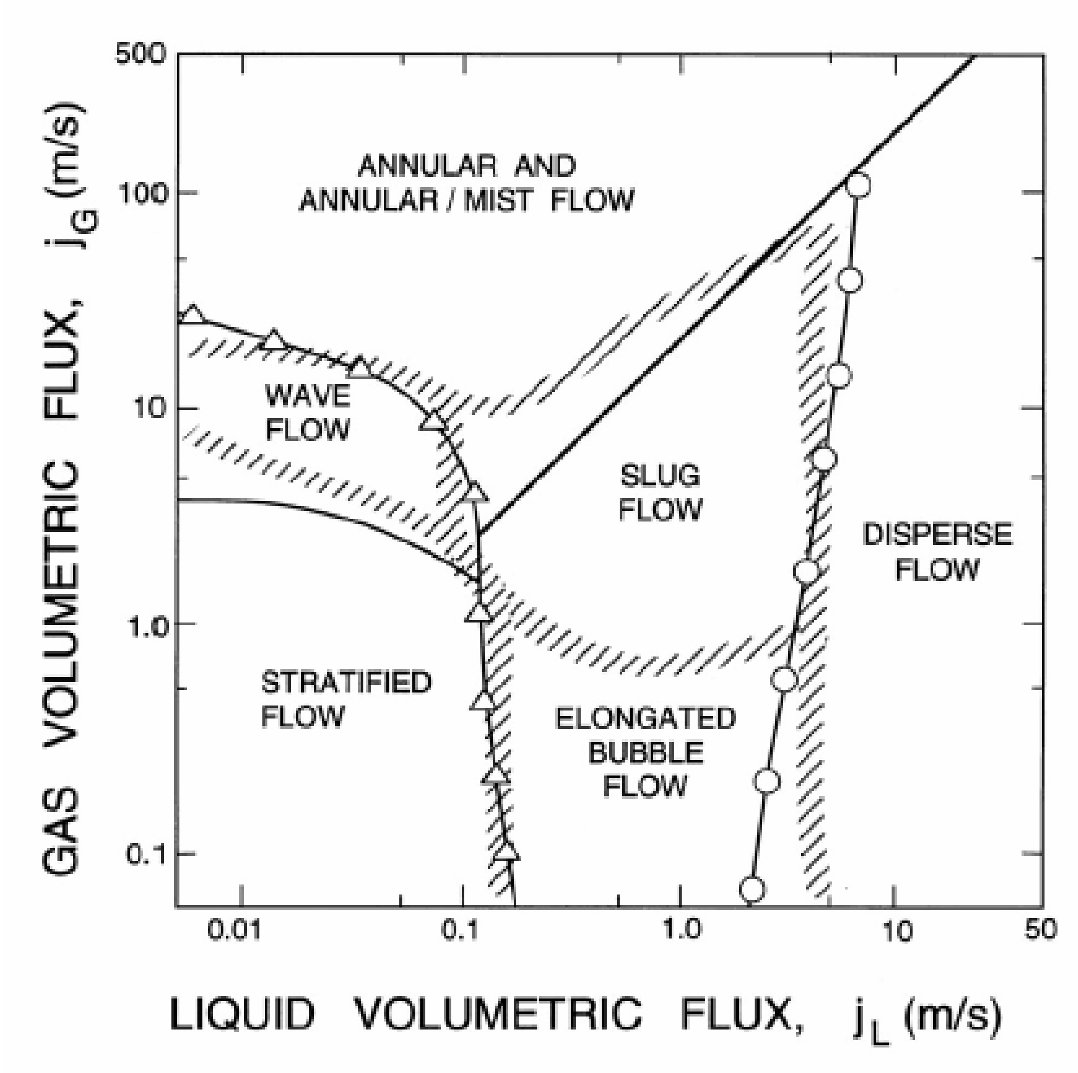
- From a practical engineering point of view one of the major design difficulties in dealing with multiphase flow is that the mass, momentum, and energy transfer rates and processes can be quite sensitive to the geometric distribution or topology of the phases within the flow
- The geometry may strongly effect the interfacial area available for mass, momentum or energy exchange between the phases → The flow within each phase will clearly depend on that geometric distribution
- Hence we recognize that there is a complicated two-way coupling between the flow in each of the phases and the geometry of the flow
- A particular type of geometric distribution of the components is called a **flow regime** (or **flow pattern**)

Air / Water Mixtures in Horizontal Pipe

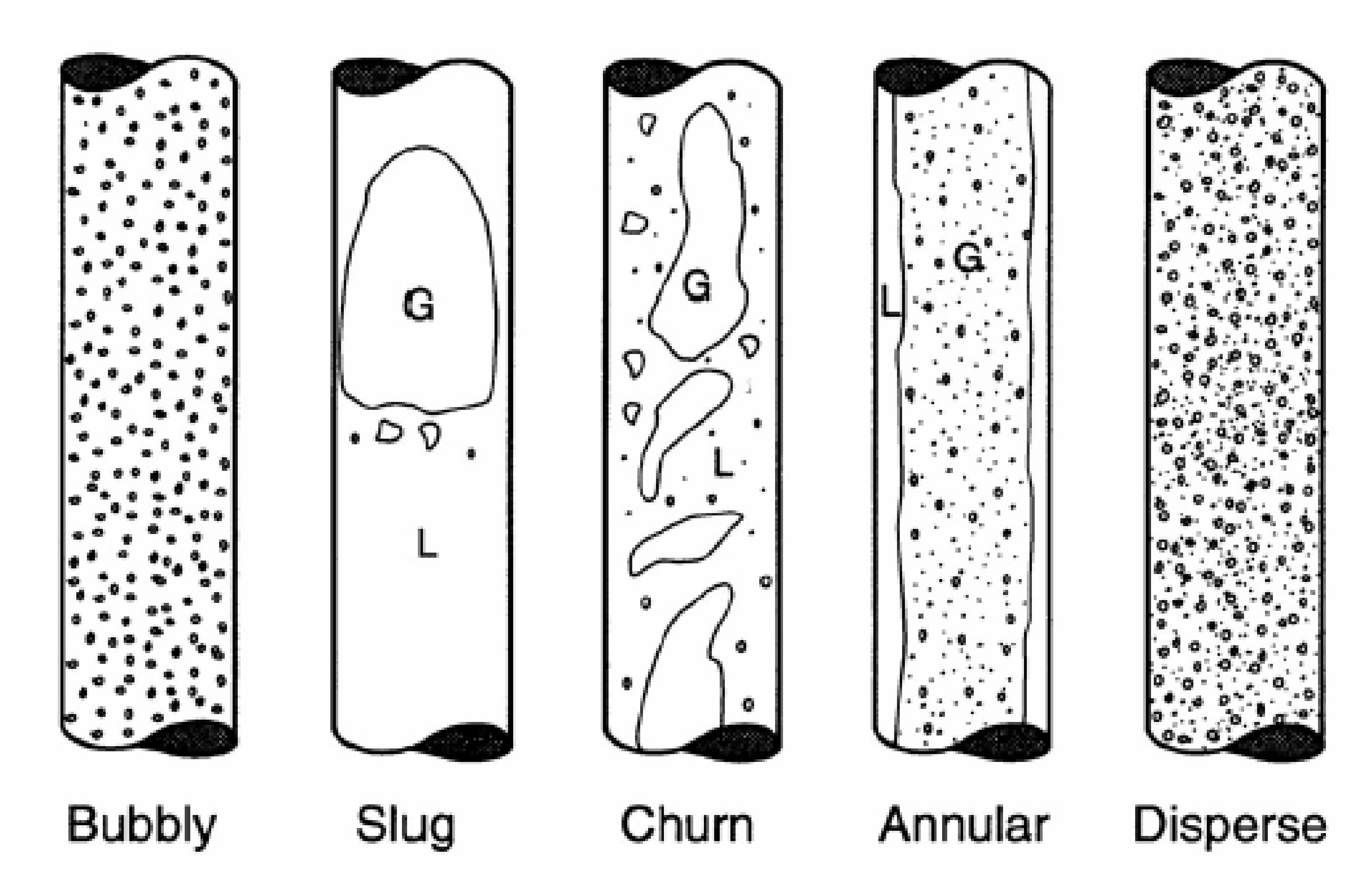
- Entrance lengths for multiphase flow patterns are not well established
- Several possible flow patterns may depend on the initial conditions



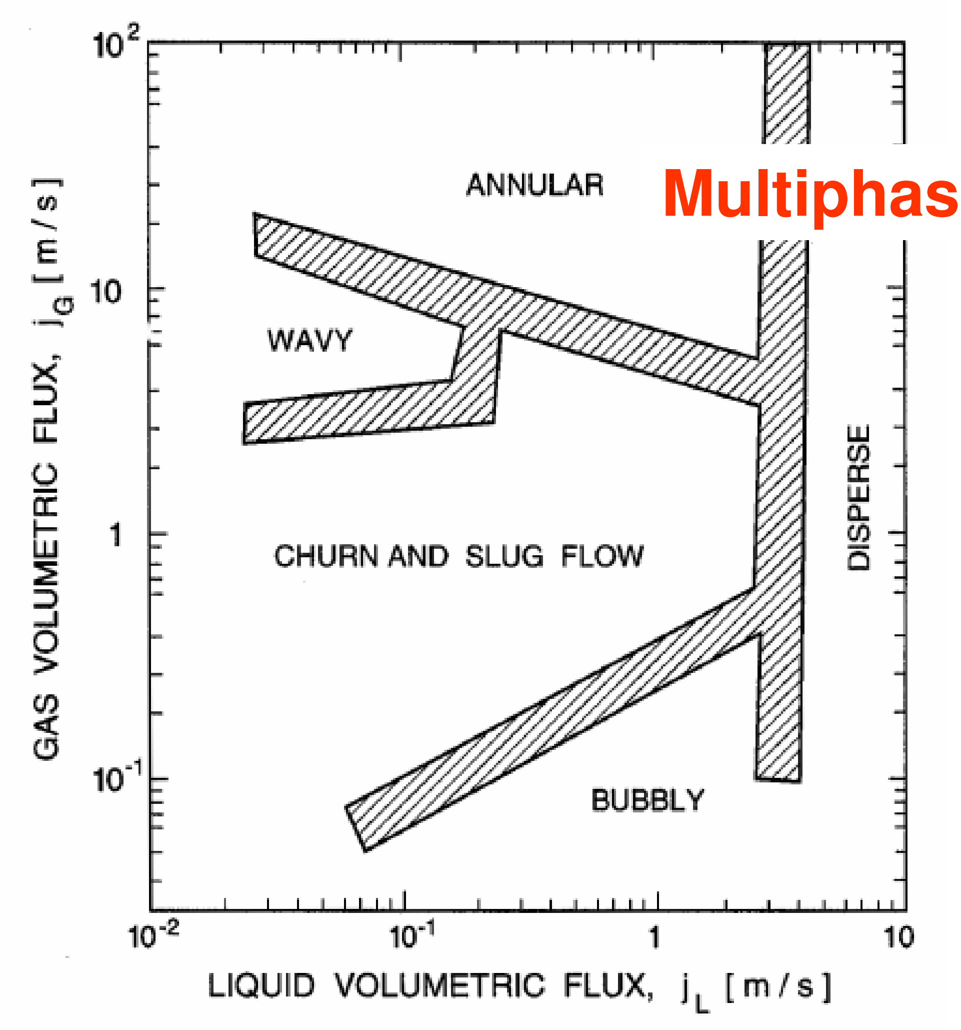
Flow Regime Map in Horizontal Pipe



Air / Water Mixtures in Vertical Pipe



Flow Regime Map in Vertical Pipe



Multiphase Transition

Instabilities of Disperse Flows…

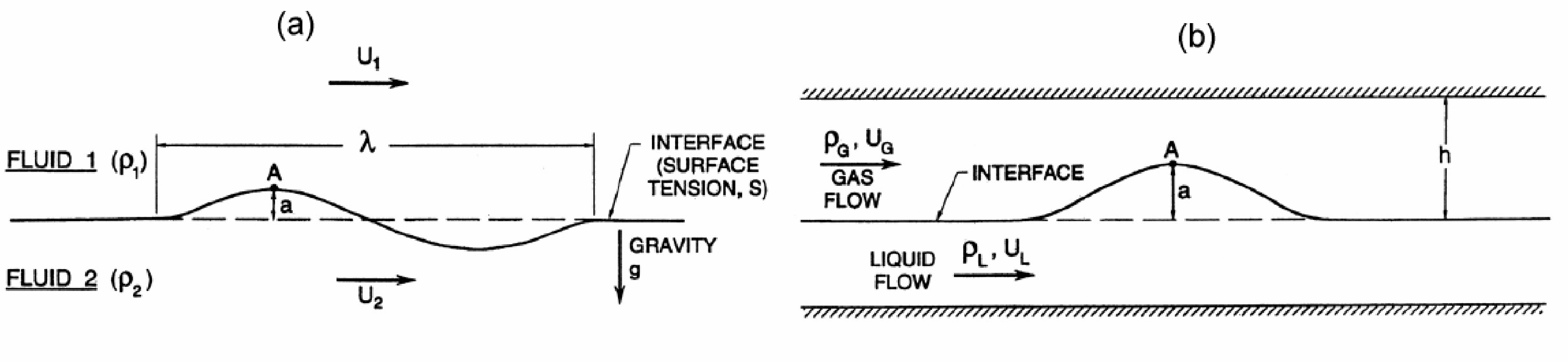
… actually lead to **multiphase transitions (D → S)**:

