SYNCHRONOUS LATERAL EXCITATION ON LIVELY FOOTBRIDGES:

MODELLING AND APPLICATION TO THE T-BRIDGE IN JAPAN

Fiammetta VENUTI
PhD
Politecnico di Torino
Torino, IT

Luca Bruno
Associate Professor
Politecnico di Torino
Torino, IT

Summary

A mathematical and computational model used to simulate Crowd-Structure Interaction in lively footbridges is presented in this work. The model is based on the mathematical and numerical decomposition of the coupled multiphysical non-linear system into two interacting subsystems. The model is conceived to simulate the synchronous lateral excitation phenomenon caused by pedestrians walking on footbridges. First, the model is applied to simulate a crowd event on an actual footbridge, the T-bridge in Japan. Then, two sensitivity analysis are performed on the same benchmark to evaluate some properties of the model. The model shows a good agreement with data in literature and could be a useful tool for designers and engineers in the different phases of footbridges design.

Keywords: coupled system; crowd-structure interaction; synchronization; footbridge; lateral vibration; computational simulation.

1. Introduction

In the last few decades, several footbridges have shown a great sensitivity to human induced vibration in the lateral direction (e.g. [1][2]). The phenomenon, known as synchronous lateral excitation, can take place every time a great number of pedestrians walk upon a surface that laterally oscillates with a frequency near to the lateral walking frequency. The deck lateral motion can be triggered by an external action (e.g. wind) or by the pedestrians themselves, who synchronise among each other in the presence of high crowd density. Hence, a second kind of synchronization can take place: when a pedestrian walks on a laterally moving surface, because of the attempt to maintain the body balance, he walks with more spread legs and adapts his frequency to that of the moving surface, that is, he synchronizes with the structure. Therefore, the lateral motion of the upper part of the torso increases and the resulting lateral force grows in turn. This phenomenon is amplified if the pedestrian walks within a crowd, since the synchronization among pedestrians increases the effects of the pedestrian-structure synchronization. The synchronous lateral excitation phenomenon has never caused structural failures, since it is self-limited, that is, when the vibrations exceed a limit value, pedestrians stop walking or touch the handrails, causing the vibration to decay. In order to provide a deep knowledge of this kind of phenomena, an intense research activity, reviewed in [3], has been done in the recent years, providing the scientific background of some recently published design guidelines [4][5].

The most relevant data concerning pedestrian behaviour have been obtained with an empirical approach. Laboratory tests involving a pedestrian walking on both a motionless platform [6] and a laterally moving treadmill [2][7], permitted the lateral force exerted by one pedestrian to be measured and interesting information about the synchronization between the pedestrian and the structure to be obtained. Moreover, the behaviour of a pedestrian within a crowd has been investigated by means of in situ experiments [2][8] and through the observation of the videos recorded during crowd events [1].

To the authors' knowledge, so far the structural effects of walking people have been evaluated through the proposal of different load models (e.g. those in [4][9]), i.e. the crowd is simply viewed as a load. The first attempt to model the crowd and the structure as parts of a complex dynamical system has been proposed by the writing authors in [10][11], where the overall modelling framework has been drawn. The same approach has been adopted without meaningful variations and improvements in [12]. It is based on the decomposition of the coupled multiphysical crowd-structure dynamical system into two subsystems, the Crowd and the Structure, interacting between each other by means of forcing terms. Subsequent works has been devoted by the authors to the a-part development of each single model component [13][14].

In the present work the updated components are collected in the initial modelling framework and the latter is implemented in an *ad-hoc* developed multi-physics numerical code. The whole improved model is applied to an actual crowd event on a real footbridge, the T-bridge (Japan). The available and detailed *in-situ* measurements of both crowd conditions and structural response allow the complete comparison between them and the computational results.

2. Mathematical model

The main features of the developed model lie in the mathematical and numerical partitioning of the coupled system into two physical subsystems and in the two-way interaction between them, according to the so-called partitioned approach first proposed by Park and Felippa [15] and generally applied to Fluid-Structure Interaction problems. In the following, each part of the model is described referring to the framework schematized in Fig. 1.

