Quantitative Methods

An Introduction for Business Management

Paolo Brandimarte

A Wiley-Interscience Publication
JOHN WILEY & SONS, INC.
New York / Chichester / Weinheim / Brisbane / Singapore / Toronto

Contents

Preface *xix*

Part I Motivations and Foundations

1 Quantitative Methods: Should We Bother? *3*
1.1 A decision problem without uncertainty: product mix *6*
1.1.1 The case of similar products *6*
1.1.2 The case of heterogeneous products *10*
1.2 The role of uncertainty *17*
1.2.1 A problem in supply chain management *17*
1.2.2 Squeezing information out of known facts *25*
1.2.3 Variations on coin flipping *28*
1.3 Endogenous vs. exogenous uncertainty: Are we alone? *35*
1.3.1 The effect of organized markets: pricing a forward contract *38*
1.4 Quantitative models and methods *40*
1.4.1 Descriptive vs. prescriptive models *41*

1.5 Quantitative analysis and problem solving *42*
Problems *43*
For further reading *44*
References *45*

2 Calculus *47*
2.1 A motivating example: economic order quantity *49*
2.1.1 Task 1: representing the total cost function *52*
2.1.2 Task 2: plotting the total cost function *53*
2.1.3 Task 3: finding the best decision *55*
2.2 A little background *56*
2.2.1 Real vs. integer numbers *56*
2.2.2 Intervals on the real line *57*
2.2.3 The sum notation *58*
2.2.4 Permutations and combinations *59*
2.3 Functions *63*
2.3.1 Linear functions *65*
2.3.2 Polynomial functions *67*
2.3.3 Rational functions *68*
2.3.4 Exponential functions *71*
2.4 Continuous functions *75*
2.5 Composite functions *76*
2.6 Inverse functions *79*
2.6.1 The logarithm *81*
2.7 Derivatives *82*
2.7.1 Definition of the derivative *83*
2.7.2 Continuity and differentiability *86*
2.8 Rules for calculating derivatives *89*
2.8.1 Derivative of functions obtained by sum, multiplication, and division *91*
2.8.2 Derivative of composite functions *92*
2.8.3 Derivative of inverse functions *94*
2.9 Using derivatives for graphing functions *95*
2.10 Higher-order derivatives and Taylor expansions *98*
2.10.1 Sensitivity analysis *102*
2.11 Convexity and optimization *107*
2.11.1 Local and global optimality *107*
2.11.2 Convex sets *109*

2.11.3 Convex functions 110
2.11.4 The role of convexity 113
2.11.5 An application to economics: optimal pricing 115
2.12 Sequences and series 118
2.13 Definite integrals 121
2.13.1 Motivation: definite integrals as an area 121
2.13.2 Calculating definite integrals 123
2.13.3 Improper integrals 125
2.13.4 A business view of definite integral 126
Problems 128
For further reading 130
References 130

3 Linear Algebra 133
3.1 A motivating example: binomial option pricing 134
3.2 Solving systems of linear equations 141
3.2.1 Substitution of variables 143
3.2.2 Gaussian elimination 143
3.2.3 Cramer's rule 145
3.3 Vector algebra 148
3.3.1 Operations on vectors 149
3.3.2 Inner products and norms 153
3.3.3 Linear combinations 155
3.4 Matrix algebra 157
3.4.1 Operations on matrices 158
3.4.2 The identity matrix and matrix inversion 161
3.4.3 Matrices as mappings on vector spaces 162
3.4.4 Laws of matrix algebra 164
3.5 Linear spaces 165
3.5.1 Spanning sets and market completeness 166
3.5.2 Linear independence, dimension, and basis of a linear space 168
3.5.3 Matrix rank 170
3.6 Determinant 172
3.6.1 Determinant and matrix inversion 173
3.6.2 Properties of the determinant 174
3.7 Eigenvalues and eigenvectors 174

3.7.1 Eigenvalues and eigenvectors of a symmetric matrix 177
3.8 Quadratic forms 180
3.9 Calculus in multiple dimensions 183
3.9.1 Partial derivatives: gradient and Hessian matrix 184
3.9.2 Taylor's expansion for multivariable functions 187
3.9.3 Integrals in multiple dimensions 189
Problems 191
For further reading 192
References 193