- the turbulent mixing (and the consequent coalescence) is larger than the separation processes driven by the density difference causing phase separation (by gravity or, in a non-uniform or turbulent flow, by the Lagrangian fluid accelerations);
- the wave perturbations may destroy a homogeneous, quiescent multiphase mixture because it may be internally unstable as a result of gravitationally-induced relative motion → As they grow in amplitude these wave-like volume fraction perturbations seem to evolve in several ways depending on the type of flow and the manner in which it is initiated (Jackson instability)

Instability of Separated Flows…

… actually lead to **multiphase transitions (S → D)**:

1. the interface may become unstable when waves form on the interface between the two fluid streams → these are called Kelvin–Helmoltz instabilities and they depend on the considered driving force:
   a) buoyancy force due to gravity (Rayleigh-Taylor instability);
   b) Bernoulli effect that implies a change in the pressure acting on the interface caused by a change in velocity (Bernoulli instability).



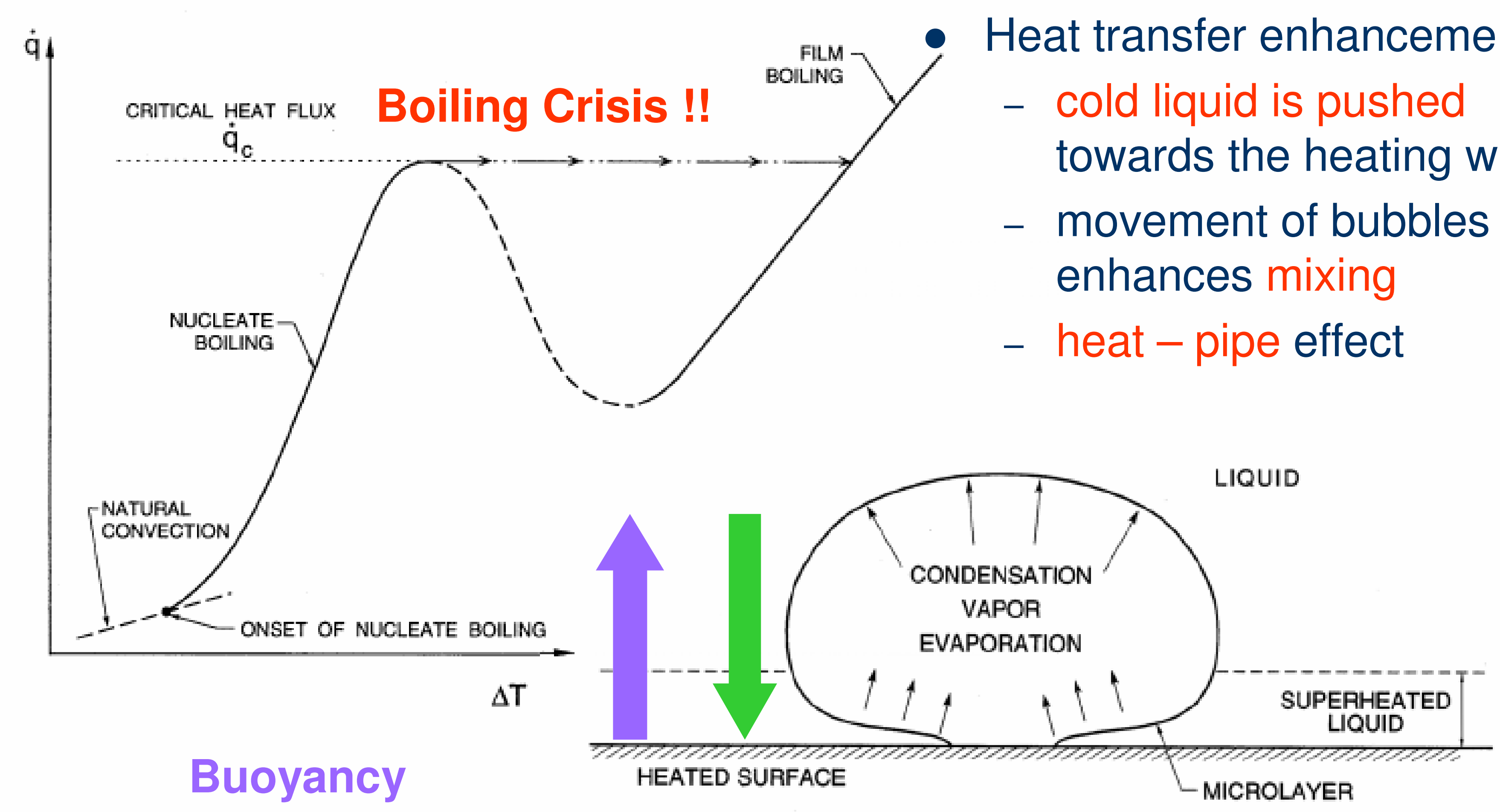
Multiphase Heat Transfer

In all cases the heat flux per unit area through the solid surface is denoted by $\dot{q}$; the wall temperature is denoted by T_w and the bulk liquid temperature by T_b (or T_L). The temperature difference $\Delta T = T_w - T_b$ is a ubiquitous feature of all these problems. Moreover, in almost all cases the pressure differences within the flow are sufficiently small that the saturated liquid/vapor temperature, T_e, can be assumed uniform. Then, to a first approximation, boiling at the wall occurs when $T_w > T_e$ and $T_b \leq T_e$. When $T_b < T_e$ and the liquid must be heated to T_e before bubbles occur, the situation is referred to as sub-cooled boiling. On the other hand condensation at the wall occurs when $T_w < T_e$ and $T_b \geq T_e$. When $T_b > T_e$ and the vapor must be cooled to T_e before liquid appears, the situation is referred to as super-heated condensation.

Boiling (Main) Phenomena

- Horizontal Surfaces
  - **Pool Boiling**: it is when a pool of liquid is heated from below through a horizontal surface
  - **Nucleate Boiling**: it is when temperature difference activates increasingly smaller (and therefore more numerous) sites
  - **Film Boiling**: it is when at or near boiling crisis a film of vapor is formed that coats the surface and substantially impedes heat transfer
- Vertical Surfaces
  - Film Boiling: qualitatively similar to that on a horizontal surface except for the upward liquid and vapor velocities caused by natural convection

Pool Boiling



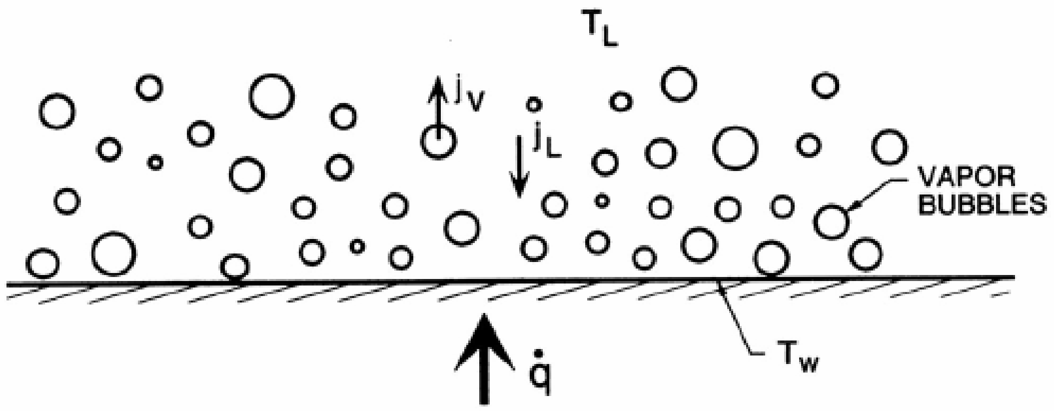
- Heat transfer enhancement
  - cold liquid is pushed towards the heating wall
  - movement of bubbles enhances mixing
  - heat – pipe effect

Buoyancy

Surface Tension

Nucleate Boiling

- Nucleation: increased temperature difference activates increasingly smaller (and therefore more numerous) sites
- The ebullition cycle consists of
  - the **bubble growth period**, which is directly related to the rate of heat supply per single bubble
  - the **moment of detachment**, when the upward buoyancy forces exceed the surface tension forces at the bubble-wall contact line
  - the **waiting period**, due to the temporary local cooling of the wall

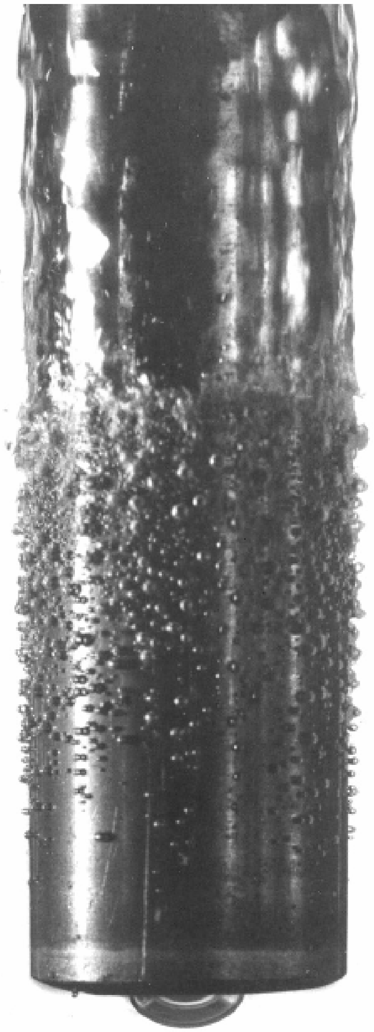


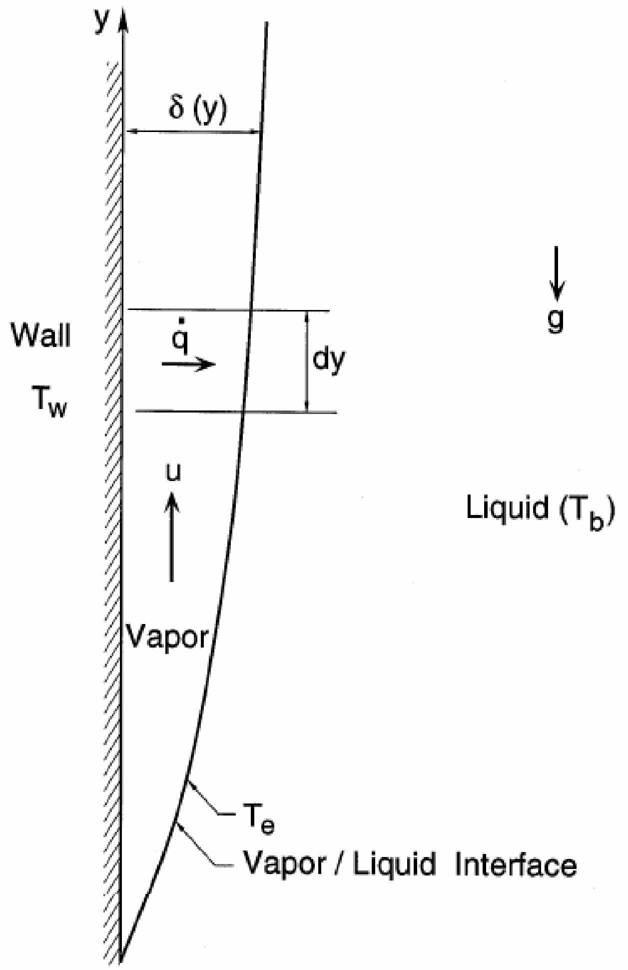
Film Boiling

- At or near boiling crisis a film of vapor is formed that coats the surface and substantially impedes heat transfer → This vapor layer presents the primary resistance to heat transfer
- These flows are usually quite unsteady since the vapor / liquid interface is unstable (Rayleigh-Taylor instability) → Vapor bubbles are introduced into the liquid and travel upwards while liquid droplets are also formed and fall down through the vapor toward the hot surface (the droplets do not normally touch the hot surface because the vapor created on the droplet surface nearest the wall creates a lubrication layer that suspends the droplet – the Leidenfrost effect)