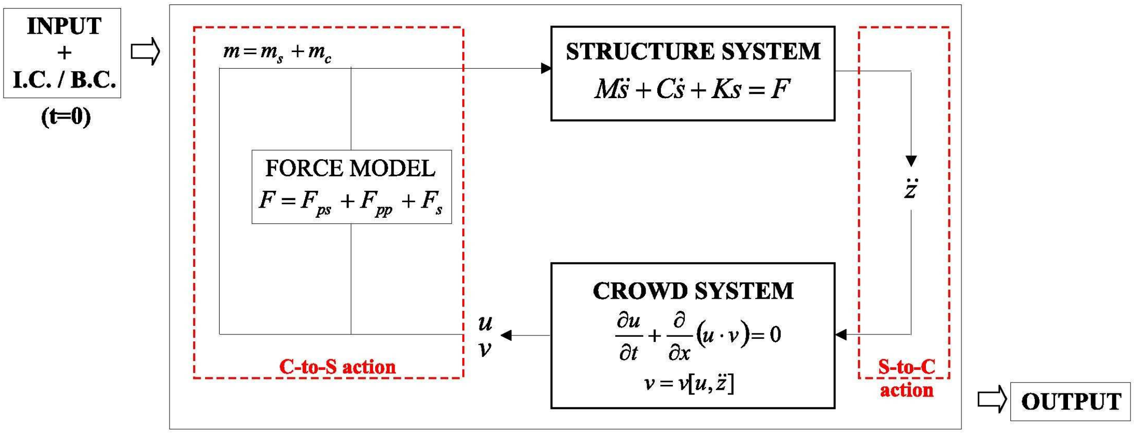


Fig. 1 Scheme of the time-domain coupled model

In particular, the two subsystems of interest, the crowd and the structure, will be referred to with the subscripts *C* and *S*, respectively.

2.1 The Structure subsystem

The structure system is modelled by a 3D model. The structural dynamics is described by the non-linear Ordinary Differential Equation (ODE) of motion:

$$[M_S + M_C(u)]\ddot{s} + C\dot{s} + Ks = F(u, \ddot{z}) \qquad (1)$$

where $s, \dot{s}, \ddot{s}$ are the structural displacements, velocities and accelerations: M_S, C and K are the structural mass, damping and stiffness; M_C, and u are the crowd mass and density, F is the applied lateral force, $\ddot{z}$ the deck lateral acceleration.

Non linearity arises from two terms: first, the forcing term F is a function of both the crowd density and the lateral acceleration of the deck; second, the overall mass M is given by the sum of the structure and the crowd mass. The latter derives from the solution of the equation governing the Crowd subsystem, that is in turn dependent on the solution of the ODE (1).

2.2 The Crowd subsystem

The crowd system is described by a 1D first-order macroscopic model, that is, the crowd flow is assumed to be a continuous fluid and its dynamics is described through the derivation of an evolution equation for the mass density, which is closed by a phenomenological relation that links the crowd velocity v to the crowd density in the form:

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}(uv) = 0$$
$$v = v_M \left\{ 1 - \exp\left[-\gamma \left(\frac{1}{u} - \frac{1}{u_M} \right) \right] \right\} \tag{2}$$

where x and t are the space and time independent variables, v_M is the mean maximum velocity, u_M is the maximum admissible density and γ is a coefficient that sensitises the relation to different travel purposes (leisure/shopping, commuters/events, rush hour/business). Both v_M and u_M are made sensitive to the geographic area and the travel purpose. In such a way the model is sensitised to both biometrical and psychological factors that are known to strongly affect crowd behaviour. The complete description of the closure equation model is given in an accompanying paper [16].

2.3 Structure-to-Crowd interaction

In order to account for the Structure-to-Crowd interaction, the closure equation has to be adapted to sensitize the walking speed to the deck lateral motion. The following assuptions are retained from phenomenological observation:

- the motion of the platform, described by its acceleration $\ddot{z}$, reduces the walking velocity;
- the pedestrians adjust their step to the platform motion with a synchronization time delay $\Delta\tau$, which is expected to be greater than the time interval between two succeeding footfalls;
- after the pedestrians have stopped because of excessive lateral vibrations at time t_s,
- a stop-and-go time interval Δt_r should elapse before they start walking again.

According to these hypotheses, the term v_M in the closure equation (2) is multiplied by a corrective factor $g(\zeta)$, which takes into account the sensitivity of v to the platform acceleration, where ζ is the envelope of the acceleration time history. $g(\zeta)$ is equal to unity for acceleration under the threshold of motion perception $\ddot{z}_c$ =0.2 m/s^2 and linearly decreases to zero for acceleration above $\ddot{z}_M$ =2.1 m/s^2. The closure equation is described in more details in [16].

