A NEW LOAD MODEL OF THE PEDESTRIANS LATERAL ACTION

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Summary

This paper proposes a new load model to predict the lateral force exerted by pedestrians walking on lively footbridges. The aim of the model is to take into account some important features of the synchronous lateral excitation phenomenon, which so far has not been fully understood or modelled, e.g. the distinction between synchronization among pedestrians, due to crowd density, and between the pedestrians and the structure, caused by deck oscillations; the triggering of the lock-in phenomenon and its self-limited nature. The proposed load model has been tested with reference to two crowd events that were recorded on the T-bridge in Japan (1993) and on the Millennium Bridge in London (2001). The results obtained with the present model are compared to results predicted by other load models found in literature and are then further discussed.

Keywords: crowd-structure interaction; footbridges; synchronous lateral excitation; pedestrian load; lock-in.

1. Introduction

The problem of determining the lateral action exerted by pedestrians walking on vibrating footbridges has become of great importance in recent years in order to study the phenomenon of synchronous lateral excitation. The problem has been generally tackled using an empirical approach. Laboratory tests involving a single pedestrian walking on a moving platform (e.g. [1]) allow important data to be obtained on the lateral force exerted by one pedestrian and to estimate the mean probability that individuals will synchronise their step to the platform motion [2]. Moreover, the observation of videos recorded during crowd events on actual bridges [3] permits a qualitative estimate to be made of the synchronization phenomena that occur between people walking in a crowd.

Several models have been proposed using the aforementioned data to predict the lateral force exerted by pedestrians. Most of the models found in literature (e.g. the ones reviewed in [4]) are deterministic time-domain models, based on the assumption that both feet produce exactly the same periodic force. The load models proposed in the design codes also belong to this category. For example, three different load models are proposed in [5] to estimate the force exerted by a single pedestrian, by a group of pedestrians or by a crowd uniformly distributed along the footbridge deck, respectively.

To the authors' knowledge, many of the force models proposed have so far experienced some difficulties in taking into account some not negligible aspects of the problem, that is:

- the dependence of the pedestrian force on the two-way interaction between two systems, the crowd and the structure, that can be described by two variables, the pedestrian density and the footbridge lateral vibration. In this paper the deck acceleration is retained as the vibration variable which mostly affects the pedestrian force;
- the possibility of a inhomogeneous distribution of the crowd along the deck due to bottlenecks, congested traffic or other non-linear traffic phenomena;
- the existence of two kinds of synchronization, as recently pointed out in [6] and [7], one between the pedestrians and the structure and the other among the pedestrians. The latter takes place when the relative movement of the pedestrians is constrained because of high crowd density;
- the presence of different frequency components in the overall force;
- triggering of the lock-in phenomena and the resulting self-limited oscillations.

The aim of the present work is to propose a force model that gives due weight to the aforementioned features of the phenomenon. The proposed model assumes that crowd density and deck acceleration are given, for instance by means of *in situ* measurements, computational simulations with a crowd-structure interaction model [7][8] or a priori chosen within a worst-case scenario approach. On one hand, the model is conceived to provide an accurate description of the phenomena and, on the other, to become a useful tool for footbridge designers and engineers, in order to predict footbridge behaviour under pedestrian loads.

2. Formulation of the model

The model is conceived on the basis of a macroscopic description of crowd dynamics, which means that the pedestrians are not viewed as single individuals but as clusters characterized by a mean walking velocity and a mean step frequency. Nevertheless, an adaptation of the model to a microscopic or statistical description of the crowd is possible.

The proposed force model can be ascribed to the category of time-domain models. It is based on the assumption that the force exerted by a number n of pedestrians walking along a portion of the bridge span is given by the sum of three components:

$$F = F_{ps} + F_{pp} + F_s \qquad (1)$$

where F_{ps} is the term due to the synchronization between the pedestrians and the structure, F_{pp} is due to the synchronization among pedestrians and F_s is the part due to uncorrelated pedestrians.