Film Boiling in Vertical Surfaces

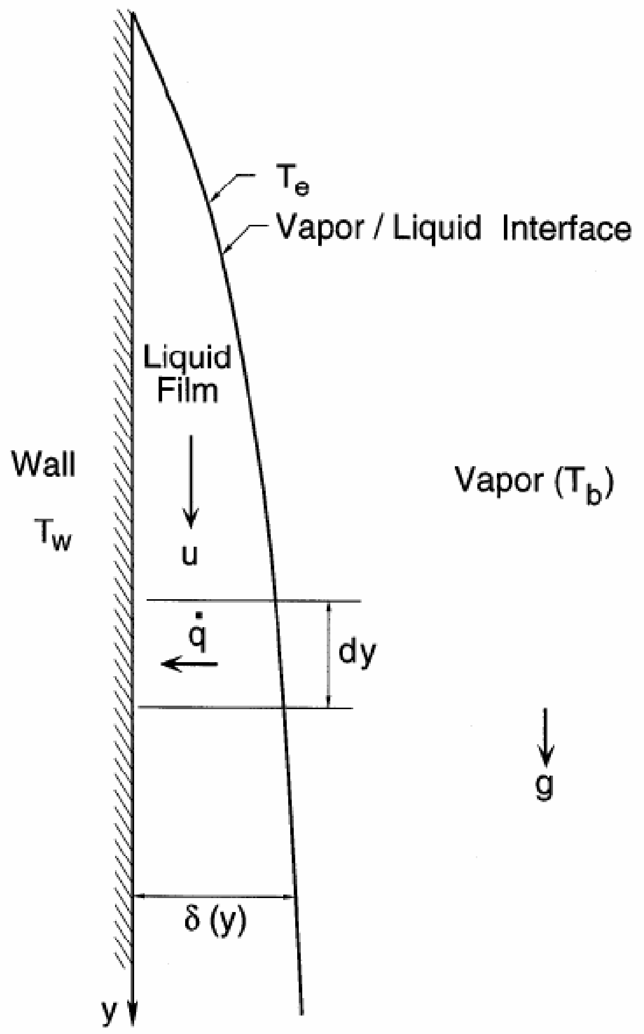
- Boiling on a heated vertical surface is qualitatively similar to that on a horizontal surface except for the upward liquid and vapor velocities caused by natural convection
- For a well-defined elevation, boiling crisis can appear





Film Condensation in Vertical Surfaces

- The spectrum of flow processes associated with condensation on a solid surface are almost a mirror image of those involved in boiling
- Drop condensation on the underside of a cooled on a vertical surface is very analogous to nucleate boiling → When the population of droplets becomes large they run together to form condensation films
- The phenomenon is most apparent as the misting up of windows or mirrors.



Discrete Phase Models (DPMs)

- In addition to solving transport equations for the continuous phase, this approach consists of simulating a discrete second phase in a Lagrangian frame of reference, made of discrete particles (which may be taken to represent droplets or bubbles) dispersed in the continuous phase
- Essentially the key idea is to compute the trajectories of these discrete phase entities, as well as heat and mass transfer to/from them → The coupling between the phases and its impact on both the discrete phase trajectories and the continuous phase flow can be included by means of proper closure models
- Acceptable for low volume fraction (less than 10 %)

Very Dilute Multiphase Flows

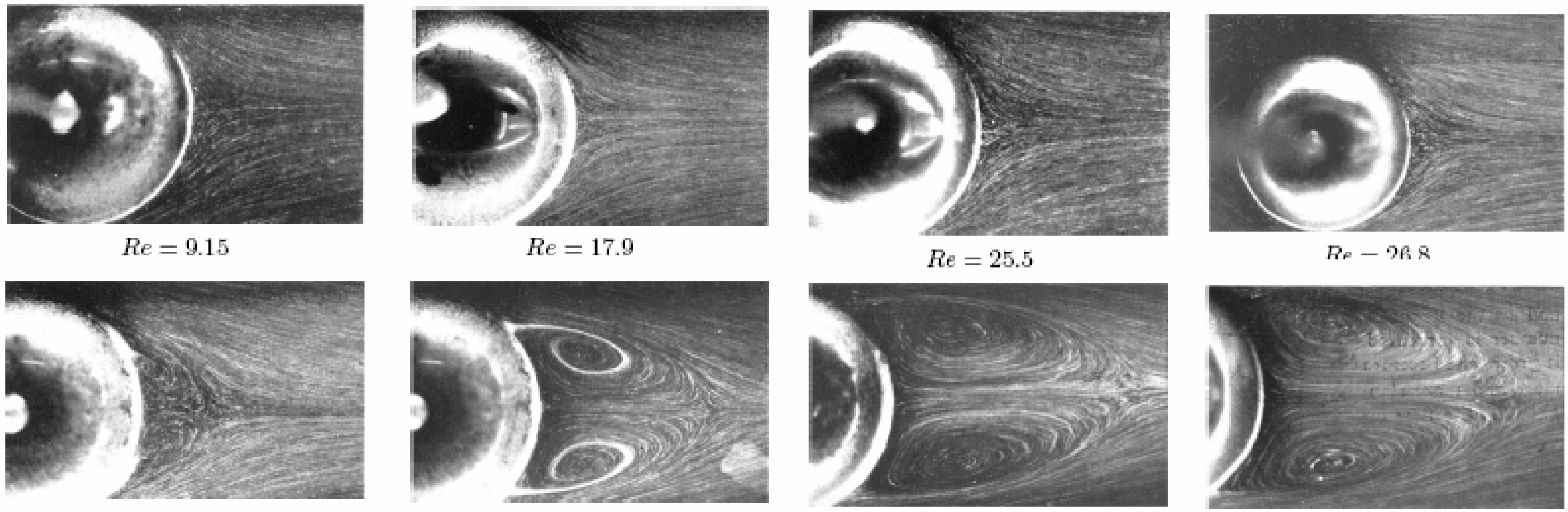
- One-way coupling: quite accurate solutions are then obtained by solving a single phase flow for the continuous phase and inputting those fluid velocities into equations of motion for the particles.
  1. It is sufficient to solve for the velocity and pressure of the continuous suspending fluid while ignoring the particles or disperse phase.
  2. Once this solution is given, one could then solve an equation of motion for the particle to determine its trajectory.
- Two-way coupling: as the concentration of the disperse phase is increased, complications can effect both the continuous phase flow and the disperse phase motions

Flows around a Sphere (Low Re)

Suspending Flow

$$\frac{\partial u_j}{\partial x_j} = 0$$

$$\rho_C \left\{ \frac{\partial u_i}{\partial x_j} \right\} = -\frac{\partial p}{\partial x_i} - \rho_C \nu_C \frac{\partial^2 u_i}{\partial x_j \partial x_j}$$

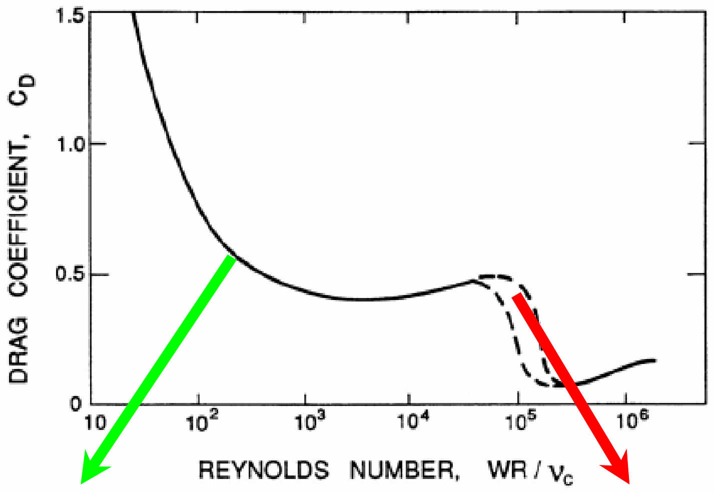


$Re = 9.15$ $Re = 17.9$ $Re = 25.5$ $Re = 26.8$

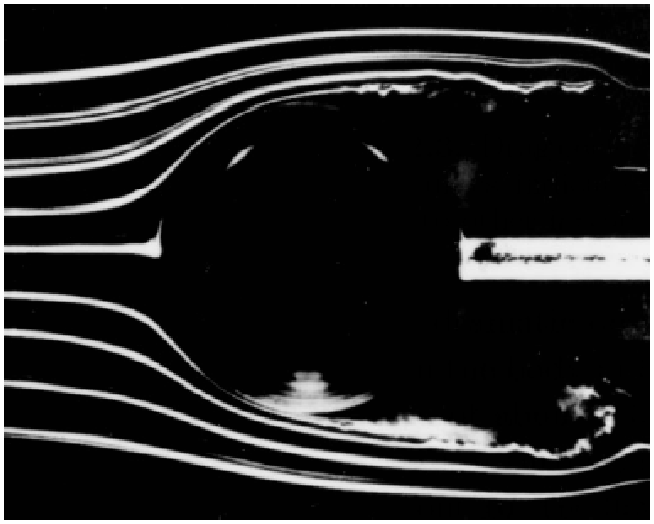
$Re = 37.7$ $Re = 73.6$ $Re = 118$ $Re = 133$

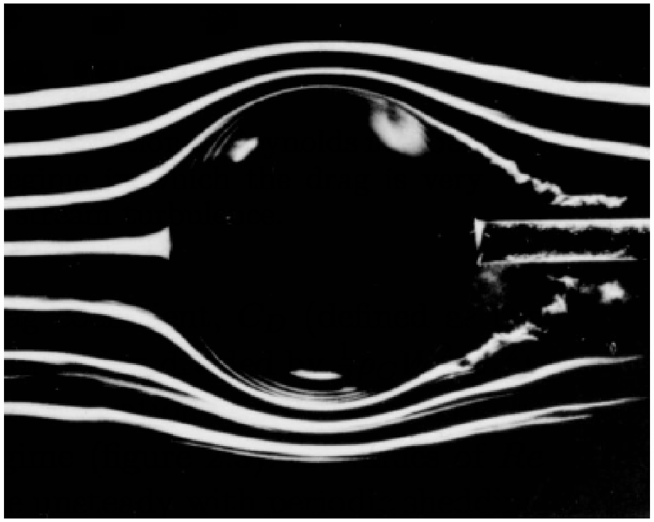
Flows around a Sphere (High Re)