2.4 Crowd-to- Structure interaction

The Crowd-to-Structure interaction takes place in two ways. On one hand, the mass M is constantly updated by adding the pedestrian mass M_C to the structural mass M_S. On the other hand, the lateral force $F(t)$ exerted by the pedestrians is expressed as a function of both the crowd density u and the lateral acceleration of the deck $\ddot{z}$.

A complete description of the macroscopic force model is provided in [17]: herein, only a few basic points are given. The lateral force F exerted by a cluster of n pedestrians walking along a portion of the bridge span is given by the sum of three terms:

$$F = F_{ps} + F_{pp} + F_s \tag{3}$$

where F_{ps} is the component due to n_{ps} pedestrians synchronized to the structure, F_{pp} is due to n_{pp} pedestrians synchronized to each other and F_s is due to n_s uncorrelated pedestrians. The number of synchronized pedestrians is determined by introducing two synchronization coefficients. The pedestrian-structure synchronization coefficient S_{ps} is a function of the deck lateral acceleration $\ddot{z}$ and of the frequency ratio $f_r = f_{pl} / f_s$, where f_{pl} is the step lateral frequency, which depends on the walking velocity, and f_s is the structural frequency. The pedestrian-pedestrian synchronization coefficient S_{pp} is a function of the crowd density. While F_{ps} varies in time with the same frequency of the structure, the other two force components varies with the step lateral frequency.

3. Computational approach

The solution of the mathematical model is obtained by means of computational simulation performed in the space and time domains. The coupled system is decomposed by means of differential partitioning, that is, the system is first decomposed into subsystems (or fields) and then each field is spatially discretised separately [15]. The differential partitioning allows each field to be treated with discretisation techniques and solution algorithms that are known to perform well for the isolated system and allows the use of non-matching grids.

The 1D crowd field is discretised in space by means of Finite Difference Method. The solution of the mass conservation Partial Differential Equation is approximated by means of the Lax-Friederichs scheme in its conservation form [18]. The Finite Element (FE) Method is employed for the space discretisation of the 3D structural multi-degree-of-freedom (MDOF) model. A Beta-Newmark step-by-step integration method is employed to solve the set of ODEs that describes the structural subsystem [19].

The two subsystems are characterized by non-matching grids in space while they share the same discretisation in time. The structure space grid is coarser than the crowd one, since the structure deformed shape requires less nodes to be described than the pedestrian traffic phenomena, because only first global lateral modes of the deck are expected to be mainly excited. As far as the time step Δt is concerned, it has to be chosen in order to guarantee both the Courant-Friederichs-Lewy (CFL) stability condition for the PDE [18] and the desired accuracy solution of the ODE. It follows that the time step is $\Delta t \le 1/(20 f_{s,\max})$, where $f_{s,max}$ is the frequency of the highest mode of interest.

4. Description of the case study: the T-bridge (Japan)

The proposed model has been tested by simulating a crowd event on the T-bridge (Toda Park Bridge, Toda City, Japan). The T-bridge was chosen since it has been widely described by Fujino and coworkers in several papers (e.g. [1][9][20]) and particular attention has been devoted to the description of the crowd conditions during the events.

In the following, only some characteristics are recalled. The T-bridge is a cable-stayed footbridge with a two-span continuous steel box girder, a two-plane multistay cable system with 11 stays per plane and a 61.4 m-high tower made of reinforced concrete. The total bridge length L is about 180 m and the road deck width B is 5.25 m. The girder is fixed longitudinally and transversely at the tower position. Concrete was poured inside the box girder on the side span to reduce the up-lift force at the end support. The bridge mass is 800 kg/m^2 and the damping ratio around 0.7%. The footbridge connects a boat race stadium to a bus terminal. Therefore, at the end of boat races the bridge is crossed by a great number of pedestrians, sometimes more than 20000, who leave the stadium to reach the bus terminal. In these situations, lateral vibrations of the girder of up to 1 cm were recorded.