Part II Elementary Probability and Statistics

4 Descriptive Statistics: On the Way to Elementary Probability 197
4.1 What is statistics? 198
4.1.1 Types of variable 200
4.2 Organizing and representing raw data 203
4.3 Summary measures 209
4.3.1 Location measures: mean, median, and mode 209
4.3.2 Dispersion measures 214
4.4 Cumulative frequencies and percentiles 219
4.4.1 Quartiles and boxplots 225
4.5 Multidimensional data 226
Problems 229
For further reading 230
References 231

5 Probability Theories 233
5.1 Different concepts of probability 234
5.2 The axiomatic approach 238
5.2.1 Sample space and events 239
5.2.2 The algebra of events 240
5.2.3 Probability measures 242
5.3 Conditional probability and independence 244

5.4 Total probability and Bayes' theorems *248*
Problems *253*
For further reading *253*
References *254*

6 Discrete Random Variables *255*
6.1 Random variables *256*
6.2 Characterizing discrete distributions *258*
6.2.1 Cumulative distribution function *259*
6.2.2 Probability mass function *261*
6.3 Expected value *263*
6.3.1 Expected value vs. mean *263*
6.3.2 Properties of expectation *265*
6.3.3 Expected value of a function of a random variable *266*
6.4 Variance and standard deviation *268*
6.4.1 Properties of variance *269*
6.5 A few useful discrete distributions *272*
6.5.1 Empirical distributions *272*
6.5.2 Discrete uniform distribution *273*
6.5.3 Bernoulli distribution *278*
6.5.4 Geometric distribution *281*
6.5.5 Binomial distribution *283*
6.5.6 Poisson distribution *287*
Problems *289*
For further reading *290*
References *291*

7 Continuous Random Variables *293*
7.1 Building intuition: from discrete to continuous random variables *295*
7.2 Cumulative distribution and probability density functions *299*
7.3 Expected value and variance *301*
7.4 Mode, median, and quantiles *302*
7.4.1 Mode *303*
7.4.2 Median and quantiles for continuous random variables *303*
7.4.3 Quantiles for discrete random variables *305*

7.4.4 An application: the newsvendor problem again 306
7.5 Higher-order moments, skewness, and kurtosis 309
7.6 A few useful continuous probability distributions 312
7.6.1 Uniform distribution 312
7.6.2 Triangular and beta distributions 313
7.6.3 Exponential distribution 314
7.6.4 Normal distribution and its quantiles 317
7.6.5 Empirical distributions 323
7.7 Sums of independent random variables 324
7.7.1 The square-root rule 327
7.7.2 Distributions obtained from the normal 328
7.7.3 Central limit theorem 332
7.7.4 The law of large numbers: a preview 333
7.8 Miscellaneous applications 335
7.8.1 The newsvendor problem with normal demand 335
7.8.2 Setting the reorder point in inventory control 336
7.8.3 An application to finance: value at risk (VaR) 337
7.9 Stochastic processes 338
7.10 Probability spaces, measurability, and information 342
Problems 348
For further reading 350
References 350

8 Dependence, Correlation, and Conditional Expectation 353
8.1 Joint and marginal distributions 354
8.2 Independent random variables 359
8.3 Covariance and correlation 360
8.3.1 A few properties of covariance 363
8.3.2 Sums of random variables 363
8.3.3 The correlation coefficient 366
8.4 Jointly normal variables 368
8.5 Conditional expectation 371
8.5.1 Computing expectations by conditioning 373

8.5.2 The memoryless property of the exponential distribution 375
8.5.3 Markov processes 378
Problems 379
For further reading 380
References 381