Suspending Flow

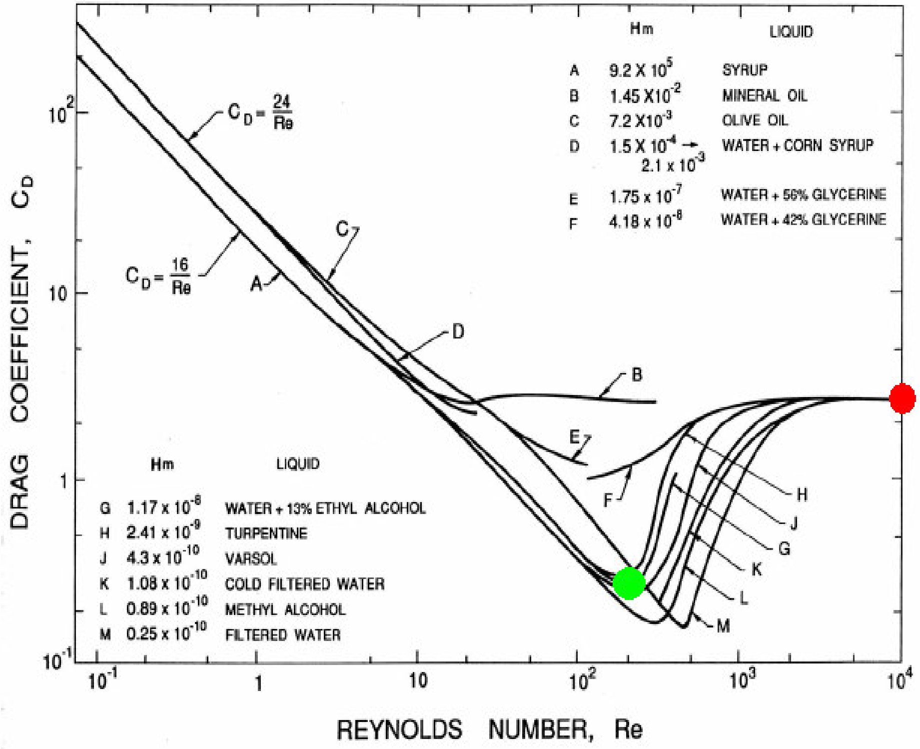


Drag Crisis



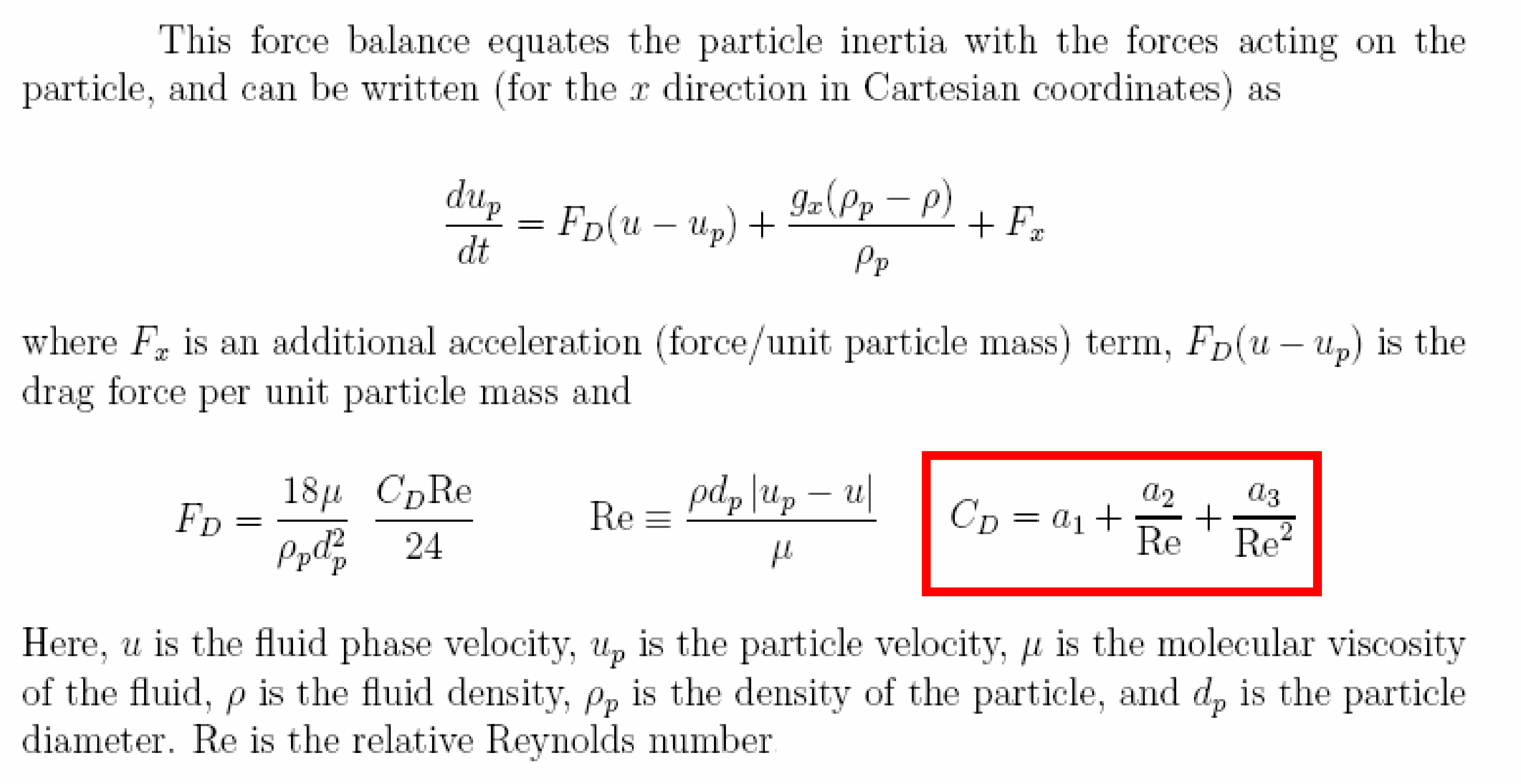


Deformation due to Translation



Drag coefficients, C_D, for bubbles as a function of the Reynolds number, Re, for a range of Haberman-Morton numbers, Hm, as shown. Data from Haberman and Morton (1953).

Particle Force Balance in Fluent®



This force balance equates the particle inertia with the forces acting on the particle, and can be written (for the x direction in Cartesian coordinates) as

$$\frac{du_p}{dt} = F_D(u - u_p) + \frac{g_x(\rho_p - \rho)}{\rho_p} + F_x$$

where F_x is an additional acceleration (force/unit particle mass) term, $F_D(u - u_p)$ is the drag force per unit particle mass and

$$F_D = \frac{18\mu}{\rho_p d_p^2} \frac{C_D \mathrm{Re}}{24} \qquad \mathrm{Re} \equiv \frac{\rho d_p \left| u_p - u \right|}{\mu} \qquad \boxed{C_D = a_1 + \frac{a_2}{\mathrm{Re}} + \frac{a_3}{\mathrm{Re}^2}}$$

Here, u is the fluid phase velocity, u_p is the particle velocity, μ is the molecular viscosity of the fluid, ρ is the fluid density, ρ_p is the density of the particle, and d_p is the particle diameter. Re is the relative Reynolds number

Particle Growth and Collapse

- Unlike solid particles or liquid droplets, gas / vapor bubbles can grow or collapse in a flow and in doing so manifest a host of phenomena with technological importance (in particular for combustion)
- Let us consider the fundamental dynamics of a growing or collapsing bubble in an infinite domain of liquid that is at rest far from the bubble
- The assumption of spherical symmetry is violated in several important processes, but it is necessary to first develop the baseline → dynamics of clouds of bubbles or of bubbly flows

Rayleigh – Plesset Equation

(1) Instantaneous driving term

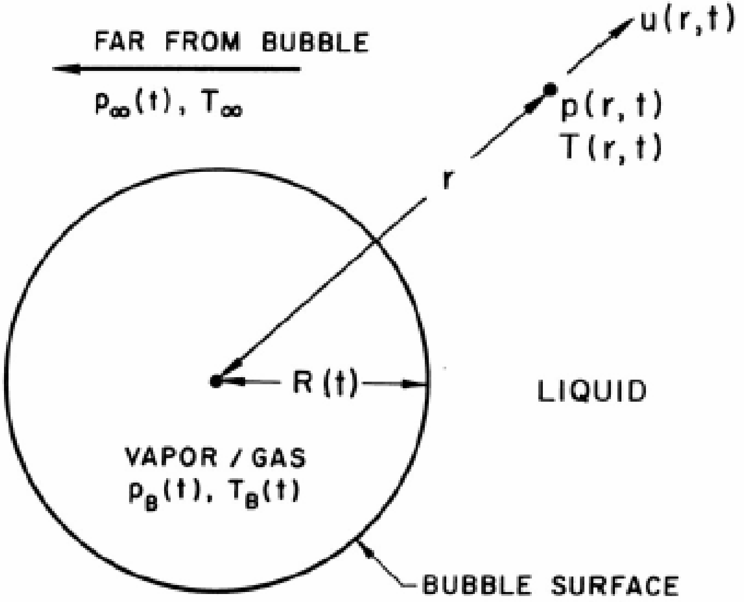
(2) Thermal term

(3) Non – condensable gases

(4) Inertial term

(5) Viscous resistance

(6) Surface tension



$$\frac{p_B(t)-p_\infty(t)}{\rho_L} = \overset{(1)}{\boxed{\frac{p_V(T_\infty)-p_\infty(t)}{\rho_L}}} + \overset{(2)}{\boxed{\frac{p_V(T_B)-p_V(T_\infty)}{\rho_L}}} + \overset{(3)}{\boxed{\frac{p_{Go}}{\rho_L}\left(\frac{T_B}{T_\infty}\right)\left(\frac{R_o}{R}\right)^3}}$$

$$= \underset{(4)}{R\frac{d^2R}{dt^2} + \frac{3}{2}\left(\frac{dR}{dt}\right)^2} + \underset{(5)}{\frac{4\nu_L}{R}\frac{dR}{dt}} + \underset{(6)}{\frac{2S}{\rho_L R}}$$

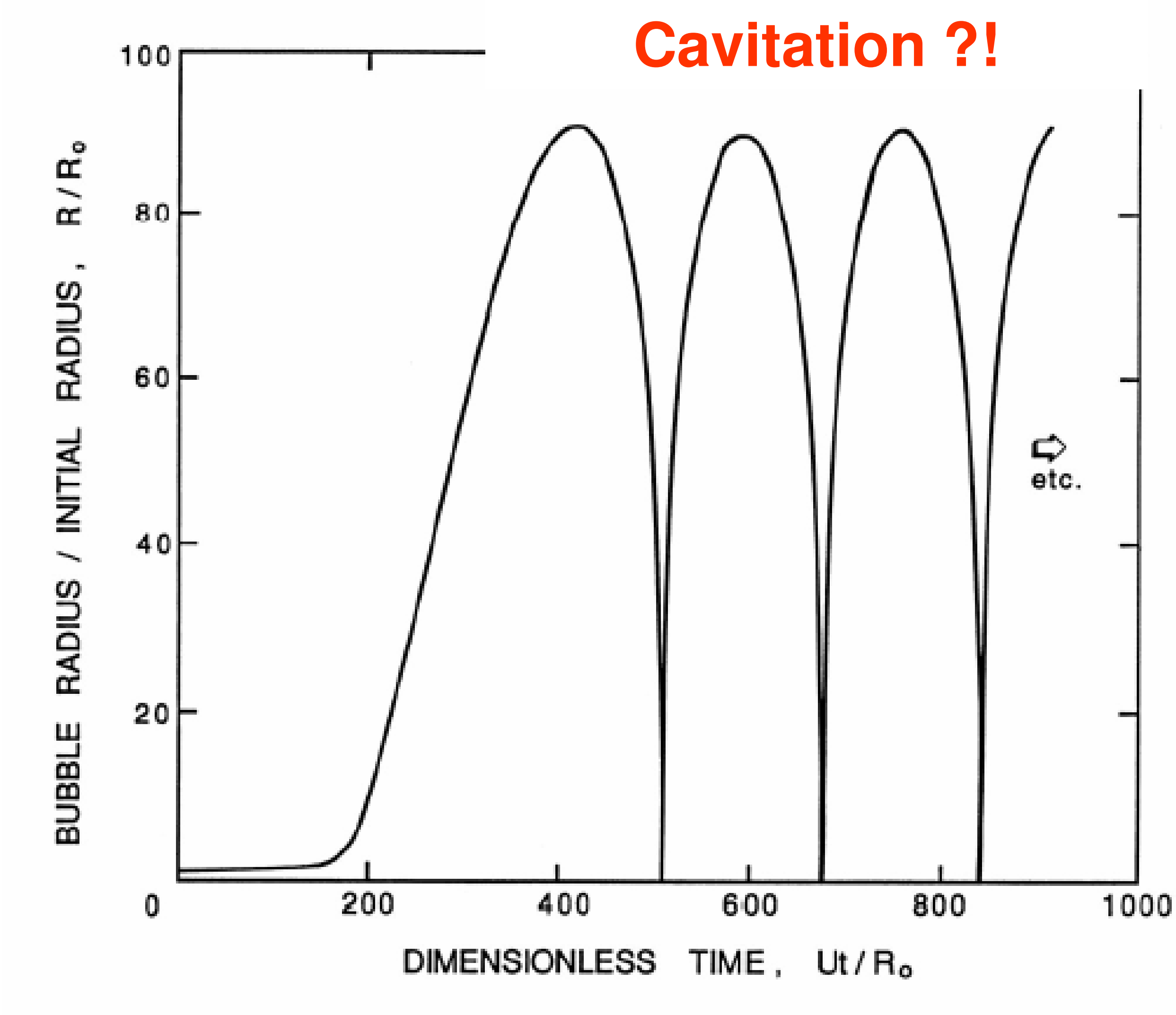
(1) Inertially Controlled Dynamics

- First we consider some of the characteristics of bubble dynamics in the absence of any significant thermal effects → This kind of bubble dynamic behavior is termed inertially controlled to distinguish it from the thermally controlled behavior discussed later → Under these circumstances the temperature in the liquid is assumed uniform