4.1 Description of the structural model

In order to reduce the degrees of freedom of the T-bridge FE model and simplify the structural analysis, a single-girder (spine) model is used, that is, the bridge deck is modelled using a single central spine with offset rigid links to provide cable anchor nodes. The deck stiffness is assigned to the spine elements and the deck translational mass (lumped mass approach) is assigned to the spine nodes. The towers and the deck are modelled with elastic beam elements and the cables with truss elements. The FE model geometry is represented in Fig. 2, while Fig. 3 compares the first structural modes reported in [1] with the ones obtained with the present model.

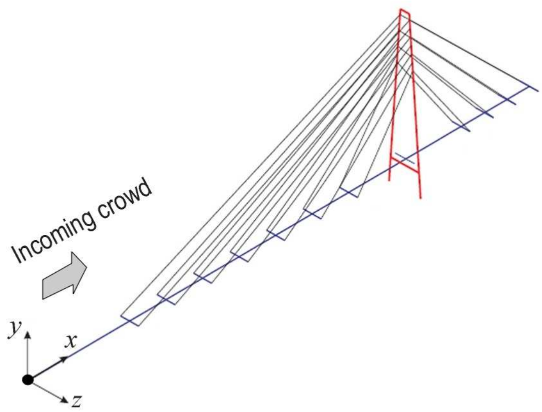


Fig. 2 FE model with direction of the incoming crowd

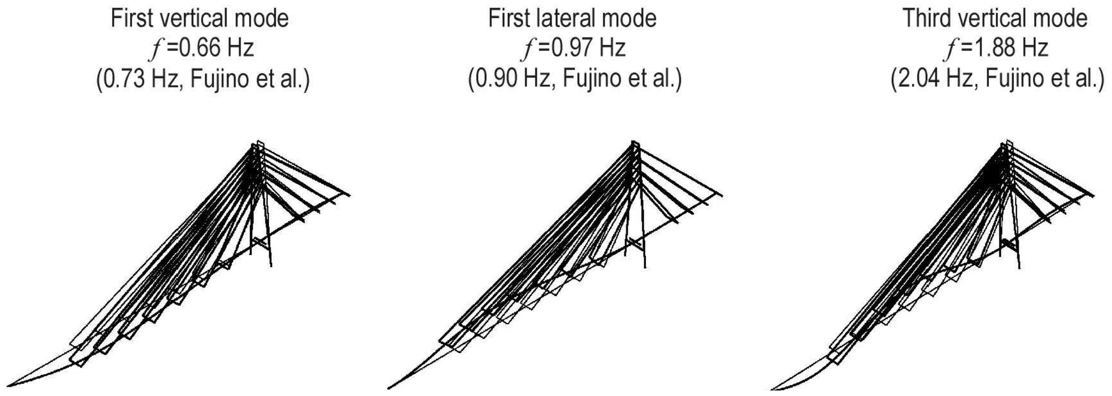
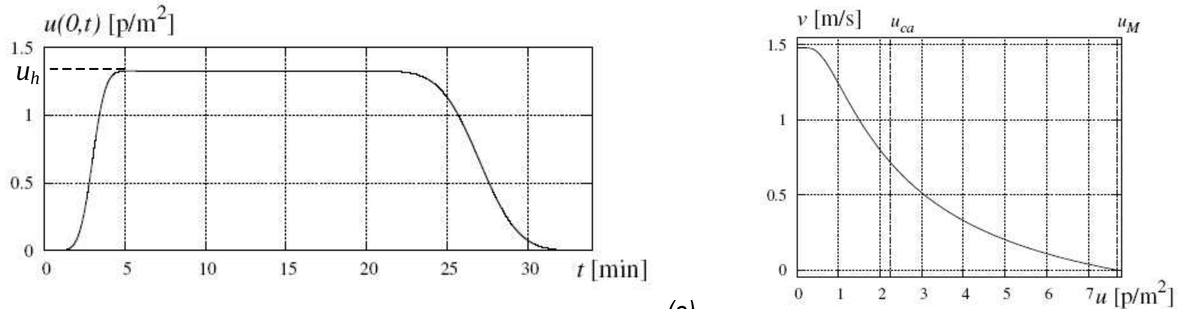


Fig. 3 T-bridge first vertical and lateral modes

4.2 Description of the crowd condition along the deck

The different crowd conditions recorded on the T-bridge have been described in a qualitative way in several papers. The most crowded event is reported in [1], when more than 20000 people left the stadium and crossed the bridge in about 20 minutes. In the most congested situation, about 2000 people walked simultaneously on the bridge. Less crowded situations are described in the other papers [9][20]: a maximum number of around 12000 people crossed the bridge in 12 to 20 minutes, with a crowd density varying between 0.8 and 1.5 ped/m^2.

