Errata and comments for the book: "Numerical methods in finance: a MATLAB-based introduction"

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Please refer to the web page (`www.polito.it/~brandimarte`) for updates and supplements to this list.

Any comment is welcome. My e-mail address is: `brandimarte@polito.it`.

Some errors in the list below were pointed out by Byunggyoo Kim, Enrico Moretto, and Aldo Tagliani. I gladly thank them for their help.

Page 8, equation (1.2). C_t should read C_k.

Page 52, last displayed equation. The boundary condition for the put option should read: $f(S,t) = \max\{X - S, 0\}$, and not $f(S,t) = \max\{S - X, 0\}$.

Page 53, 3rd displayed equation.

$$d1 - \sigma\sqrt{T},$$

should read

$$d_1 - \sigma\sqrt{T},$$

Page 59, line -6. The statement: *"In this way, an up-step followed by a down-step yields the same price as a down-step followed by an up-step"* is a bit misleading. This happens whatever choice you take for the parameters u and d. The choice $u = 1/d$ is simply one possible approach to supply the third condition.

Page 80, line -5. "..., but In other cases ..." should read "..., but in other cases ..."

Page 81, first displayed equation. There is a duplicated line in the definition of the Hilbert matrix $\mathbf{H}$:

$$\mathbf{H} = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \cdots & \frac{1}{n} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \cdots & \frac{1}{n+1} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \cdots & \frac{1}{n+1} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \cdots & \frac{1}{n+2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{1}{n} & \frac{1}{n+1} & \frac{1}{n+2} & \cdots & \frac{1}{2n-1} \end{bmatrix}$$

should read:

$$\mathbf{H} = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \cdots & \frac{1}{n} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \cdots & \frac{1}{n+1} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \cdots & \frac{1}{n+2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{1}{n} & \frac{1}{n+1} & \frac{1}{n+2} & \cdots & \frac{1}{2n-1} \end{bmatrix}.$$

Page 84, line 2. "... and $F_1 = \text{frac}(2F_{k-1})$" should read "... and $F_k = \text{frac}(2F_{k-1})$."

Page 117, line -5 in the `L11` function, figure 2.10. The exponential discount factor has the wrong sign: it should read `exp(-r*deltaT)` and not `exp(r*deltaT)`.

Page 99, line 2. The statement "Indeed, it can be shown that the condition number gives a measure of how close a matrix is to be singular" is a bit misleading and it should be taken with care. On the one hand, the following theorem holds for a non-singular matrix $\mathbf{A} \in \mathbb{R}^{n,n}$:

$$\frac{1}{K(\mathbf{A})} = \min \left\{ \frac{\| \mathbf{A} - \mathbf{B} \|}{\| \mathbf{A} \|} : \mathbf{B} \in \mathbb{R}^{n,n} \text{ and not singular} \right\}$$

for a compatible matrix norm. So, the condition number is, in a sense, related to the relative distance between $\mathbf{A}$ and the set of singular matrices of the same order n. However, the following examples show that the relationship between the determinant and the condition number of a matrix is not trivial.

Consider first a symmetric matrix of order $n = 101$, and assume its eigenvalues are $\lambda_1 = 1$ and $\lambda_j = 10^{-1}$ for $j = 2, \ldots, 101$. Then

$$\det(\mathbf{A}) = \prod_{i=1}^{101} \lambda_i = 10^{-100}.$$

But if we consider the condition number $K_2(\mathbf{A})$, based on the spectral norm, we have $K_2(\mathbf{A}) = \lambda_1 \cdot \lambda_2^{-1} = 10$. Hence, even if the determinant is very small, the condition number is not so large in absolute terms.

On the other hand, consider the matrix:

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & -1 & \cdots & -1 & -1 \\ 0 & 1 & -1 & \cdots & -1 & -1 \\ 0 & 0 & 1 & \cdots & -1 & -1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & -1 \\ 0 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix}$$

We have $\det(\mathbf{A}) = 1$, but the condition number of this matrix is rather large (try computing it with MATLAB). In fact, it can be shown that a lower bound for the condition number based on the $\| \cdot \|_1$ norm is 2^{n-2}, where n is the order of the matrix.

So, it is safer to say that the condition number is an upper bound on the amplification of relative errors in the data. While this bound may be overly pessimistic in some cases, it can be tight, i.e., it can shown that the inequality at the bottom of page 89 may be satisfied as an equality in some other cases.

Page 219, line 6. The statement: *"From equation (4.3) we see that the absolute error is actually the half-length"* is not quite correct.