$$\frac{p_V(T_\infty) - p_\infty(t)}{\rho_L} = R\frac{d^2R}{dt^2} + \frac{3}{2}\left(\frac{dR}{dt}\right)^2 + \frac{4\nu_L}{R}\frac{dR}{dt} + \frac{2S}{\rho_L R}$$

$$p_V(T_\infty)R^3 = \text{cost}$$

Isothermal Oscillating Bubbles



(2) Thermally Controlled Dynamics

- When thermal deriver in Rayleigh – Plesset equation is dominant, then an additional relationship between the temperature and radius dynamics must be provided
- In some cases, the effects due to decompression in the bubble growth can be neglected and the practical formula are highly simplified
- Several other factors can complicate and alter the dynamics of thermally controlled bubbles:
  - the presence of a nearby wall;
  - nonequilibrium effects (high evaporation rates);
  - surface instabilities (bubble surfaces are usually rough and irregular);
  - convective heat transfer;...

Complete System of Equations

- We need to correlate **T(t)** with **R(t)** → the generalized conduction equation in the liquid phase is highly non–linear → no exact analytical solution exists
- Also the relationship between **p(t)** and **R(t)** is complicated by the fact that the actual temperature is changing in time

$$\frac{p_V(T_\infty) - p_\infty(t)}{\rho_L} + \frac{p_V(T_B) - p_V(T_\infty)}{\rho_L} = R\frac{d^2R}{dt^2} + \frac{3}{2}\left(\frac{dR}{dt}\right)^2 + \frac{4\nu_L}{R}\frac{dR}{dt} + \frac{2S}{\rho_L R}$$

$$\frac{\partial T}{\partial t} + \frac{dR}{dt}\left(\frac{R}{r}\right)^2 \frac{\partial T}{\partial r} = \frac{\mathcal{D}_L}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right)$$

$$\frac{dR}{dt} = \frac{k_L}{\rho_V \mathcal{L}} \left(\frac{\partial T}{\partial r}\right)_{r=R}$$

(3) Diffusion Controlled Dynamics

- In most of the circumstances, it is assumed that the events occur too rapidly for significant mass transfer of non–condensable gas to occur between the bubble and the liquid → Actually all of the gas–filled microscopic bubbles that are present in a sub–saturated liquid (and particularly in water) should dissolve away if the ambient pressure is sufficiently high (Henry’s law)
- The process of mass transfer can be modeled by means of a proper concentration equation

$$\frac{\partial c}{\partial t} + \frac{dR}{dt}\left(\frac{R}{r}\right)^2 \frac{\partial c}{\partial r} = \frac{D}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial c}{\partial r}\right)$$

Cavitation

- Cavitation occurs in flowing liquid systems when the pressure falls sufficiently low in some region of the flow so that vapor bubbles are formed.
- Parsons (1906) recognized the role played by vaporization → The phenomenon has been a subject of intensive research ever since
  - because of the adverse effects it has on performance,
  - because of the noise it creates,
  - and, most surprisingly, the damage it can do to nearby solid surfaces.
- Cavitation number: $\sigma = \dfrac{p_\infty - p_V(T_\infty)}{\frac{1}{2}\rho_L U_\infty^2}$

Heat & Mass Transfer in Fluent®

- Inert heating/cooling

$$m_p c_p \frac{dT_p}{dt} = hA_p(T_\infty - T_p) + \boxed{\frac{dm_p}{dt}h_{\mathrm{fg}}} + A_p \epsilon_p \sigma({\theta_R}^4 - {T_p}^4)$$

- Droplet vaporization

$$N_i = k_c(C_{i,s} - C_{i,\infty})$$

- Droplet boiling

$$\frac{d(d_p)}{dt} = \frac{4k_\infty}{\rho_p c_{p,\infty} d_p}(1 + 0.23\sqrt{\mathrm{Re}_d}) \ln\left[1 + \frac{c_{p,\infty}(T_\infty - T_p)}{h_{\mathrm{fg}}}\right]$$

- Devolatilization

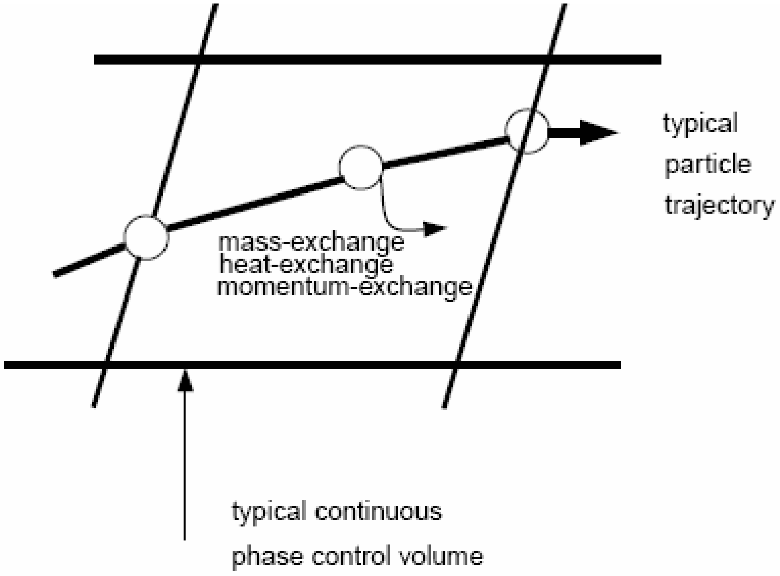
$$-\frac{dm_p}{dt} = k[m_p - (1 - f_{v,0})(1 - f_{w,0})m_{p,0}]$$

- Surface combustion

$$\frac{dm_p}{dt} = -4\pi d_p D_{i,m} \frac{Y_{\mathrm{ox}} T_\infty \rho}{S_b(T_p + T_\infty)}$$

Back Coupling in Fluent®

- Tracking of heat, mass, and momentum gained or lost by the particles with known trajectories allows one to take into account the effects with regards to the continuous (back coupling or two – way coupling)
- The changes of the particles which pass through each control volume are summed up



$$M = \frac{\Delta m_p}{m_{p,0}} \; \dot{m}_{p,0}$$

$$F = \sum \left(\frac{18 \mu C_D \mathrm{Re}}{\rho_p d_p^2 24} (u_p - u) + F_{\mathrm{other}} \right) \dot{m}_p \Delta t$$

$$Q = \left[\frac{\bar{m}_p}{m_{p,0}} \, c_p \Delta T_p + \frac{\Delta m_p}{m_{p,0}} \left(-h_{\mathrm{fg}} + h_{\mathrm{pyrol}} + \int_{T_{\mathrm{ref}}}^{T_p} c_{p,i} \, dT \right) \right] \dot{m}_{p,0}$$

Continuous Phase Models (CPMs)

- The dispersed and the continuous phases are combined together and modeled as a new, continuous phases

Definitions: Volumetric Quantities

Volumetric fluxes (volume flow per unit area) of individual components will be denoted by j_{Ai}, j_{Bi}

The *total volumetric flux*, j_i is then given by

$$\boxed{j_i = j_{Ai} + j_{Bi} + \ldots = \sum_N j_{Ni}}$$

Mass fluxes are similarly denoted by G_{Ai}, G_{Bi} or G_i. Thus if the densities of individual components are denoted by ρ_A, ρ_B it follows that

$$G_{Ai} = \rho_A j_{Ai} \;;\; G_{Bi} = \rho_B j_{Bi} \;;\; G_i = \sum_N \rho_N j_{Ni}$$

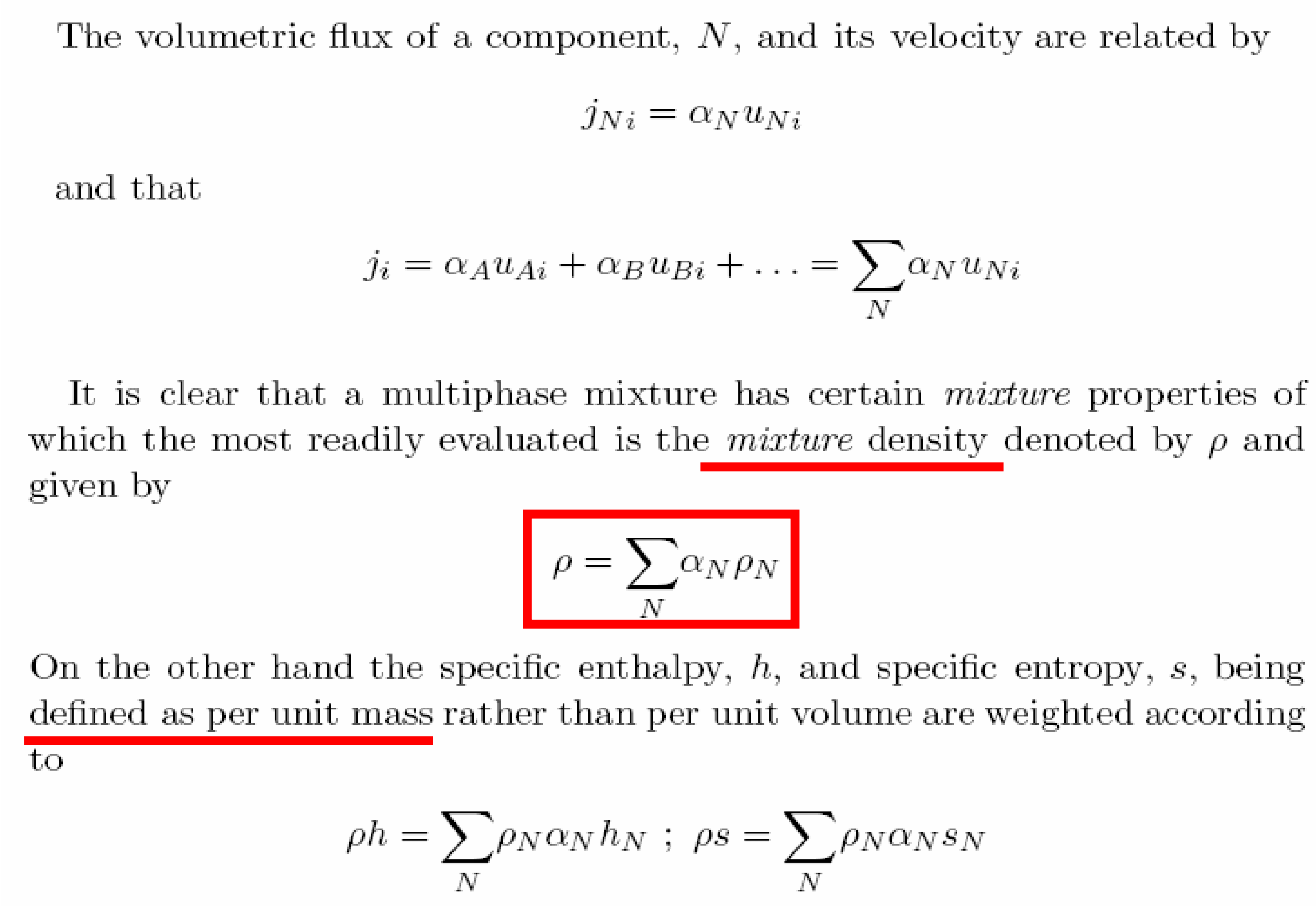
Velocities of the specific phases are denoted by u_{Ai}, u_{Bi} or, in general, by u_{Ni}.