The simulated condition represents an average of the events reported in literature. The initial condition on the density is $u(0,x)$=0.01 ped/m^2, while the boundary condition (BC) at the inlet $u(t,0)$ (Fig. 4a) has been built in order to allow about 14000 pedestrians to pass over the bridge in 23 minutes, with a maximum density of 1.33 ped/m^2. The velocity-density relation has been adapted for the case of Asia and rush-hour traffic (Fig. 4b), that is, u_M=7.7 ped/m^2, v_M=1.48 m/s and γ=0.273 u_M [16].



(a) *(b)*

Fig. 4 Crowd boundary condition at the inlet (a) and closure equation (b)

5. Simulation of an actual event

The results of the computational simulation are compared to the measurements reported in [1] for five time windows (Fig. 5). As for the structure results, the figure reports the time history of the deck lateral displacement in the node corresponding to the position of the installed accelerometers, the structure first lateral frequency f_s averaged over the period and the dominant frequency f, obtained through the PSD of the displacement in the considered period. As far the crowd results are concerned, the instantaneous spatial distributions of the crowd density are reported, as well as the mean walking frequency f_{pl}.

Looking at the results, a very good agreement between the simulation and the measured data is evident. As far as the deck response in x=0.3L is concerned, the peaks at about t=3' and t=30' are due to the travelling load effects for advancing and leaving crowd flow, respectively, while the largest deck oscillations at 10'<t<25' are due to congested crowd regime. The analysis of the crowd regimes is tackled in an accompanying paper [16]: herein it is worth to point out that these regimes roughly correspond to the windowing proposed by Fujino et al. in [1].The maximum amplitude of the lateral deck displacement, of about 9 mm, matches the measurements very well. The maximum percentage of pedestrians synchronized with the structure is about 21%, which is in very good agreement with the observation data in [1], where a percentage equal to 20% is estimated. Similar considerations can be made for all the considered variables. It is worth to point out that the dominant frequency of the deck vibration is always closer to the walking frequency than to the structure frequency. This means that the force components due to synchronized-among-each-other or uncorrelated pedestrians are dominant with respect to the one due to pedestrians synchronized with the structure, as also shown in Table 1.

Table 1 Maximum amplitudes of the force components [N/ped] between 16 and 23 min.