F_{ps} has the same frequency f_s as the excited lateral structural mode, while the other two terms have the same frequency f_{pl} as the lateral pedestrian footstep. f_{pl}, which is half the walking frequency f_p, is assumed to vary as a function of the walking velocity v (Fig. 1), which is in turn dependent on the crowd density u [9]. The f_p-v relation is derived through a fitting of the experimental data in [10], that is:

$$f_{pl} = (0.35v^3 - 1.59v^2 + 2.93v)/2 \qquad (2)$$

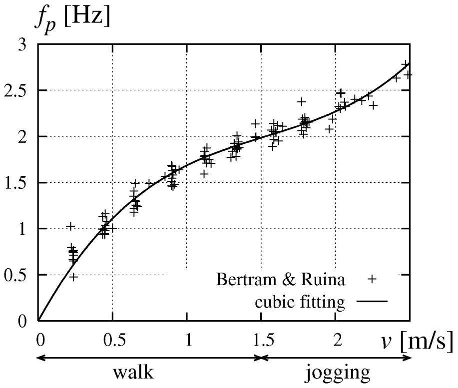

Fig. 1 Relation between step frequency and walking velocity

Each term of the overall force is weighted on the basis of phenomenological considerations, by means of three weights, n_{ps}, n_{pp} and n_s, that can be considered, respectively, as the number of pedestrians in the cluster that are synchronized with the structure, synchronized to each other and uncorrelated:

$$\begin{aligned} n_{ps} &= nS_{ps} \\ n_{pp} &= nS_{pp}(1 - S_{ps}) \\ n_s &= n - n_{ps} - n_{pp} \end{aligned} \qquad (3)$$

where S_{ps} and S_{pp} are the synchronization coefficients, which both vary in the [0 1] range. Thanks to the distinction of pedestrians in three categories, the model is able to represent the triggering of lock-in: even though no one is synchronized with the structure, the presence of a high crowd density results in a lateral force that triggers the lateral vibration of the bridge.

S_{ps} represents the degree of coupling between the crowd and the structure. This is a function of two variables: the structure lateral acceleration ζ (to be intended as the envelope of the acceleration time history $|\ddot{z}(t)|$) and the ratio $f_r = f_{pl} / f_s$, where f_r is defined in the [0 2] domain: the lower bound depends on the minimum value of f_{pl} (f_{pl} =0), when the walking velocity is null; the upper bound was obtained from the laboratory tests reported in [1], according to which pedestrians are not influenced by structural oscillations under 0.6 Hz and the maximum recorded lateral walking frequency is 1.1 Hz.

The variation of S_{ps} versus ζ (Fig. 2a) is given by a fitting of the experimental data in [2], by means of the interpolating function:

$$S_{ps}(\zeta) = 1 - \exp[-b(\zeta - \ddot{z}_c)] \qquad (4)$$

where $\ddot{z}_c$ =0.2 m/s² is the threshold of motion perception defined in [8]. Pedestrians start to synchronize with the structure for values of ζ higher than $\ddot{z}_c$, and they are completely synchronized when ζ reaches the maximum value $\ddot{z}_M$ =2.1 m/s² [11].