The relative velocity between the two phases A and B will be denoted by u_{ABi} such that

$$u_{Ai} - u_{Bi} = u_{ABi}$$

The volume fraction of a component or phase is denoted by α_N and, in the case of two phases, A and B, it follows that $\alpha_B = 1 - \alpha_A$.

Definitions: Mass Weighted Quantities



The volumetric flux of a component, N, and its velocity are related by

$$j_{Ni} = \alpha_N u_{Ni}$$

and that

$$j_i = \alpha_A u_{Ai} + \alpha_B u_{Bi} + \ldots = \sum_N \alpha_N u_{Ni}$$

It is clear that a multiphase mixture has certain *mixture* properties of which the most readily evaluated is the *mixture* density denoted by ρ and given by

$$\boxed{\rho = \sum_N \alpha_N \rho_N}$$

On the other hand the specific enthalpy, h, and specific entropy, s, being defined as per unit mass rather than per unit volume are weighted according to

$$\rho h = \sum_N \rho_N \alpha_N h_N \;;\; \rho s = \sum_N \rho_N \alpha_N s_N$$

Definitions: Drift Quantities

The *drift velocity* of a component is defined as the velocity of that component in a frame of reference moving at a velocity equal to the total volumetric flux, j_i, and is therefore given by, u_{NJi}, where

$$u_{NJi} = u_{Ni} - j_i$$

Even more frequent use will be made of the *drift flux* of a component which is defined as the volumetric flux of a component in the frame of reference moving at j_i. Denoted by j_{NJi} this is given by

$$j_{NJi} = j_{Ni} - \alpha_N j_i = \alpha_N \left(u_{Ni} - j_i \right) = \alpha_N u_{NJi}$$

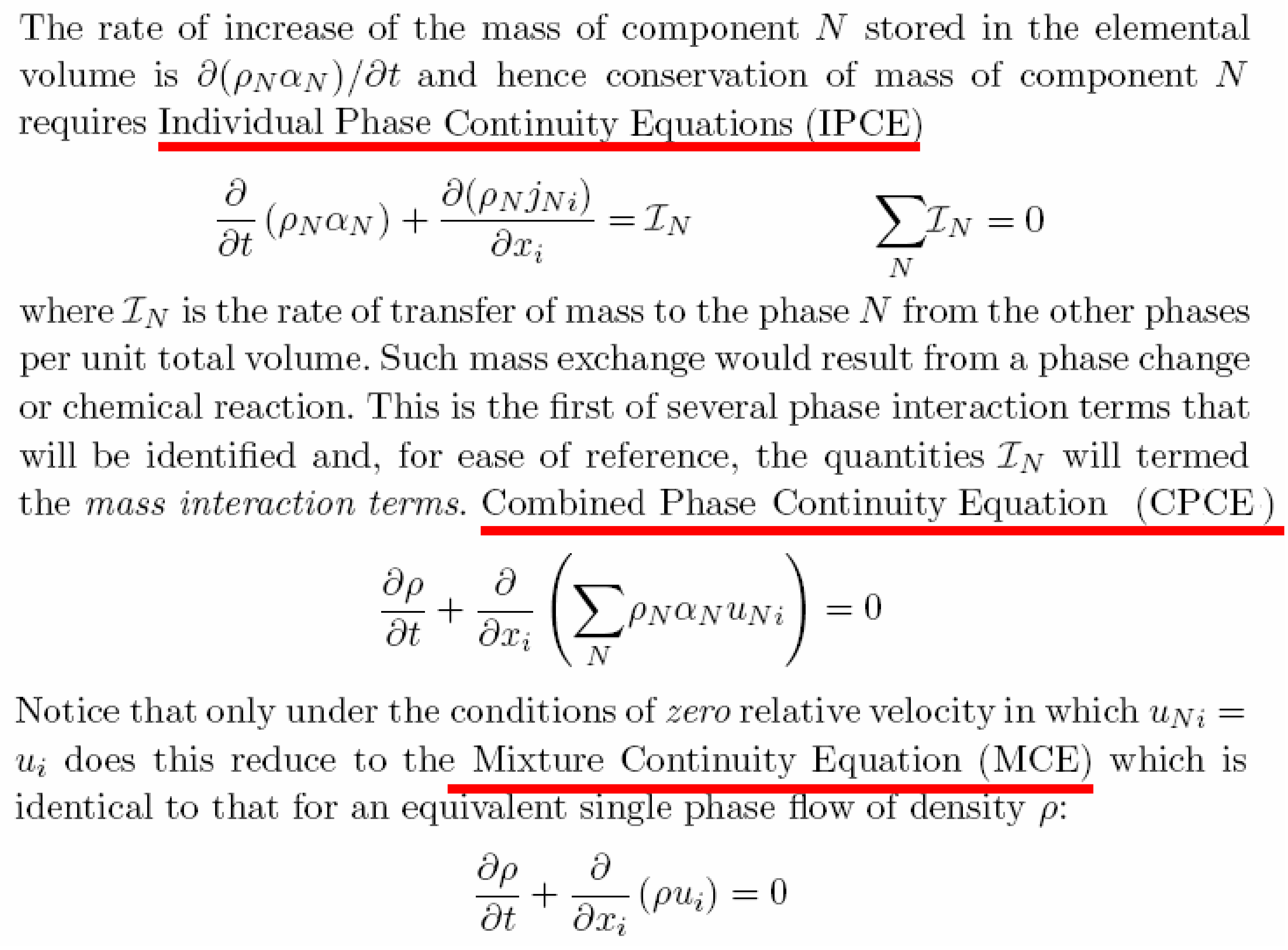
It is particularly important to notice that the sum of all the drift fluxes must be zero

$$\boxed{\sum_N j_{NJi} = \sum_N j_{Ni} - j_i \sum_N \alpha_N = j_i - j_i = 0}$$

Equations of Motion

- We need to assume that there exists an infinitesimal volume of dimension, such that
  - it was not only very much smaller than the typical distance over which the flow properties varied significantly (necessary in order to define derivatives of the flow properties within the flow field);
  - but also very much larger than the size of the individual phase elements, i.e. the disperse phase particles, drops or bubbles (each averaging volume contain representative samples of each of the components or phases).
- Above two conditions are rarely both satisfied → The averaging volumes contain a finite number of finite-sized particles and therefore flow properties vary significantly from point to point

Individual Phase Continuity Equation



The rate of increase of the mass of component N stored in the elemental volume is $\partial(\rho_N \alpha_N)/\partial t$ and hence conservation of mass of component N requires Individual Phase Continuity Equations (IPCE)

$$\frac{\partial}{\partial t}(\rho_N \alpha_N) + \frac{\partial(\rho_N j_{Ni})}{\partial x_i} = \mathcal{I}_N \qquad \sum_N \mathcal{I}_N = 0$$

where $\mathcal{I}_N$ is the rate of transfer of mass to the phase N from the other phases per unit total volume. Such mass exchange would result from a phase change or chemical reaction. This is the first of several phase interaction terms that will be identified and, for ease of reference, the quantities $\mathcal{I}_N$ will termed the *mass interaction terms*. Combined Phase Continuity Equation (CPCE)

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}\left(\sum_N \rho_N \alpha_N u_{Ni}\right) = 0$$

Notice that only under the conditions of *zero* relative velocity in which $u_{Ni} = u_i$ does this reduce to the Mixture Continuity Equation (MCE) which is identical to that for an equivalent single phase flow of density ρ:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}(\rho u_i) = 0$$

Individual Phase Momentum Equation

Individual Phase Momentum Equation (IPME) becomes

$$\frac{\partial}{\partial t}(\rho_N \alpha_N u_{Nk}) + \frac{\partial}{\partial x_i}(\rho_N \alpha_N u_{Ni} u_{Nk})$$

$$= \alpha_N \rho_N g_k + \mathcal{F}_{Nk} - \delta_N \left\{ \frac{\partial p}{\partial x_k} - \frac{\partial \sigma^D_{Cki}}{\partial x_i} \right\} \qquad \sum_N \mathcal{F}_{Nk} = 0$$

where $\delta_D = 0$ for the disperse phase and $\delta_C = 1$ for the continuous phase.

Thus we identify the second of the interaction terms, namely the *force interaction*, $\mathcal{F}_{Nk}$. If the momentum equations for each of the components are added together the resulting Combined Phase Momentum Equation (CPME) becomes

$$\frac{\partial}{\partial t}\left(\sum_N \rho_N \alpha_N u_{Nk}\right) + \frac{\partial}{\partial x_i}\left(\sum_N \rho_N \alpha_N u_{Ni} u_{Nk}\right)$$

$$= \rho g_k - \frac{\partial p}{\partial x_k} + \frac{\partial \sigma^D_{Cki}}{\partial x_i}$$

Note that this equation 1.46 will only reduce to the equation of motion for a single phase flow in the absence of relative motion, $u_{Ck} = u_{Dk}$.

