The synchronous lateral excitation phenomenon: modelling framework and application

Fiammetta Venuti [a], Luca Bruno [a]

[a] *Politecnico di Torino, Department of Structural Engineering and Geotechnics, Viale Mattioli 39, I-10125, Torino*

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Abstract

A mathematical model conceived to simulate the mechanics of synchronous lateral excitation induced by pedestrians on footbridges is presented in this work. The model is based on the mathematical and numerical decomposition of the coupled multiphysical non-linear system into two interacting subsystems: the Structure system, whose dynamics is described by the non-linear equation of motion; the Crowd system, which is described by a first-order hydrodynamic model governed by the mass conservation equation. The model was applied to the simulation of a crowd event recorded on the T-bridge in Japan and results are commented on. *To cite this article: F. Venuti, L. Bruno, C. R. Mecanique ....*

Résumé

Le phénomène de synchronisation forcée latérale : modélisation et application. Un modèle mathématique conçu pour la simulation du phénomène de synchronisation forcée latérale d'une foule de piétons en marche sur une passerelle est présenté dans la présente étude. Le modèle consiste en la décomposition mathématique et numérique du système couplé nonlinéaire multi-physique en deux sous systèmes en interaction : le système Structure, décrit par des équations fondamentales nonlinéaires de la dynamique ; le système foule, qui est décrit par un modèle hydrodynamique du première ordre géré par l'équation de conservation de la masse. Le modèle est appliqué à la simulation de la foule traversant la passerelle du Toda Park au Japon. *Pour citer cet article : F. Venuti, L. Bruno, C. R. Mecanique ....*

Key words: Synchronization ; Crowd-structure interaction ; Footbridges

Mots-clés : Synchronisation ; Interaction piétons-structure ; passerelles piétonnes

Email addresses: `fiammetta.venuti@polito.it` (Fiammetta Venuti), `luca.bruno@polito.it` (Luca Bruno).

1. Introduction

The synchronous lateral excitation has been the object of a great amount of studies in the last few years (e.g. [1]), after a famous footbridge, the London Millennium bridge, was closed the day it opened because of excessive lateral vibrations [2]. The phenomenon occurs each time a footbridge with a lateral frequency under 1.3 Hz is loaded with a sufficient number of pedestrians [2]: if the footbridge laterally vibrates, the pedestrians tend to synchronize their step to the deck motion (lock-in), with a consequent increase of the exerted lateral force; in addition, the problem is amplified if the number of pedestrians is high, since they unconsciously synchronize to each other.

The main consequence of the human-induced lateral excitation is a loss of comfort for the users. Structural failure has never occurred because of its self-limited nature, that is, when the pedestrians can no more maintain the body balance because of excessive vibration, they stop walking or just touch the handrails, causing the vibration decrement. Nevertheless, the closure of the footbridge to provide countermeasures can represent a severe social and economic cost. This reason has motivated the research of a way to prevent the problem by accounting for it in the footbridge design phase.

The lateral action of pedestrians has so far been taken into account by proposing different load models (e.g. [2]-[4]), which have the advantage of being synthetic and conceived for practical use. On the other hand, they do not permit to take into account some relevant features of the phenomenon, such as the triggering of the lock-in and its self-limited nature, the different effects of the two kind of synchronization (i.e. between the pedestrians and the structure and among pedestrians) and the presence of different frequency components in the overall force exerted by pedestrians.

In order to overcome these limitations, an innovative approach was presented for the first time by the present authors in [5] and [6]: it is based on the partitioning of the coupled system into two subsystems and on the two-way interaction between them. One of its most important features is that the crowd is modelled not simply as a load, but as a dynamical system, which interacts with the structural system. After [5] and [6], the same approach has been adopted without meaningful variations and improvements in [7].

The authors have subsequently developed the proposed framework in each of its parts in [8] and [9], with particular attention to the modelling of the interacting terms. In this Note the resulting complete coupled model is briefly described and an application to a real crowd event is presented.