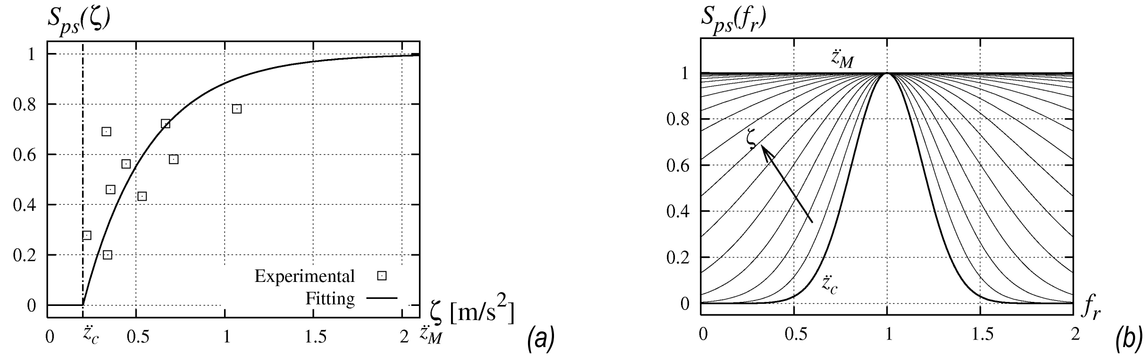

Fig. 2 S_{ps} versus ζ and f_r

S_{ps} (f_r) is supposed to have a normal distribution, with a variance that grows when ζ increases. This means that, for increasing values of ζ, the pedestrians who walk with a step frequency that is different from f_s gradually become involved in the synchronization phenomenon. For ζ = $\ddot{z}_M$, everyone is synchronized with the structure, whatever the value of f_r (Fig. 2b). S_{ps} (f_r) is defined as:

$$S_{ps}(f_r) = \exp[-\eta(f_r - 1)^2]$$
$$\eta(\zeta) = 50\exp(-20\zeta/\pi) \qquad (5)$$

The synchronization coefficient S_{ps} (ζ,f_r) is given by the product of equations (4) and (5).

The coefficient S_{pp} (Fig. 3) represents the degree of synchronization among pedestrians and, because of the lack of experimental data, it has been defined in a qualitative way as a function of the crowd density u [ped/m²]:

$$S_{pp}(u) = \frac{1}{2}\left\{1 + erf\left[a\left(u - \frac{u_{sync} + u_c}{2}\right)\right]\right\} \qquad (6)$$

where a=3.14, u_c=0.3 ped/m² is the upper limit for unconstrained free walking [7] and u_{sync} is the density that corresponds to the total synchronization of pedestrians. Its value is estimated to be 1.8 ped/m², according to the maximum densities recorded on crowd events (e.g. [3]).

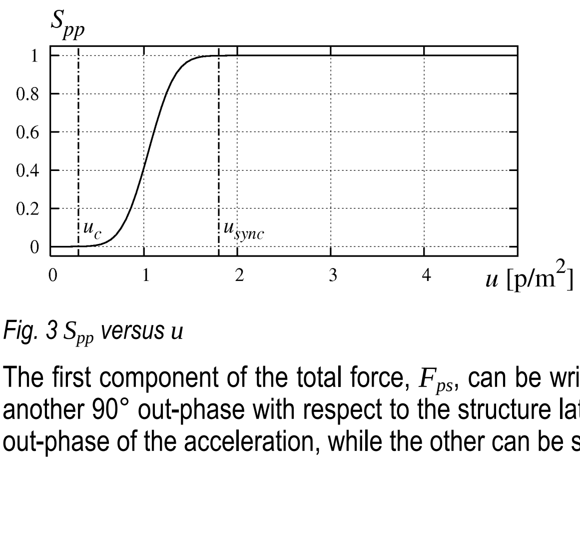

Fig. 3 S_{pp} versus u

The first component of the total force, F_{ps}, can be written, analogously to [1], as the sum of a component in-phase and another 90° out-phase with respect to the structure lateral displacement. The in-phase component can be seen as 180° out-phase of the acceleration, while the other can be seen as in-phase with the lateral velocity:

$$F_{ps} = n_{ps}[\overline{F}(\zeta)\sin(2\pi f_s t + \pi) + \overline{F}(\nu)\cos(2\pi f_s t)] \qquad (7)$$