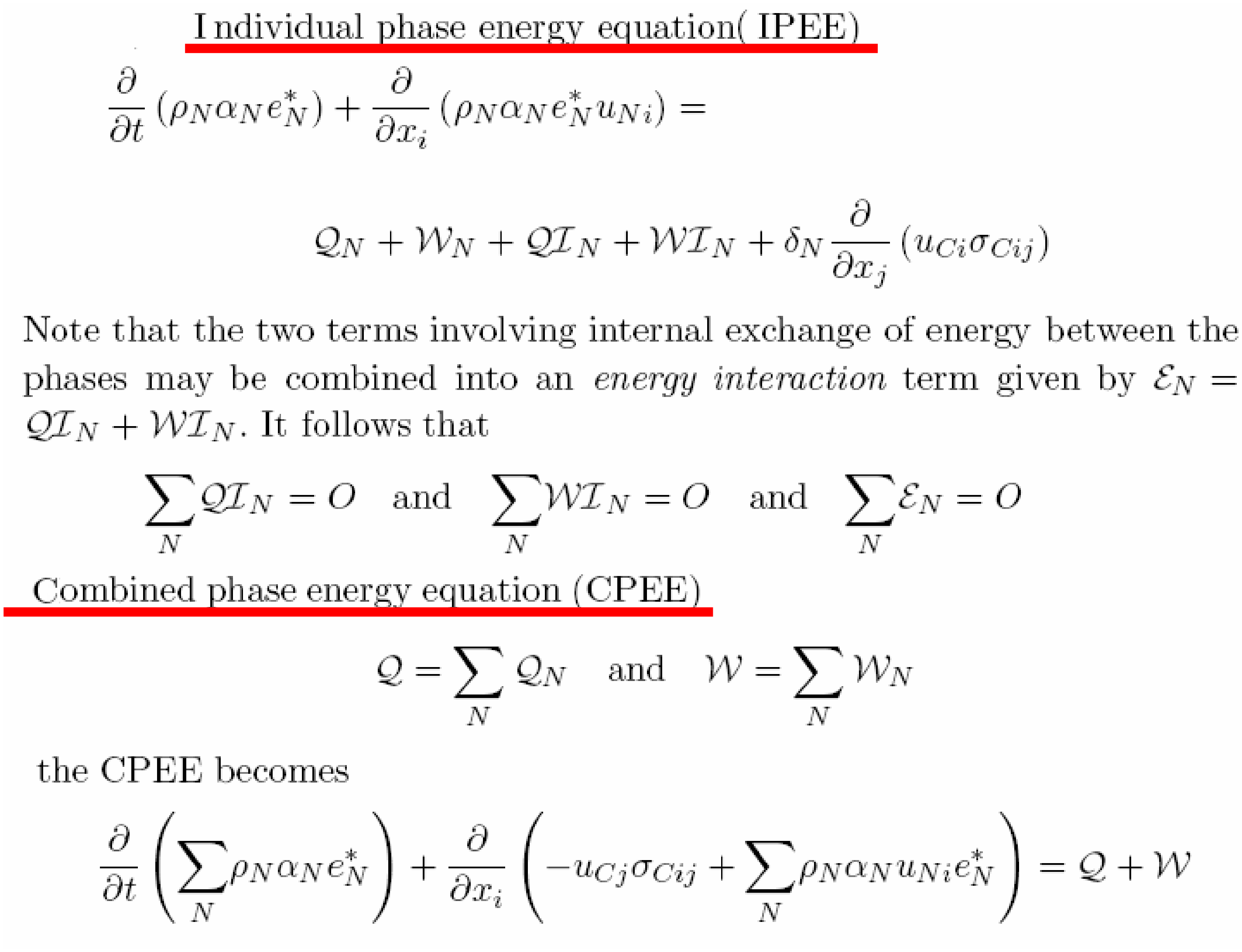
Difficulties for Energy Equation

- In single-phase flows it is usually adequate to assume that the fluid is in an equilibrium thermodynamic state at all points in the flow and that an appropriate thermodynamic constraint (for example, constant and locally uniform entropy or temperature) may be used to relate the pressure, density, temperature, entropy, …
- In many multiphase flows the different phases and/or components are often **not** in equilibrium and consequently thermodynamic equilibrium arguments that might be appropriate for single phase flows are no longer valid → Under those circumstances it is important to evaluate the heat and mass transfer occurring between the phases and/or components

Relaxation Times

- **Velocity relaxation time t_U**: it is a characteristic time that typifies the natural attenuation of velocity differences between the phases → Downstream of some disturbance that creates a relative velocity, the drag will tend to reduce that difference
- **Temperatures relaxation time t_T**: it is characteristic time associated with the equilibration of temperatures through the process of heat transfer between the phases → It can be obtained by equating the rate of heat transfer from the continuous phase to the particle with the rate of increase of heat stored in the particle: the heat transfer to the particle can occur as a result of conduction, convection, radiation,…

Individual Phase Energy Equation



Individual phase energy equation(IPEE)

$$\frac{\partial}{\partial t}(\rho_N \alpha_N e_N^*) + \frac{\partial}{\partial x_i}(\rho_N \alpha_N e_N^* u_{Ni}) =$$

$$\mathcal{Q}_N + \mathcal{W}_N + \mathcal{QI}_N + \mathcal{WI}_N + \delta_N \frac{\partial}{\partial x_j}(u_{Ci}\sigma_{Cij})$$

Note that the two terms involving internal exchange of energy between the phases may be combined into an *energy interaction* term given by $\mathcal{E}_N = \mathcal{QI}_N + \mathcal{WI}_N$. It follows that

$$\sum_N \mathcal{QI}_N = O \quad \text{and} \quad \sum_N \mathcal{WI}_N = O \quad \text{and} \quad \sum_N \mathcal{E}_N = O$$

Combined phase energy equation (CPEE)

$$\mathcal{Q} = \sum_N \mathcal{Q}_N \quad \text{and} \quad \mathcal{W} = \sum_N \mathcal{W}_N$$

the CPEE becomes

$$\frac{\partial}{\partial t}\left(\sum_N \rho_N \alpha_N e_N^*\right) + \frac{\partial}{\partial x_i}\left(-u_{Cj}\sigma_{Cij} + \sum_N \rho_N \alpha_N u_{Ni} e_N^*\right) = \mathcal{Q} + \mathcal{W}$$

Homogeneous Flows

In the absence of relative motion the governing mass and momentum conservation equations for inviscid, homogeneous flow reduce to the single-phase form,

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}(\rho u_j) = 0$$

$$\rho \left[\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \right] = -\frac{\partial p}{\partial x_i} + \rho g_i$$

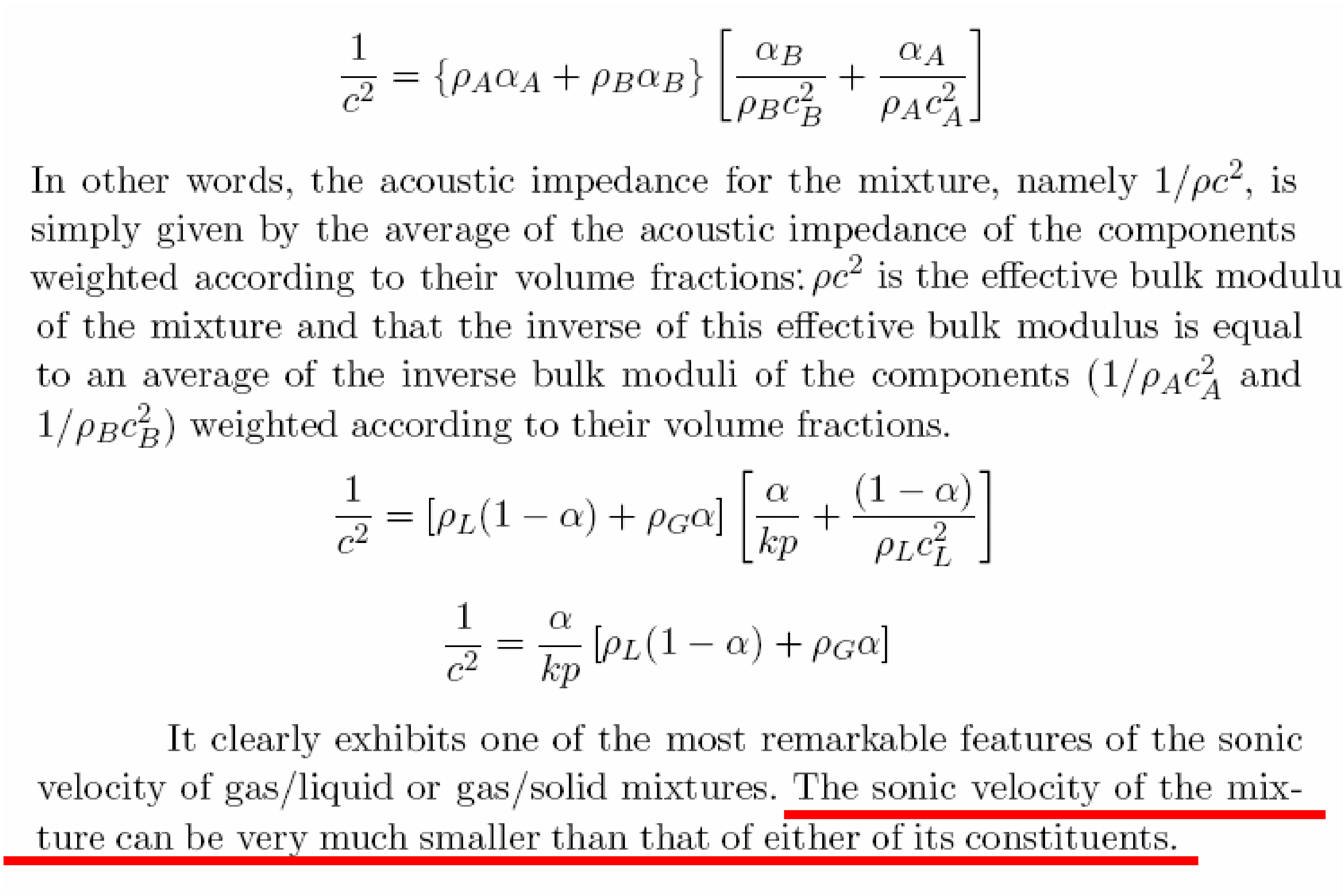
As in single phase flows the existence of a barotropic relation, $p = f(\rho)$, would complete the system of equations. In some multiphase flows it is possible to establish such a barotropic relation, and this allows one to anticipate (with, perhaps, some minor modification) that the entire spectrum of phenomena observed in single-phase gas dynamics can be expected in such a two-phase flow.

Attention will be confined to the identification of a barotropic relation (if any) and focused on some flows in which there are major departures from the conventional gas dynamic behavior.

Condensation Shocks

Condensation *fronts* in the flow around a transonic F/A-18 Hornet operating in humid conditions. U.S. Navy photograph by Ensign John Gay.

Non – Ideal Phenomena



$$\frac{1}{c^2} = \{\rho_A\alpha_A + \rho_B\alpha_B\}\left[\frac{\alpha_B}{\rho_B c_B^2} + \frac{\alpha_A}{\rho_A c_A^2}\right]$$

In other words, the acoustic impedance for the mixture, namely $1/\rho c^2$, is simply given by the average of the acoustic impedance of the components weighted according to their volume fractions: ρc^2 is the effective bulk modulus of the mixture and that the inverse of this effective bulk modulus is equal to an average of the inverse bulk moduli of the components ($1/\rho_A c_A^2$ and $1/\rho_B c_B^2$) weighted according to their volume fractions.

$$\frac{1}{c^2} = [\rho_L(1-\alpha) + \rho_G\alpha]\left[\frac{\alpha}{kp} + \frac{(1-\alpha)}{\rho_L c_L^2}\right]$$

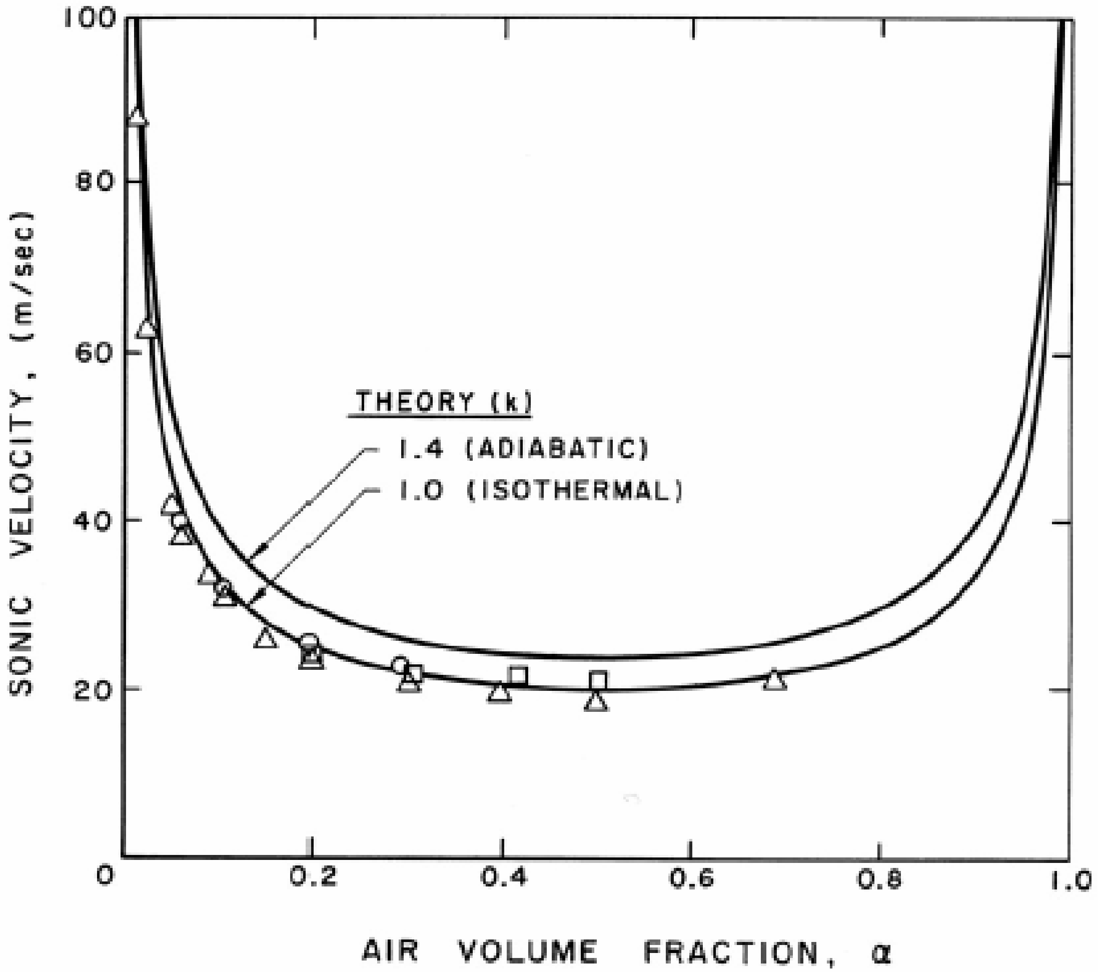
$$\frac{1}{c^2} = \frac{\alpha}{kp}[\rho_L(1-\alpha) + \rho_G\alpha]$$

It clearly exhibits one of the most remarkable features of the sonic velocity of gas/liquid or gas/solid mixtures. The sonic velocity of the mixture can be very much smaller than that of either of its constituents.