2. Mathematical model

The main features of the model lie in the mathematical and numerical partitioning of the coupled system into two physical subsystems and in the two-way interaction between them, according to the general approach first proposed by Park and coworkers [10]. The two subsystems, the Crowd and the Structure, will be referred to with the subscripts c and s, respectively.

In the following, each part of the model is described referring to the framework schematized in Fig. 1. It is worth stressing that the model is herein expressed in its dimensional form in order to better point out the physical meaning of its components. Neverthless, the dimensionless form can be obtained by scaling all the variables with respect to reference quantities [11], that is, the maximum mean pedestrian velocity v_M, the maximum admissible crowd density u_M and the footbridge length L.

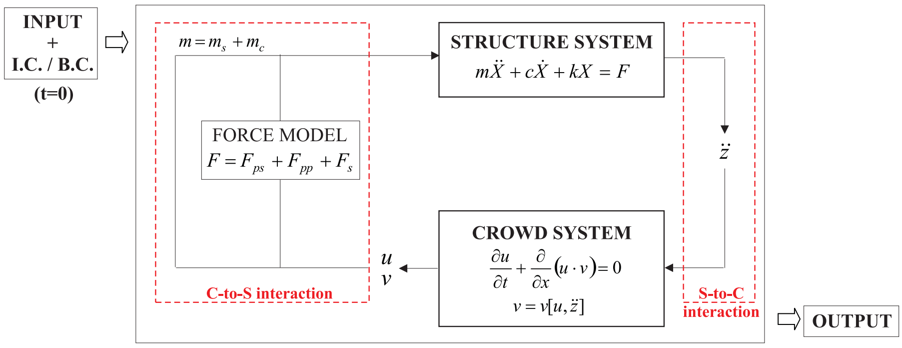


Figure 1. Scheme of the time-domain coupled model

2.1. *The Structure subsystem*

The Structure subsystem is modelled by a 3D multi-degree-of-freedom (MDOF) model. The structural dynamics is described by the non-linear equation of motion:

$$[m_s + m_c(u)]\ddot{X} + c\dot{X} + kX = F(u, \ddot{z}), \tag{1}$$

where $X = \{x, y, z\}$ are the longitudinal (i.e. along the span length), vertical and lateral displacements; m_s, c and k are the structural mass, damping and stiffness; m_c is the crowd mass; u is the crowd density; F is the applied load.

It should be noticed that the Ordinary Differential Equation (ODE) (1) is non-linear for two reasons: first, the forcing term F is a function of both the crowd density and the lateral acceleration of the deck; second, the overall mass m is given by the sum of the structure and the crowd mass, which is computed by the solution of the equation governing the Crowd subsystem, in turn dependent on the solution of the ODE (1), as will be explained in the next sections.

2.2. *The Crowd subsystem*

The Crowd subsystem is described by a monodimensional first-order macroscopic model [11,6], that is, the crowd flow is assumed to be a continuous fluid and its dynamics is described through the derivation of an evolution equation for the mass density $u = u(x, t)$, considered as a macroscopic quantity of the flow. The derivation of the model refers to the main conservation equation, which is closed by a phenomenological relation that links the crowd velocity $v(x, t)$ to the crowd density, the so-called fundamental relation $v = v(u)$, in the form proposed by the authors in [8]:

$$\begin{aligned} &\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}(uv) = 0 \\ &v = v_M \left\{ 1 - \exp\left[-\gamma \left(\frac{1}{u} - \frac{1}{u_M} \right) \right] \right\} \end{aligned} \tag{2}$$

where x and t are the space and time variables and γ is a coefficient that sensitises the relation to different travel purposes (leisure/shopping, commuters/events, rush hour/business), obtained through a fitting of the data in [12] and [13]. Both v_M and u_M are made sensitive to the geographic area and the travel purpose by means of coefficients [8], determined through the observation data reported in [14]. In such a way the model is sensitized to psychological factors that are known to strongly affect crowd behaviour.