where ν is the envelope of the deck lateral velocity time history. The amplitudes of the two components (Fig. 4) are defined by means of piecewise functions. The first branch comes from a quadratic fitting of the experimental data concerning the medium Dynamic Load Factors (DLF) of the in-phase and out-phase components [1]. The second branch is defined qualitatively, based on the following assumptions: **i)** the DLFs reach their maximum when the velocity and the acceleration exceed their serviceability limit (that is, $\dot{z}_s$ =0.25 m/s and $\ddot{z}_s$ =1.35 m/s² [11]); **ii)** the DLFs decrease to zero when the velocity and the acceleration reach maximum values, above which pedestrians stop walking, i.e. $\dot{z}_M$ =0.44 m/s and $\ddot{z}_M$ =2.1 m/s² [11]. This trend also guarantees that the amplitude of F_{ps} is self-limited as is the overall structural response. It should be pointed out that the threshold values suggested in [11] are based on a very limited number of observed cases. In particular, the maximum velocity and acceleration refer to those recorded on the Millennium Bridge, while the serviceability values are based on the field tests performed on the M-bridge in Japan. Recalling that the ratio between the acceleration and the velocity amplitudes depends on the natural frequency of vibration f_s (i.e. $|\zeta/\nu|=2\pi f_s$), it is clear that the threshold values on the velocity and on the acceleration are simultaneously reached only for a particular frequency.

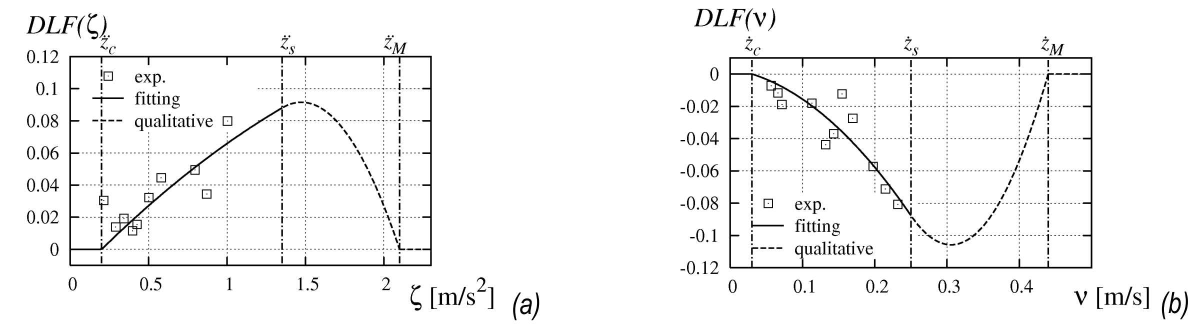

Fig. 4 DLFs of the components in phase with ζ (a) and ν (b)

The second force component, F_{pp}, is defined as:

$$F_{pp} = n_{pp}\overline{F}_s \sin(2\pi f_{pl} t) \qquad (8)$$

where $\overline{F}_s$ is the medium amplitude of the force exerted by a single pedestrian in the case of a motionless deck, whose DLF=0.04 [12].

Finally, the component F_S is determined according to the model proposed by Matsumoto et al. [13], who found that the force due to n uncorrelated pedestrians is $\sqrt{n}$ higher than the force due to a single pedestrian. Therefore, F_S becomes:

$$F_{pp} = \sqrt{n_s}\,\overline{F}_s \sin(2\pi f_{pl} t) \qquad (9)$$

3. Applications

The properties of the force model have been evaluated by means of a sensitivity study on the two main variables, ζ in the [0 $\ddot{z}_M$] m/s² interval and u in the [0 u_M] ped/m² range. The simulations are performed assuming a deck acceleration time history with constant amplitude ζ:

$$\ddot{z}(t) = \zeta \sin(2\pi f_s t) \qquad (10)$$

It is worthwhile pointing out that this type of simulation can be compared to an experimental test on a moving treadmill, except that the single pedestrian is substituted by a cluster of pedestrians.