F	F_{ps}	F_{pp}	F_s	n_{ps}/n	n_{pp}/n	n_s/n
28.8	1.1	26.9	2.1	0.21	0.9	0.13

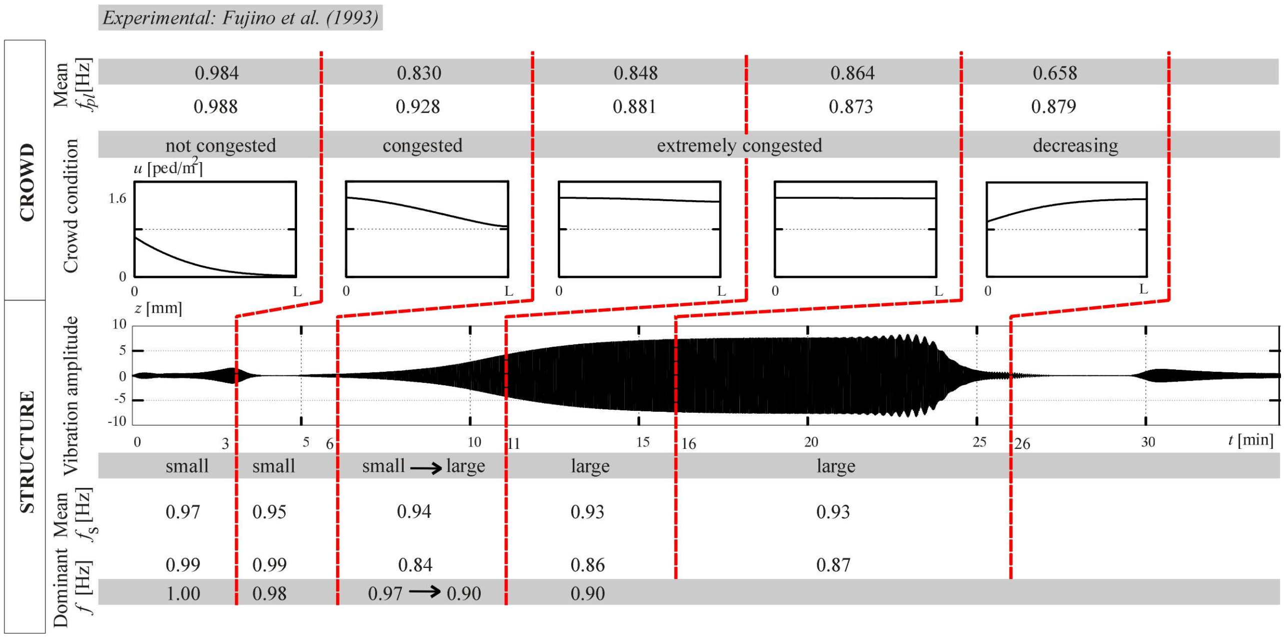


Fig. 5 Comparison between the simulation results and the experimental data reported in [1], highlighted in gray

6. Sensitivity studies on the model parameters

6.1 Sensitivity study on the interacting terms

The computational approach offers not only the opportunity to simulate mechanical systems in real conditions, but also to evaluate their behaviour under unphysical states in order to set apart the effects of each component of a coupled interacting system. Bearing in mind this goal, the same benchmark was subjected to a sensitivity analysis on the model interacting terms. In practice, the various interacting terms were turned off in the complete model by successive subtraction, in order to perform the following simulations: 1) complete Crowd-Structure Interaction, introduced in 5; 2) constant overall mass, that is, the structural mass M_s is retained while the crowd added mass M_c is discarded; 3) the sensitivity to the deck vibration is not taken into account in the Structure-to-Crowd interaction term, that is, $v=v(u)$, instead of $v=v(u, \ddot{z})$; 4) the sensitivity to the deck vibration is not taken into account in the Crowd-to-Structure interaction term, that is, S_{ps}=0 and consequently F_{ps}=0; 5) a combination of cases 3) and 4), i.e. the deck vibration effects are completely discarded; 6) the influence of the density u in the lateral force is not considered, that is, S_{pp} and consequently F_{pp} =0, so that all the pedestrians are uncorrelated.

The structural responses obtained from the six configurations are compared in Fig. 6. The following considerations can be made: **i)** the maximum amplitude of the deck displacement is reached with a complete crowd-structure-interaction simulation. The change in the footbridge modal properties, due to the crowd added mass, causes the first lateral frequency to be closer to the walking frequency. As a consequence, the pedestrian-structure synchronization increases and the vibration grows in turn; **ii)** the simulations from 2 to 5 give identical results. The absence of the crowd added mass causes the deck lateral acceleration $\ddot{z}$ to never exceed the critical value $\ddot{z}_c$. Therefore, S_{ps} is always null, v is only dependent on u and the four simulations are, in fact, the same as simulation 5; **iii)** when all the pedestrians are forced to be uncorrelated (simulation 6), the maximum vibration amplitude dramatically reduces to 1.3 mm. This is in line with the result obtained by Fujino and coworkers in their pioneering work [1]: the application of a simplified single-DOF model of the T-bridge with an applied force due to uncorrelated pedestrians produced a vibration amplitude of 1 mm.

The sensitivity study highlights the leading role of the pedestrian added mass, which is about 11.6% of the deck weight, on the overall structural behaviour. The simulations without the added mass M_c (2-5) follow the qualitative trend of the observed structural response, but the vibration amplitude is underestimated by about 33%. In the case of the T-bridge, the amplitude of the lateral force exerted by pedestrians is mainly due to the synchronization among pedestrians: if the synchronization is neglected, that is, all the pedestrians are uncorrelated, the obtained vibration amplitude is further underestimated by about 90% of the recorded value.