9 Inferential Statistics 383
9.1 Random samples and sample statistics 385
9.1.1 Sample mean 387
9.1.2 Sample variance 391
9.2 Confidence intervals 393
9.2.1 A counterexample 396
9.2.2 An important remark about confidence levels 399
9.2.3 Setting the sample size 400
9.3 Hypothesis testing 402
9.3.1 One-tail tests 409
9.3.2 Testing with p-values 412
9.4 Beyond the mean of one population 414
9.4.1 Testing hypotheses about the difference in the mean of two populations 415
9.4.2 Estimating and testing variance 419
9.4.3 Estimating and testing proportions 422
9.4.4 Estimating covariance and related issues 424
9.4.5 Correlation analysis: testing significance and potential dangers 429
9.4.6 Estimating skewness and kurtosis 433
9.5 Checking the fit of hypothetical distributions: the chi-square test 435
9.6 Analysis of variance 436
9.6.1 One-way ANOVA 437
9.6.2 Two-way ANOVA 440
9.7 Monte Carlo simulation 443
9.7.1 Discrete-event vs. discrete-time simulation 446
9.7.2 Random-number generation 449
9.7.3 Methodology of a simulation study 451

9.8 Stochastic convergence and the law of large numbers 452
9.8.1 Convergence in probability 453
9.8.2 Convergence in quadratic mean 455
9.8.3 Convergence in distribution 456
9.8.4 Almost-sure convergence 456
9.8.5 Slutsky's theorems 459
9.9 Parameter estimation 460
9.9.1 Features of point estimators 461
9.9.2 The method of moments 462
9.9.3 The method of maximum likelihood 463
9.10 Some more hypothesis testing theory 466
9.10.1 Size and power of a test 467
9.10.2 Likelihood ratio tests 468
Problems 470
For further reading 473
References 474

10 Simple Linear Regression 477
10.1 Least-squares method 479
10.1.1 Alternative approaches for model fitting 482
10.1.2 What is linear, exactly? 484
10.2 The need for a statistical framework 484
10.3 The case of a nonstochastic regressor 490
10.3.1 Properties of estimators 491
10.3.2 The standard error of regression 497
10.3.3 Statistical inferences about regression parameters 498
10.3.4 The R^2 coefficient and ANOVA 502
10.3.5 Analysis of residuals 509
10.4 Using regression models 512
10.5 A glimpse of stochastic regressors and heteroskedastic errors 516
10.6 A vector space look at linear regression 521
Problems 524
For further reading 525
References 526

11 Time Series Models 527

11.1 Before we start: framing the forecasting process 530
11.2 Measuring forecast errors 532
11.2.1 In- and out-of-sample checks 536
11.3 Time series decomposition 537
11.4 Moving average 542
11.4.1 Choice of time window 545
11.5 Heuristic exponential smoothing 546
11.5.1 Stationary demand: three views of a smoother 547
11.5.2 Stationary demand: initialization and choice of α 550
11.5.3 Smoothing with trend 555
11.5.4 Smoothing with multiplicative seasonality 557
11.5.5 Smoothing with trend and multiplicative seasonality 560
11.6 A glance at advanced time series modeling 561
11.6.1 Moving-average processes 564
11.6.2 Autoregressive processes 566
11.6.3 ARMA and ARIMA processes 570
11.6.4 Using time series models for forecasting 575
Problems 577
For further reading 578
References 579

Part III Models for Decision Making

12 Deterministic Decision Models 583
12.1 A taxonomy of optimization models 584
12.1.1 Linear programming problems 590
12.1.2 Nonlinear programming problems 598
12.1.3 Convex programming: difficult vs. easy problems 601
12.2 Building linear programming models 608
12.2.1 Production planning with assembly of components 611
12.2.2 A dynamic model for production planning 616
12.2.3 Blending models 620
12.2.4 Network optimization 624

12.3 A repertoire of model formulation tricks 627
12.3.1 Alternative regression models 628
12.3.2 Goal programming 630
12.3.3 Multiobjective optimization 631
12.3.4 Elastic model formulations 635
12.3.5 Column-based model formulations 637
12.4 Building integer programming models 638
12.4.1 Knapsack problem 639
12.4.2 Modeling logical constraints 640
12.4.3 Fixed-charge problem and semicontinuous decision variables 641
12.4.4 Lot-sizing with setup times and costs 643
12.4.5 Plant location 644
12.4.6 An optimization model for portfolio tracking and compression 646
12.4.7 Piecewise linear functions 649
12.5 Nonlinear programming concepts 653
12.5.1 The case of equality constraints: Lagrange multipliers 654
12.5.2 Dealing with inequality constraints: Karush–Kuhn–Tucker conditions 659
12.5.3 An economic interpretation of Lagrange multipliers: shadow prices 664
12.6 A glance at solution methods 670
12.6.1 Simplex method 670
12.6.2 LP-based branch and bound method 677
12.6.3 The impact of model formulation 683
Problems 687
For further reading 690
References 691