The sensitivity study has been performed by referring to two case-studies, the T-bridge and the south span of the London Millennium Bridge, since they are fully documented. Therefore, the ν-u relation, described in the accompanying paper [9], has been characterized for the two cases as for the geographic area and the travel purpose, that is: Asia and rush hour traffic for the T-bridge (TB); Europe and leisure traffic for the Millennium Bridge (MB). A summary of the input data is reported in Table 1.

Table 1 Input data

Footbridge	v_M [m/s]	u_M [ped/m^2]	γ	f_s [Hz]
T-bridge	1.48	7.7	0.273 u_M	0.9
Millennium Bridge	1.18	6	0.245 u_M	0.8

Fig. 5 a-c-e and Fig. 6 a-c-e show the weights of the force components, scaled with respect to the number of pedestrians n, versus ζ and u. The following considerations can be made for both the case-studies:

- n_{ps} is much more sensitive to the deck acceleration ζ than to the crowd density u and grows monotonically as ζ increases and as u decreases; it reaches the highest value for $\zeta > \ddot{z}_s$, whatever the value of u (Fig. 5a and Fig. 6a);
- n_{pp} has the opposite trend to n_{ps}: it grows as u increases and as ζ decreases. For $u > u_{sync}$ (S_{pp}=1), n_{pp} is complementary to n_{ps} (Fig. 5c and Fig. 6c);
- n_s is obtained from subtraction of the other two terms, therefore it reaches the maximum value when ζ and u are under their critical values and is null for $\zeta > \ddot{z}_s$ and $u > u_{sync}$, i.e. when all the pedestrians are synchronized (Fig. 5e and Fig. 6e).