Mixture Sound Speed

Water (L)

Air (G)



Continuous Phase Models in Fluent®

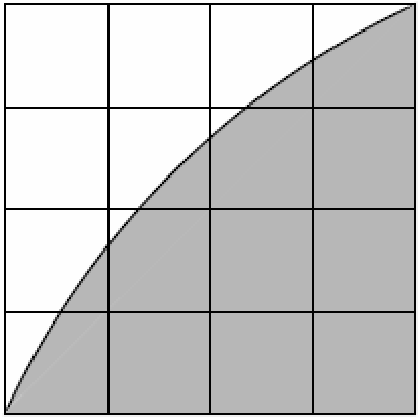
- **Volume of Fluid (VOF) Model**, which is preferable for stratified or free-surface flows
- **Mixture Model**, which is preferable if there is a wide distribution of the dispersed phases (i.e., if the particles vary in size and the largest particles do not separate from the primary flow field), because it is less computationally expensive
- **Eulerian Model**, which is preferable if interphase drag laws are known for the considered application, even though this approach requires a larger set of equations to be solved and hence it implies a larger computational demand

Volume of Fluid (VOF) Model

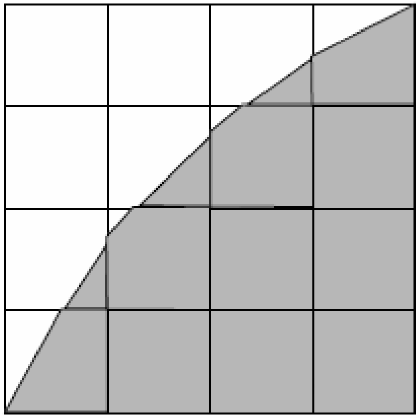
- The VOF model can model two or more immiscible fluids by solving a single set of momentum equations and tracking the volume fraction of each of the fluids throughout the domain
- The VOF formulation relies on the fact that two or more fluids (or phases) are not interpenetrating → For each additional phase that you add to your model, a variable is introduced: the volume fraction of the phase in the computational cell (in each control volume, the volume fractions of all phases sum to unity)

$$\frac{1}{\rho_{\mathrm{q}}}\left[\frac{\partial}{\partial t}(\alpha_q\rho_q)+\nabla\cdot(\alpha_q\rho_q\vec{v}_q)=S_{\alpha_q}+\sum_{p=1}^{n}(\dot{m}_{\mathrm{pq}}-\dot{m}_{\mathrm{qp}})\right]$$

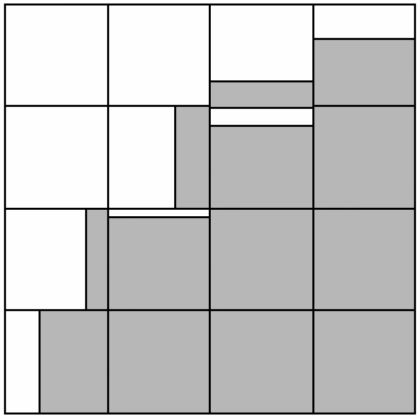
VOF: Interpolation near the Interface



actual interface shape



interface shape represented by the geometric reconstruction (piecewise-linear) scheme



interface shape represented by the donor-acceptor scheme

Mixture Model

- The mixture model, like the VOF model, uses a single–fluid approach. It differs from the VOF model in two respects:
  - the mixture model allows the phases to be interpenetrating → the volume fractions q and p for a control volume can therefore be equal to any value between 0 and 1;
  - the mixture model allows the phases to move at different velocities, using the concept of slip (or drift) velocities (note that the phases can also be assumed to move at the same velocity, and the mixture model is then reduced to a homogeneous multiphase model).

$$\frac{\partial}{\partial t}(\rho_m \vec{v}_m) + \nabla\cdot(\rho_m \vec{v}_m \vec{v}_m) = -\nabla p + \nabla\cdot\left[\mu_m\left(\nabla\vec{v}_m + \nabla\vec{v}_m^{\mathrm{T}}\right)\right] +$$

$$\rho_m\vec{g} + \vec{F} + \nabla\cdot\boxed{\left(\sum_{k=1}^{n}\alpha_k\rho_k\vec{v}_{\mathrm{dr},k}\vec{v}_{\mathrm{dr},k}\right)} \qquad \vec{v}_{\mathrm{dr},k} = \vec{v}_k - \vec{v}_m$$

Drift Velocity

$$\vec{v}_{\mathrm{dr},p} = \vec{v}_{pq} - \sum_{k=1}^{n} c_k \vec{v}_{qk}$$

$$\vec{v}_{pq} = \vec{v}_p - \vec{v}_q \qquad c_k = \frac{\alpha_k \rho_k}{\rho_m} \qquad \vec{v}_{\mathrm{dr},k} = \vec{v}_k - \vec{v}_m$$

the relative velocity is given by:

$$\boxed{\vec{v}_{pq} = \frac{\tau_p}{f_{\mathrm{drag}}} \frac{(\rho_p - \rho_m)}{\rho_p} \vec{a}}$$

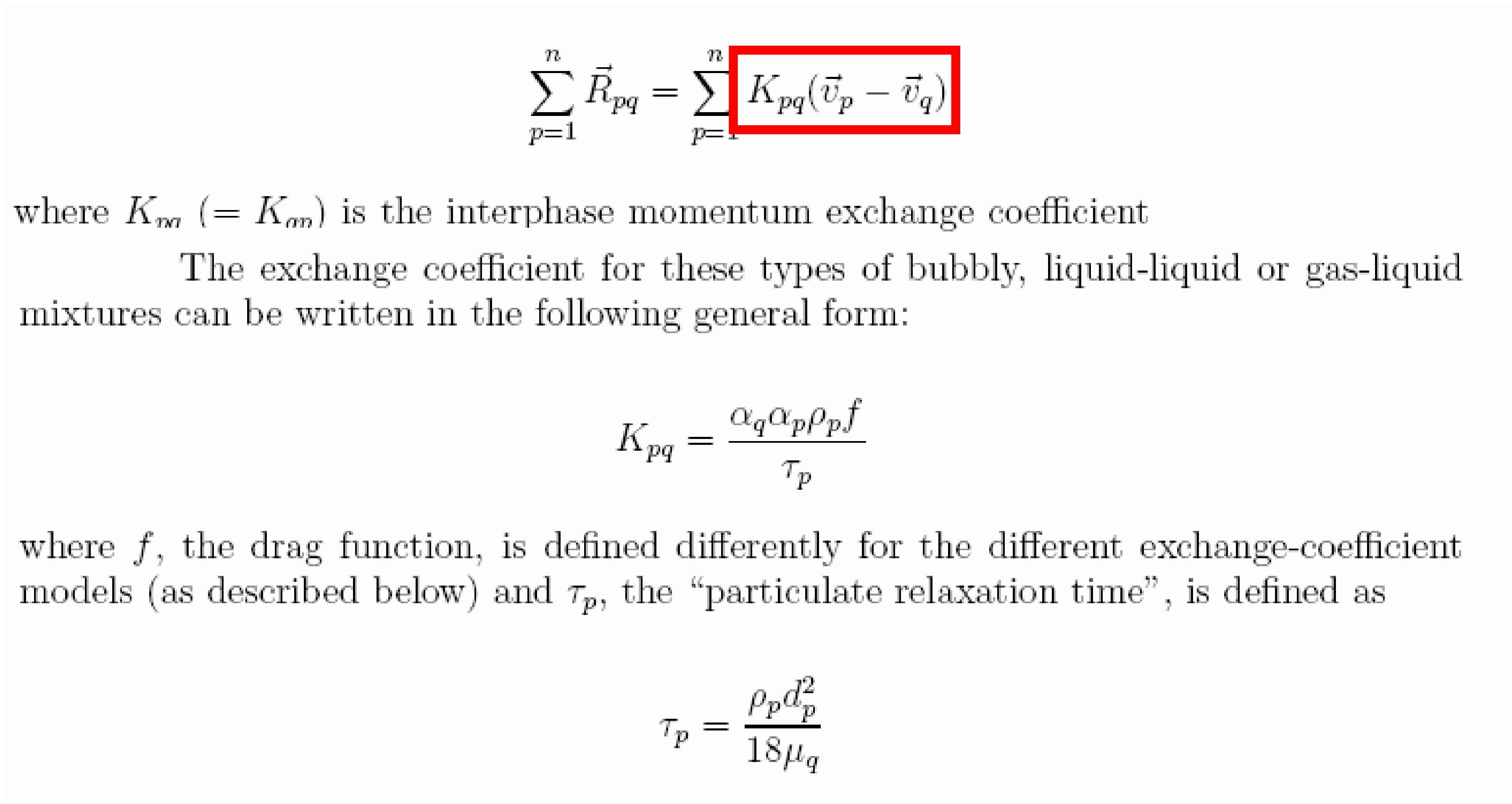
where τ_p is the particle relaxation time

$$\tau_p = \frac{\rho_p d_p^2}{18 \mu_q}$$

Eulerian Model

- The Eulerian multiphase model allows for the modeling of multiple separate, yet interacting phases. The essential features of this model are:
  - a set of conservation equations for momentum, continuity and (optionally) energy is individually solved for each phase;
  - the single phase volume fractions are introduced and some proper scalar equations are considered (individual phase continuity equations);
  - some proper coupling terms are introduced in the previous equations among the phases → the mechanisms for the exchange of momentum, heat, and mass between the phases are modeled by means of phenomenological correlations (interphase exchange coefficients)

Interphase Exchange Laws



$$\sum_{p=1}^{n} \vec{R}_{pq} = \sum_{p=1}^{n} \boxed{K_{pq}(\vec{v}_p - \vec{v}_q)}$$

where K_{pq} $(= K_{qp})$ is the interphase momentum exchange coefficient

The exchange coefficient for these types of bubbly, liquid-liquid or gas-liquid mixtures can be written in the following general form:

$$K_{pq} = \frac{\alpha_q \alpha_p \rho_p f}{\tau_p}$$

where f, the drag function, is defined differently for the different exchange-coefficient models (as described below) and τ_p, the "particulate relaxation time", is defined as

$$\tau_p = \frac{\rho_p d_p^2}{18\mu_q}$$

Summary

- A fully satisfactory description and understanding of multiphase flows is still missing and this explains the large set of phenomenological correlations used to characterize different flows in different geometrical configurations
- The key problems are the following:
  - different phases and/or components are often not in equilibrium (non negligible relaxation times), and this rises difficulties in evaluating the heat and mass transfer occurring among the phases (usual thermodynamic assumptions may not apply);
  - moreover, flow properties, such as the continuous phase velocity, vary significantly from point to point and averaged flow properties are not suitable to compute the mean of the gradients

Further Readings

- V.P. Carey, Liquid – Vapor Phase Change Phenomena, Taylor & Francis, 1984.
- C.E. Brennen, Fundamentals of Multiphase Flow, Cambridge University Press, 2005. http://caltechbook.library.caltech.edu/51/01/content.htm
- M.L. Corradini, Fundamentals of Multiphase Flow, Department of Engineering Physics University of Wisconsin, USA. http://wins.engr.wisc.edu/teaching/mpfBook

Acknowledgements

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