13 Decision Making Under Risk 693
13.1 Decision trees 694
13.1.1 Expected value of perfect information 698
13.2 Risk aversion and risk measures 701
13.2.1 A conceptual tool: the utility function 703
13.2.2 Mean–risk optimization 708
13.2.3 Quantile-based risk measures: value at risk 712

13.3 Two-stage stochastic programming models 718
13.3.1 A two-stage model: assembly-to-order production planning 721
13.3.2 The value of the stochastic solution 726
13.3.3 A mean–risk formulation of the assembly-to-order problem 727
13.4 Multistage stochastic linear programming with recourse 728
13.4.1 A multistage model: asset–liability management 730
13.4.2 Asset–liability management with transaction costs 734
13.4.3 Scenario generation for stochastic programming 736
13.5 Robustness, regret, and disappointment 737
13.5.1 Robust optimization 738
13.5.2 Disappointment and regret in decision making 739
Problems 742
For further reading 746
References 747

14 Multiple Decision Makers, Subjective Probability, and Other Wild Beasts 751
14.1 What is uncertainty? 753
14.1.1 The standard case: decision making under risk 754
14.1.2 Uncertainty about uncertainty 755
14.1.3 Do black swans exist? 756
14.1.4 Is uncertainty purely exogenous? 757
14.2 Decision problems with multiple decision makers 758
14.3 Incentive misalignment in supply chain management 760
14.4 Game theory 766
14.4.1 Games in normal form 767
14.4.2 Equilibrium in dominant strategies 769
14.4.3 Nash equilibrium 773
14.4.4 Simultaneous vs. sequential games 776
14.5 Braess' paradox for traffic networks 781
14.6 Dynamic feedback effects and herding behavior 783

14.7 Subjective probability: the Bayesian view 785
14.7.1 Bayesian estimation 786
14.7.2 A financial application: The Black–Litterman model 792
Problems 798
For further reading 799
References 800

Part IV Advanced Statistical Modeling

15 Introduction to Multivariate Analysis 805
15.1 Issues in multivariate analysis 805
15.1.1 Visualization 806
15.1.2 Complexity and redundancy 806
15.1.3 Different types of variables 807
15.1.4 Adapting statistical inference procedures 807
15.1.5 Missing data and outliers 809
15.2 An overview of multivariate methods 811
15.2.1 Multiple regression models 812
15.2.2 Principal component analysis 812
15.2.3 Factor analysis 813
15.2.4 Cluster analysis 813
15.2.5 Canonical correlation 813
15.2.6 Discriminant analysis 814
15.2.7 Structural equation models with latent variables 815
15.2.8 Multidimensional scaling 816
15.2.9 Correspondence analysis 817
15.3 Matrix algebra and multivariate analysis 817
15.3.1 Covariance matrices 819
15.3.2 Measuring distance and the Mahalanobis transformation 821
For further reading 823
References 824

16 Advanced Regression Models 825
16.1 Multiple linear regression by least squares 826

16.2 Building, testing, and using multiple linear regression models 828
16.2.1 Selecting explanatory variables: collinearity 828
16.2.2 Testing a multiple regression model 832
16.2.3 Using regression for forecasting and explanation purposes 835
16.3 Logistic regression 837
16.3.1 A digression: logit and probit choice models 839
16.4 A glance at nonlinear regression 842
16.4.1 Polynomial regression 844
16.4.2 Data transformations 845
Problems 846
For further reading 846
References 846