Page 219, equation (4.5). There are two typos, in the second and fourth line:

$$\begin{aligned} 1 - \alpha &\approx P \left\{ \frac{| \bar{X}(n) - \mu |}{| \bar{X}(n) |} \leq \frac{H}{| \bar{X}(n) |} \right\} \\ &= P \left\{ | \bar{X}(n) - \mu | \leq \gamma \, | \bar{X}(n) | \right\} \end{aligned}$$

$$
\begin{aligned}
&= P\left\{\mid \bar{X}(n)-\mu \mid \leq \gamma \mid \bar{X}(n)-\mu+\mu \mid\right\} \\
&\leq P\left\{\mid \bar{X}(n)-\mu \mid \leq \gamma \mid \bar{X}(n)-\mu \mid + \mid \mu \mid\right\} \\
&= P\left\{\frac{\mid \bar{X}(n)-\mu \mid}{\mid \mu \mid} \leq \frac{\gamma}{1-\gamma}\right\},
\end{aligned}
$$

should read:

$$
\begin{aligned}
1-\alpha &\approx P\left\{\frac{\mid \bar{X}(n)-\mu \mid}{\mid \bar{X}(n) \mid} \leq \frac{H}{\mid \bar{X}(n) \mid}\right\} \\
&\leq P\left\{\mid \bar{X}(n)-\mu \mid \leq \gamma \mid \bar{X}(n) \mid\right\} \\
&= P\left\{\mid \bar{X}(n)-\mu \mid \leq \gamma \mid \bar{X}(n)-\mu+\mu \mid\right\} \\
&\leq P\left\{\mid \bar{X}(n)-\mu \mid \leq \gamma \mid \bar{X}(n)-\mu \mid +\gamma \mid \mu \mid\right\} \\
&= P\left\{\frac{\mid \bar{X}(n)-\mu \mid}{\mid \mu \mid} \leq \frac{\gamma}{1-\gamma}\right\},
\end{aligned}
$$

Page 230. The illustration of variance reduction by importance sampling in computing $F(N) = \sum_{i=1}^{N} h(x_i)$ is not very convincing. Please refer to the related supplement on the web page.

Page 257, first displayed equation. The line

$$\phi_{ij} = \phi(i\,\delta x, j\,\delta y).$$

has been duplicated.

Page 259, line 8. The statement:

> Now, using the computational scheme (5.14), for $j = 1$ we have

should read

> Now, using the computational scheme (5.14), for $j = 0$ we have

Page 270, last statement before subsection 5.3.2. An additional comment is in order. We are forced to keep δt very small because the condition $\rho \leq 0.5$ implies $\delta t \leq 0.5(\delta x)^2$. Since δx is relatively small to ensure a good approximation, $(\delta x)^2$ is even smaller.

Page 316, equations (7.2), (7.3).

$$
\begin{aligned}
\mathrm{E}[\ln S(t)/\ln S(0)] &= \nu t \\
\mathrm{Var}[\ln S(t)/\ln S(0)] &= \sigma^2 t
\end{aligned}
$$

should read

$$
\begin{aligned}
\mathrm{E}[\ln(S(t)/S(0))] &= \nu t \\
\mathrm{Var}[\ln(S(t)/S(0))] &= \sigma^2 t
\end{aligned}
$$

Page 321, last statement. The statement here is a bit debatable. While it is true that volatility may influence the difficulty in getting accurate estimates, this cannot be concluded from the results illustrated.

Page 332, figure 7.11. The first line of code:

```
[Call,Put] = blsprice(S0,X,r,T,sigma);
```

is useless and should be omitted.

Page 333, last displayed equation. The equality:

$$\mathrm{E}\left[I(\mathbf{S})(X-S_M)^+ \mid j^*, S_{j^*}\right] = e^{r(T-t^*)} B_p(S_{j^*}, X, T-t^*),$$

is true conditional on $I(\mathbf{S}) = 1$, i.e., when the barrier is crossed. This should be made clearer in the text.

Page 336, line 17. The first expression in the series of equalities should be eliminated. Substitute:

$$\begin{aligned}\mathrm{E}_f\left[I(\mathbf{S})(X-S_M)^+ \middle| j^*, S_{j^*}\right] &= \mathrm{E}_g\left[\frac{f(\mathbf{Z})I(\mathbf{S})(X-S_M)^+}{g(\mathbf{Z})} \middle| j^*, S_{j^*}\right] \\ &= \frac{f(z_1,\ldots,z_{j^*})}{g(z_1,\ldots,z_{j^*})}\,\mathrm{E}_g\left[\frac{f(Z_{j^*+1},\ldots,Z_M)}{g(Z_{j^*+1},\ldots,Z_M)} I(\mathbf{S})(X-S_M)^+ \middle| j^*, S_{j^*}\right]\end{aligned}$$

with

$$\begin{aligned}&\mathrm{E}_g\left[\frac{f(\mathbf{Z})I(\mathbf{S})(X-S_M)^+}{g(\mathbf{Z})} \middle| j^*, S_{j^*}\right] \\ &\quad= \frac{f(z_1,\ldots,z_{j^*})}{g(z_1,\ldots,z_{j^*})}\,\mathrm{E}_g\left[\frac{f(Z_{j^*+1},\ldots,Z_M)}{g(Z_{j^*+1},\ldots,Z_M)} I(\mathbf{S})(X-S_M)^+ \middle| j^*, S_{j^*}\right]\end{aligned}$$

Page 355, last displayed equation. The vector:

$$\begin{bmatrix} a_1 f_{i+1,0} \\ 0 \\ 0 \\ \vdots \\ 0 \\ c_{M-1} f_{i+1,M} \end{bmatrix} \qquad \text{should read} \qquad \begin{bmatrix} a_1 f_{i,0} \\ 0 \\ 0 \\ \vdots \\ 0 \\ c_{M-1} f_{i,M} \end{bmatrix}$$