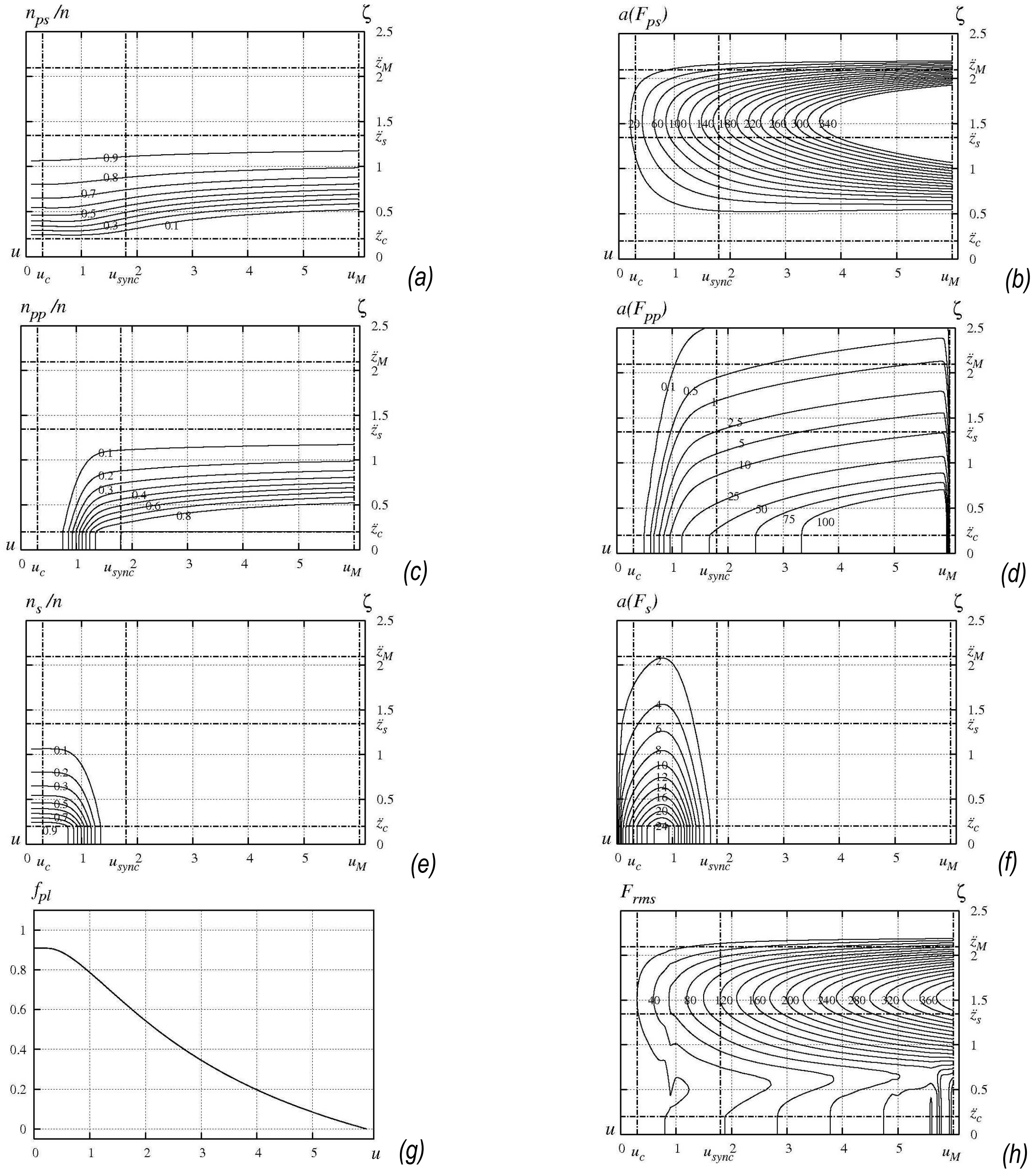

Fig. 5 Results for the Millennium Bridge

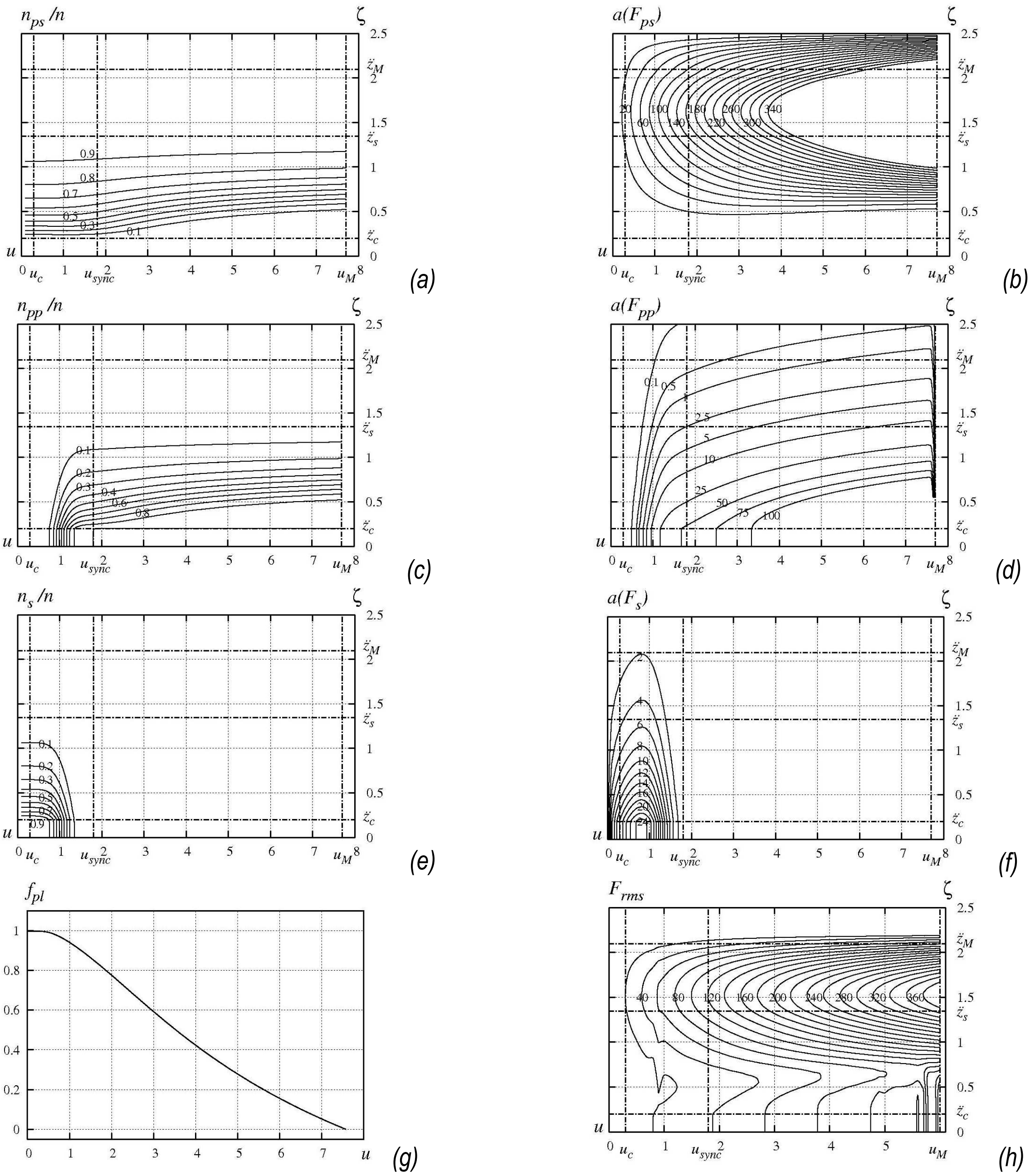

Fig. 6 Results for the T-bridge

A colour map of the three weights in the u-ζ plane (Fig. 7) can be obtained for each structure, in order to quickly estimate, in the preliminary design phase, which kind of synchronization will play the main role.

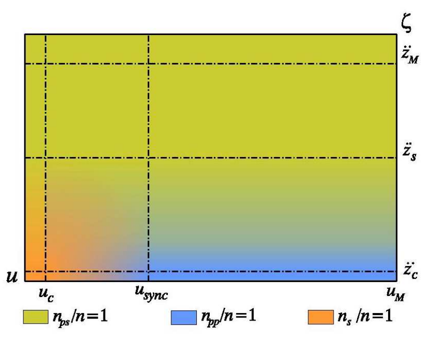

Fig. 7 Colour map of the weights of the force components

Fig. 5 b-d-f-h and Fig. 6 b-d-f-h represent the amplitudes a of the three force terms (in N/m²) and of the overall force F, which is expressed in terms of its root mean square (rms) value (F_{rms}), versus ζ and u. Some common features can be recognized in both cases:

- the evolution of F_{ps} versus ζ is influenced by both n_{ps} and by the distributions of the DLFs (see Fig. 4), which determine its non monotonic trend and its self-limited nature with respect to ζ. The dependence of F_{ps} on u is almost linear, i.e. its amplitude grows linearly as the number of pedestrians increases (Fig. 5b and Fig. 6b);
- the amplitude of F_{pp} has the same evolution as n_{pp}, since it comes from the product of n_{pp} and the constant $\overline{F}_s$. It is worthwhile pointing out that F_{pp} goes abruptly to zero when f_{pl} =0, that is, for $u=u_M$: this means that the F_{pp} component is self-limited with respect to u (Fig. 5d and Fig. 6d). A similar consideration can be drawn for F_s, which also has a non monotonic evolution versus u (Fig. 5f and Fig. 6f);
- the evolution of F_{rms} versus u and ζ is not trivial, since it comes from the sum of periodic signals with different frequencies and it therefore cannot be determined by simply summing the three component amplitudes (Fig. 5h and Fig. 6h).