17 Dealing with Complexity: Data Reduction and Clustering 849
17.1 The need for data reduction 850
17.2 Principal component analysis (PCA) 851
17.2.1 A geometric view of PCA 852
17.2.2 Another view of PCA 854
17.2.3 A small numerical example 855
17.2.4 Applications of PCA 857
17.3 Factor analysis 859
17.4 Cluster analysis 863
17.4.1 Measuring distance 863
17.4.2 Hierarchical methods 865
17.4.3 Nonhierarchical clustering: k-means 866
For further reading 867
References 868

Index 869

Preface

And there I was, waiting for the big door to open, the big door that stood between me and my archnemesis. I found little comfort and protection, if any, sitting in what seemed my thin tin tank, looking around and searching for people in my same dire straits. Then, with a deep rumble, the big steel door of the ship opened, engines were started, and I followed the slow stream of cars. I drove by rather uninterested police officers, and there it was, my archnemesis: the first roundabout in Dover.

For European continental drivers like me, used to drive on the right side of the street (and yes, I do mean *right*), the first driving experience in the Land of Albion has always been a challenge. That difficulty compounded with the lack of roundabouts in Italy at the time, turning the whole thing into sheer nightmare. Yet, after a surprisingly short timespan, maybe thanks to the understanding and discipline of the indigenous drivers, I got so used to driving there, and to roundabouts as well, that after my return to Calais I found driving back in supposedly familiar lanes somewhat confusing.

I had overcome my fear, but I am digressing, am I? Well, this book should indeed be approached like a roundabout: There are multiple entry and exit points, and readers are expected to take their preferred route among the many options, possibly spinning a bit for fun. I should also mention that, however dreadful that driving experience was to me, it was nothing compared with the exam labor of my students of the terrifying quantitative methods course. I hope that this book will help them, and many others, to overcome their fear. By the same token, I believe that the book will be useful to practitioners as well, especially those using data analysis and decision support software packages, possibly in need of a better understanding of those black boxes.

I have a long teaching experience at Politecnico di Torino, in advanced courses involving the application of quantitative methods to production planning, logistics, and finance. A safe spot, indeed, with a fairly homogeneous population of students. Add to this the experience in teaching numerical methods in quantitative finance master's programs, with selected and well-motivated students. So, you may imagine my shock when challenged by more generic and basic courses within a business school (ESCP Europe, Turin Campus), which I started teaching a few years ago. The subject was quite familiar, quantitative methods, with much emphasis on statistics and data analysis.

However, the audience was quite different, as the background of my new students ranged from literature to mathematics/engineering, going through law and economics. When I wondered about how not to leave the whole bunch utterly disappointed, the "mission impossible" theme started ringing in my ears. I must honestly say that the results have been occasionally disappointing, despite my best efforts to make the subject a bit more exciting through the use of business cases, a common mishap for teachers of technical subjects at business schools. Yet, quite often I was delighted to see apparently hopeless students struggle, find their way, and finally pass the exam with quite satisfactory results. Other students, who had a much stronger quantitative background, were nevertheless able to discover some new twists in familiar topics, without getting overly bored. On the whole, I found that experience challenging and rewarding.

On the basis of such disparate teaching experiences, this possibly overambitious book tries to offer to a hopefully wide range of readers whatever they need.

- **Part I** consists of three chapters. Chapter 1 aims at motivating the skeptical ones. Then, I have included two chapters on calculus and linear algebra. Advanced readers will probably skip them, possibly referring back to refresh a few points just when needed, whereas other students will not be left behind. Not all the material provided there is needed; in particular, the second half of Chapter 3 on linear algebra is only necessary to tackle Parts III and IV.

- **Part II** corresponds to the classical core of a standard quantitative methods course. Chapters 4–10 deal with introductory topics in probability and statistics. Readers can tailor their way through this material according to their taste. Especially in later chapters, they can safely skip more technical sections, which are offered to more mathematically inclined readers. Both Chapter 9, on inferential statistics, and Chapter 10, on linear regression, include basic and advanced sections, bridging the gap between cookbook-oriented texts and the much more demanding ones. Also Chapter 11, on time series, consists of two parts. The first half includes classical topics such as exponential smoothing methods; the second half introduces the reader to more challenging models and is included to help readers bridge the gap with the more advanced literature without getting lost or intimidated.