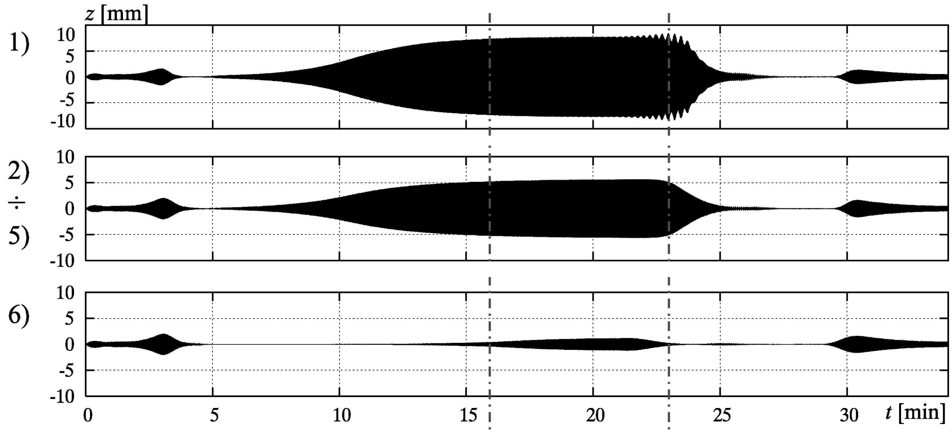


Fig. 6 Deck displacement in the six different configurations at x=L/3

6.2 Sensitivity study on the crowd density

The second sensitivity study presented herein is performed on the crowd BCs at the inlet, by varying the maximum value reached by the density u_h [0.8; 1.33; 2.0 ped/m^2] (see Fig. 4a).

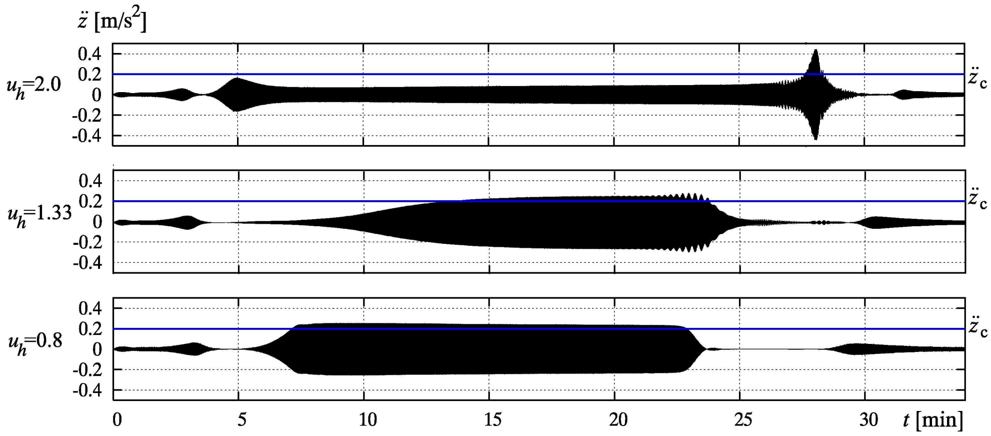


Fig. 7 Deck lateral acceleration in x=L/3

Fig. 7 shows that the maximum amplitude of the deck lateral acceleration does not correspond to the case with the highest density u_h =2 ped/m^2. Increasing values of u_h correspond to increasing amplitude of the total force but also to a decrease of f_r due to the effect of the crowd added mass. As a consequence, when u_h =0.8 ped/m^2, the total force is an order of magnitude less that in the case u_h =2 ped/m^2, but the force is almost resonant with the deck first mode and, therefore, it induces the highest structural response. The almost steady-state response for u_h =2 ped/m^2 is due to the fact that, in the same time window, the crowd density exceeds the value, above which S_{pp}=1. Therefore, all pedestrians are synchronized to each other and walk with the same frequency, which is sufficiently far from f_s to prevent resonance.

7. Concluding remarks

In this paper a crowd-structure interaction model characterized to describe the synchronous lateral excitation phenomenon has been presented. The model has been implemented in an *ad hoc* computational code, which has been used to perform simulations of actual or ideal events, using the T-bridge in Japan as a case study.

The results of the simulations highlight the major role played by the crowd added mass. In lightweight structures, the added mass has the main effect of changing the structural dynamic properties, that is, the natural frequencies, with the effect of making both the structure less or more sensitive to the lateral force exerted by pedestrians and the pedestrians less or more likely to synchronize to the structure. Therefore, as already pointed out in recent guidelines (e.g. [4]), the mass of pedestrians has to be considered together with the mass of the structure in the overall dynamical system. Quite surprisingly, a more crowded condition does not always correspond to higher deck vibrations. This result confirms once more the complexity of the coupled dynamical system. It also shows that simplified comfort criteria based on the limitation of the number of pedestrians crossing the bridge (e.g. [2]) could be not always effective to prevent the synchronous lateral excitation phenomenon.