Table 2 provides the results obtained with the model for the actual values of crowd density and deck acceleration recorded on the case studies. The correspondence between the simulated results and the actual data can clearly be seen by looking at the force frequency content. In the case of the T-bridge, the estimated step lateral frequency f_{pl} (0.86 Hz) is very close to the deck lateral frequency (0.9 Hz), therefore the two components F_{pp} and F_s also excite the first lateral mode. As for the Millennium bridge, since f_{pl} is very far from f_s, it can be stated that the overall force is mainly due to the synchronization between pedestrians and the structure.

Table 2 Results obtained with the proposed force model (forces in [N/ped])

	Input data			**Output**						
Footbridge	u [ped/m^2]	ζ [m/s^2]	f_s [Hz]	$a(F_{ps})$	$a(F_{pp})$	$a(F_s)$	f_{pl} [Hz]	F_{rms}	n_{ps}/n	n_{pp}/n
T-bridge	1.4	0.34	0.9	3.44	19.0	5.10	0.86	17.46	0.33	0.63
Millennium Bridge	1.4	1.50	0.8	94.3	0.87	1.09	0.66	67.0	0.97	0.03

Finally, let us consider the number of synchronized pedestrians. Even though both events are characterized by high crowd density, as the two structures have different slenderness, the Millennium Bridge shows higher oscillations than the T-bridge, which means a greater number of pedestrians synchronized with the structure. Since only a few pedestrians are captured in the structure-induced synchronization on the T-bridge, the crowd density plays a leading role in determining the overall force.

The single force per pedestrian [N/ped] obtained with the model is, finally, compared to that predicted with the force models proposed in [3], [14] and [2] (Table 3). It should be pointed out that, since the cited models do not distinguish between the two types of synchronization, the predicted force is considered to be only due to the pedestrian-structure synchronization and it is therefore compared to the term F_{ps}.

Table 3 Comparison between the proposed force model and those found in literature (forces in [N/ped])

Model	T-bridge	Millennium Bridge
Fujino et al. [3]	7	n.a.
Nakamura & Kawasaki [14]	4.7	n.a.
Dallard et al. [2]	n.a.	89.5
Proposed	3.44	94.3

First, it is should be noticed that each model in literature is based on the data recorded on a particular footbridge and it can not therefore be directly extended to all other structures. For instance, Nakamura and Kawasaki observed that the model they proposed for the T-bridge only agreed with that of Dallard et al. for deck lateral velocities under 0.015 m/s. The proposed model, instead, has a more general validity, since it is not based on one single event but on the phenomenological description of the components of the coupled system in their fundamental constitutive laws. The amplitudes of the term F_{ps} predicted by the proposed model are very similar to those estimated by Nakamura and Kawasaki for the T-bridge and by Dallard et al. for the Millennium bridge, respectively: this fact highlights the wide applicability of the model.

4. Concluding remarks

The proposed force model satisfies the preset objectives. The main features of the pedestrian lateral excitation phenomenon are taken into account, in particular the fact that both the crowd density and the motion of the footbridge deck influence pedestrian behaviour. The model is also accurate compared to the data and models found in literature, and can therefore be considered as a general predictive tool for the design and analysis of footbridges under pedestrian loads. Because of its versatility, it can be used for different purposes and with different degrees of accuracy: during the preliminary design phase, it allows the worst load scenario to be outlined if the expected values of crowd density and deck acceleration are *a priori* chosen; in the final design phase, it can be used to determine the load that should be applied to the structural model or to a complete computational simulation of the crowd-structure interaction [8]; once the footbridge has been built, the model is able to define the pedestrian force on the basis of the deck acceleration and crowd density measured *in situ* during actual events. It should be pointed out that some of the laws presented in this paper come from qualitative considerations and are not adequately supported by experimental data. For this reason, the model, which constitutes a valid reference framework, can certainly be improved with a proper tuning of the parameters by means of *ad hoc* conceived experimental tests.

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