- **Part III** moves on to decision models. Quite often, a course on quantitative methods is declined in such a way that it could be renamed as "business statistics," possibly including just a scent of decision trees. In my opinion, this approach is quite limited. Full-fledged decision models should find their way into the education of business students and professionals. Indeed, statistics and operations research models have too often led separate lives within academia, but they do live under the same

roof in the new trend that has been labeled "business analytics." Chapter 12 deals mostly with linear programming, with emphasis on model building; some knowledge on how these problems are actually solved, and which features make them computationally easy or hard, is also provided, but we do not certainly cover solution methods in detail, as quite robust software packages are widely available. This part also relies more heavily on the advanced sections of Chapters 2 and 3. Chapter 13 is quite important, as it merges all previous chapters into the fundamental topic of decision making under risk. Virtually all interesting business management problems are of this nature, and the integration of separate topics is essential from a pedagogical point of view. Chapter 14 concludes Part III with some themes that are unusual in a book at this level. Unlike previous chapters, this is more of an eye-opener, as it outlines a few topics, like game theory and Bayesian statistics, which are quite challenging and can be covered adequately only in dedicated books. The message is that no one should have blind faith in fact-based decisions. A few examples and real-life cases are used to stimulate critical thinking. This is not to say that elementary techniques should be disregarded; on the contrary, they must be mastered in order to fully understand their limitations and to use them consciously in real-life settings. We should always keep in mind that all models are wrong (G.E.P. Box), but some are *useful*, and that nothing is as practical as a *good* theory (J.C. Maxwell).

- **Part IV** completes the picture by introducing selected tools from multivariate statistics. Chapter 15 introduces the readers to the challenges and the richness of this field. Among the many topics, I have chosen those that are more directly related with the previous parts of the book, i.e., advanced regression models in Chapter 16, including multiple linear, logistic, and nonlinear regression, followed in Chapter 17 by data reduction methods, like principal component analysis, factor analysis, and cluster analysis. There is no hope to treat these topics adequately in such a limited space, but I do believe that readers will appreciate the relevance of the basics dealt with in earlier chapters; they will hopefully gain a deeper understanding of these widely available methods, which should not just be used as software black boxes.

Personally, I do not believe too much in books featuring a lot of simple and repetitive exercises, as they tend to induce a false sense of security. On the other hand, there is little point in challenging students and practitioners with overly complicated problems. I have tried to strike a fair compromise, by including a few of them to reinforce important points and to provide readers with some more worked-out examples. The solutions, as well as additional problems, will be posted on the book Webpage.

On the whole, this is a book about fact- and evidence-based decision making. The availability of information-technology-based data infrastructures has

made it a practically relevant tool for business management. However, this is not to say that the following simple-minded equation holds:

$$Data = Decisions$$

This would be an overly simplistic view. To begin with, there are settings in which we do not have enough data, because they are hard or costly to collect, or simply because they are not available; think of launching a brand-new and path-breaking product or service. In these cases, knowledge, under the guise of subjective assessments or qualitative insights, comes into play. Yet, some discipline is needed to turn gut feelings into something useful. Even without considering these extremes, it is a fact that knowledge is needed to turn rough data into *information*. Hence, the equation above should be rephrased as

$$Data + Knowledge = Decisions$$

Knowledge includes plenty of things that are not treated here, such as good and sensible intuition or the ability to work in a team, which must be learned on the field. I should also mention that, in my teaching, the discussion of business cases and the practical use of software tools play a pivotal role, but cannot be treated in a book like this. Yet, I believe that an integrated view of quantitative methods, resting on solid but not pedantic foundations, is a fundamental asset for both students and practitioners.