Acknowledgements

The authors wish to thank Y. Fujino and S. Nakamura for kindly providing the structural properties of the T-bridge. This research has been carried on with the financial support of IABSE Foundation and of the Italian Ministry of Education, University and Research M.I.U.R. within the project "Aeroelastic phenomena and other dynamic interaction on non-conventional bridges and footbridges".

References

[1] FUJINO Y., PACHECO B.M. , NAKAMURA S., WARNITCHAI P., Synchronization of Human Walking Observed during Lateral Vibration of a Congested Pedestrian Bridge, *Earthquake Engineering and Structural Dynamics*, No. 22, 1993, pp. 741-758.

[2] DALLARD P., FITZPATRICK T., FLINT A., LE BOURVA S., LOW A., RIDSDILL R. M., WILLFORD M., The London Millennium Footbridge, *The Structural Engineer*, vol. 79, No. 22, 2001, pp. 17-33.

[3] ŽIVANOVIC S., PAVIC A., REYNOLDS P., Vibration serviceability of footbridges under human-induced excitation: a literature review, *Journal of Sound and Vibration*, vol. 279, 2005, pp. 1-74.

[4] FIB Federation International du Beton. *Guidelines for the design of footbridges*, FIB Bulletin No. 32, Lausanne, 2006.

[5] SETRA/AFGC. *Passerelles piétonnes – Evaluation du comportement vibratoire sous l'action de piétons. Guide méthodologique*. Paris, 2006.

[6] ANDRIACCHI T. P., OGLE J. A., GALANTE J. O., Walking speed as a basis for normal and abnormal gait measurements, *Journal of Biomechanics*, vol. 10, pp. 261-268, 1997.

[7] PIZZIMENTI A.D., Experimental evaluation of the dynamic lateral loading of footbridges by walking pedestrians, *6th International Conference on Structural Dynamics*, Paris, 2005.

[8] NAKAMURA S. Field measurement of lateral vibration on a pedestrian suspension bridge. *The Structural Engineer*, No. 81(22), pp. 22-26, 2003.

[9] NAKAMURA S., KAWASAKI T. Lateral vibration of footbridges by synchronous walking, *Journal of Constructional Steel Research*, No. 62, pp. 1148-1160, 2006.

[10] VENUTI F., BRUNO L., BELLOMO N. Crowd structure interaction: dynamics modelling and computational simulations. *Proceedings Footbridge 2005*, Venice, 2005.

[11] VENUTI F., BRUNO L., BELLOMO N. Crowd dynamics on a moving platform: mathematical modelling and application to lively footbridges. *Mathematical and Computer Modelling*, No. 45, pp. 252-267, 2007.

[12] BODGI J., ERLICHER S., ARGOUL P., Lateral vibration of footbridges under crowd-loading: Continuous crowd modeling approach, *Key Engineering Materials,* No. 347, pp. 685–690, 2007.

[13] VENUTI F., BRUNO L., Pedestrian lateral action on lively footbridges: a new load model. *SEI Structural Engineering International*, no. 17(3), 2007.

[14] VENUTI F., BRUNO L., An interpretative model of the pedestrian fundamental relation. *C.R. Mecanique*, no.335, pp. 194-200, 2007.

[15] PARK K. C., FELIPPA C. A., FARHAT C.. *Partitioned analysis of coupled mechanical systems*, Report no. CU-CAS-99-06. University of Colorado, 1999.

[16] BRUNO L., VENUTI F., The pedestrian speed-density relation: modelling and application, *Proceedings Footbridge 2008*, Porto, 2008.

[17] VENUTI F., BRUNO L., A new load model of the pedestrians lateral action, *Proceedings Footbridge 2008*, Porto, 2008.

[18] LEVEQUE R. J., *Numerical methods for conservation laws*. Birkhauser, Zurich, 1992.

[19] CLOUGH R., PENZIEN J.. *Dynamics of structures*. McGraw-Hill, New York, 1987.

[20] NAKAMURA S., FUJINO Y.. Lateral vibration on a pedestrian cable-stayed bridge. *Structural Engineering International*, 12(4), pp. 295–300, 2002.