Generalized Kinetic Models in Applied Sciences. *By Luisa Arlotti, Nicola Bellomo, Elena De Angelis, and Mirosław Lachowicz.* World Scientific, River Edge, NJ, 2003. $38.00. xii+199 pp., hardcover. ISBN 981-238-560-6.

The classical Boltzmann equation provides a statistical (let us call it mesoscopic) description of gas behavior in mathematical physics. The usual equation has the form

$$\frac{\partial f}{\partial t} + v \cdot \nabla_x f = J[f, f]. \tag{1}$$

The collision operator J has a local character w.r.t. (t, x), v is the velocity, and the unknown $f = f(t, x, v)$ represents the density of particles. The level of description provided by the Boltzmann equation builds the bridge between microscopic and macroscopic levels. In the former case the evolution of a gas is prescribed by the high-dimensional system of ODEs. In the latter, the PDE description completely neglects the microscopic velocities of the particles. Only macroscopic quantities, like density ρ, momentum ρv, and internal energy E (all of which are functions of (t, x) only), are considered. The natural assumption arising from physics on the microscopic level is that the interactions among particles are due to some short range potentials. This is related to the assumption on the local character of the collision operator at the Boltzmann (mesoscopic) level.

Contrary to this situation, in the book under consideration the authors generalize the Boltzmann equation through a nonlocal (w.r.t. x) collision operator. Such studies are motivated by biological systems where, contrary to the gas dynamics case, the "particles" or "individuals" interact "globally," with the possibility of some additional non-mechanical socio-biological variable. The considerations are made on the whole space $\mathbb{R}^3$ or on the torus $\mathbb{T}^3$, which is very natural for nonlocal but spatially homogeneous collision operators.

The monograph is split into five chapters. In the first chapter the authors introduce the averaged Boltzmann equation (i.e., with nonlocal collision operator). There is also a discussion of the definitions of the dissipative averaged Boltzmann equation and the symmetrized dissipative averaged Boltzmann equation (i.e., the bilinear collision operator is assumed to be symmetric).

The second chapter is dedicated to the existence theory for the averaged Boltzmann equation. In the classical Boltzmann equation (with a local collision term), difficulties arise with a priori estimates for the collision operator. Due to this obstacle there is no global in time existence result in the usual class of solutions. The brilliant idea of DiPerna and Lions was to introduce a new, weaker notion of solutions, so-called renormalized solutions. The nonlocal character of the collision term J in the case of the averaged Boltzmann equation provides better properties of equations. For the symmetrized averaged Boltzmann equation the authors prove global in time existence of weak solutions and Lipschitz dependence on initial data. Using the form of the appropriate type of Maxwellian and the entropy inequality, which is still valid for symmetrized averaged Boltzmann equations, long-time convergence results are provided. Under some additional assumptions, information on the rate of convergence is also obtained. The last part of the chapter is devoted to mild solutions of the averaged Boltzmann equation, not necessarily symmetrized.

Chapter 3 is dedicated to the limit analysis of the averaged Boltzmann equation. Two different limits are considered: (1) from the microscopic (stochastic) ODE to the averaged Boltzmann equation, and (2) from the Boltzmann level to the macroscopic level. The authors obtain results similar to those for the classical Boltzmann equation; cf. [5]. The methods are similar, but the hydrodynamic limits yield different equations.

The fourth chapter concerns the discussion of possible applications of the generalized kinetic (Boltzmann) models. The description of this class of equations, which also contains the averaged Boltzmann equation, is provided in [6]. The authors of the monograph recall a number of papers giving examples of applications of this theory in modeling: for immunology [1, 2], social sciences [4, 7], or traffic flow modeled by kinetic equations [3]. From all these various mathematical structures the general scheme including all the particular cases is built. As before, special attention is paid to relationships among various levels of description.

This book is valuable both for mathematicians and researchers working in applied sciences. It presents analytical tools and provides a wide overview of possible applications. I can definitely recommend this monograph to everyone interested in the subject of kinetic models. It is a valuable source of new mathematical ideas and modeling methods.

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