Use of software. In writing this book, a deliberate choice has been not to link it with any software tool, even though the application of quantitative methods does require such a support in practice.[1] One the one hand, whenever you select a specific tool, you lose a share of readers. On the other hand, there is no single software environment adequately covering the wide array of methods discussed in the book. Microsoft Excel is definitely a nice environment for introducing quantitative modeling, but when it comes, e.g., to complex optimization models, its bidimensional nature is a limitation; furthermore, only dedicated products are able to cope with large-scale, real-life models. For the reader's convenience, we offer a nonexhaustive list of useful tools:

- MATLAB (`http://www.mathworks.com/`) is a numerical computing environment, including statistics and optimization toolboxes.[2] Indeed, many diagrams in the book have been produced using MATLAB (and a few using Excel).

[1]The software environments that are mentioned here are copyrights and/or trademarks of their owners. Please refer to the listed Websites.
[2]The virtues of MATLAB are well illustrated in my other book: P. Brandimarte, *Numerical Methods in Finance and Economics: A MATLAB-Based Introduction*, 2nd. ed., Wiley, New York, 2006.

- Stata (`http://www.stata.com/`) and SAS (`http://www.sas.com/`) are examples of rich software environments for statistical data analysis and business intelligence.

- Gurobi (`http://www.gurobi.com/`) is an example of a state-the-art optimization solver, which is necessary when you have to tackle a large-scale, possibly mixed-integer, optimization model.

- AMPL (`http://www.ampl.com/`) is a high-level algebraic modeling language for expressing optimization models in a quite natural way. A tool like AMPL provides us with an interface to optimization solvers, such as Gurobi and many others. Using this interface, we can easily write and maintain a complex optimization model, without bothering about low-level data structures. We should also mention that a free student version is available on the AMPL Website.

- COIN-OR (`http://www.coin-or.org/`) is a project aimed at offering a host of free software tools for Operations Research. Given the cost of commercial licenses, this can be a welcome resource for students.

- By a similar token, the R project (`http://www.r-project.org/`) offers a free software tool for statistics, which is continuously enriched by free libraries aimed at specific groups of statistical methods (time series, Bayesian statistics, etc.).

Depending on readers' feedback, I will include illustrative examples, using some of the aforementioned software packages, on the book Website. Incidentally, unlike other textbooks, this one does not include old-style statistical tables, which do not make much sense nowadays, given the wide availability of statistical software. Nevertheless, tables will also be provided on the book Website.

Acknowledgments. Much to my chagrin, I have to admit that this book would not have been the same without the contribution of my former coauthor Giulio Zotteri. Despite his being an utterly annoying specimen of the human race, our joint teaching work at Politecnico di Torino has definitely been an influence. Arianna Alfieri helped me revise the whole manuscript; Alessandro Agnetis, Luigi Buzzacchi, and Giulio Zotteri checked part of it and provided useful feedback. Needless to say, any remaining error is their responsibility. I should also thank a couple of guys at ESCP Europe (formerly ESCP-EAP), namely, Davide Sola (London Campus) and Francesco Rattalino (Turin Campus); as I mentioned, this book is in large part an outgrowth of my lectures there. I gladly express my gratitude to the authors of the many books that I have used, when I had to learn quantitative methods myself; all of these books are included in the end-of-chapter references, together with other textbooks that helped me in preparing my courses. Some illuminating examples from these sources have been included here, possibly with some adaptation. I have

provided the original reference for (hopefully) all of them, but it might be the case that I omitted some due reference because, after so many years of teaching, I could not trace all of the original sources; if so, I apologize with the authors, and I will be happy to include the reference in the list of errata. Last but not least, the suffering of quite a few cohorts of students at both Politecnico di Torino and ESCP Europe, as well as their reactions and feedback, contributed to shape this work (and improved my mood considerably).

Supplements. A solution manual for the problems in the book, along with additional ones and computational supplements (Microsoft Excel workbooks, MATLAB scripts, and AMPL models), will be posted on a Webpage. My current URL is:

- `http://staff.polito.it/paolo.brandimarte`

A hopefully short list of errata will be posted there as well. One of the many corollaries of Murphy's law says that my URL is going to change shortly after publication of the book. An up-to-date link will be maintained on the Wiley Webpage:

- `http://www.wiley.com/`

For comments, suggestions, and criticisms, my e-mail address is

- `paolo.brandimarte@polito.it`

Paolo Brandimarte
Turin, February 2011