As pointed out in [11], the hyperbolic nature of the PDE can show unrealistic shock wave phenomena, due to the fact that conditions that correspond to a steady uniform flow are instantaneously imposed in unsteady conditions. This problem is tackled by introducing a space dislocation in the closure equation, as first suggested in [11], so that it takes the form:

$$v = v(u(x + \delta, t)). \tag{3}$$

The term δ accounts for the fact that the pedestrians do not react to the local crowd conditions, but to what happens in a suitable stretch of road in front of them. A similar assumption is made in [15] for a kinetic vehicular model, where δ is referred to as *visibility length*. In the case of pedestrian traffic, δ is interpreted as the *sensory distance* d_s, described in [8], that is, the forward length needed for perception, evaluation and reaction. It is worth pointing out that d_s is in turn a function of the walking velocity, as detailed in [8].

2.3. *The Structure-to-Crowd interaction*

In order to account for the Structure-to-Crowd interaction, the closure equation has to be adapted to sensitize the walking speed to the deck lateral motion. The following assuptions are retained from phenomenological observation:

- the motion of the platform, described by its acceleration $\ddot{z}$, reduces the walking velocity;
- the pedestrians adjust their step to the platform motion with a synchronization time delay $\Delta\tau$, which is expected to be greater than the time interval between two succeeding footfalls;
- after the pedestrians have stopped because of excessive lateral vibrations at time t_s, a stop-and-go time interval Δt_r should elapse before they start walking again.

According to these hypotheses, the term v_M in Eq. (2) is multiplied by a corrective factor $g(\ddot{z})$, which takes into account the sensitivity of v to the platform acceleration $\ddot{z}$. In order to define g, let us introduce the continuous function $\zeta(x,t)$ that represents the envelope of the deck acceleration time history ([6]). Hence, the corrective factor $g(x,t)$ takes the qualitative trend:

$$g(x,t) = \begin{cases} 1 & \zeta \leq \ddot{z}_c \cap t \geq t_s + \Delta t_r \\ (\ddot{z}_M - \zeta(x, t - \Delta\tau))/(\ddot{z}_M - \ddot{z}_c) & \ddot{z}_c < \zeta < \ddot{z}_M \cap t \geq t_s + \Delta t_r \\ 0 & \zeta \geq \ddot{z}_M \cap t_s < t < t_s + \Delta t_r \end{cases}, \tag{4}$$

where $\ddot{z}_c \cong 0.2$ m/s^2 [16] corresponds to the threshold of motion perception, while $\ddot{z}_M = 2.1$ m/s^2 [17] is the maximum acceptable acceleration above which pedestrians stop walking.

2.4. *The Crowd-to-Structure interaction*

The scheme in Fig. 1 shows that the Crowd-to-Structure interaction takes place in two ways. On one hand, the mass m is constantly updated by adding the pedestrian mass m_c to the structural mass m_s; on the other hand, a force model is proposed to determine the lateral force exerted by pedestrians on the footbridge deck.

The macroscopic time-domain force model, described in details in [9] and herein briefly recalled, is based on the assumption that the force exerted by a cluster of n pedestrians walking along a portion of the bridge span is given by the sum of three components: F_{ps}, which is the term due to n_{ps} pedestrians synchronized to the structure; F_{pp}, which is due to n_{pp} pedestrians synchronized to each other; F_s, that is the part due to n_s uncorrelated pedestrians. The amplitude of each force component is expressed as the product of the corresponding single-pedestrian force [1,18] and the weights:

$$\begin{aligned} n_{ps} &= nS_{ps}, \\ n_{pp} &= nS_{pp}(1 - S_{ps}), \\ n_s &= n - n_{ps} - n_{pp}. \end{aligned} \tag{5}$$

The coefficient S_{ps} represents the degree of coupling between the crowd and the structure:

$$S_{ps}(\zeta, f_r) = \left[1 - e^{-b(\zeta - \ddot{z}_c)}\right]\left[e^{[-\eta(\zeta)(f_r-1)^2]}\right], \tag{6}$$

where f_r is the ratio between the step lateral frequency f_{pl} and the structure lateral frequency f_s, while $\eta(\zeta) = 50e^{(-20\zeta/\pi)}$. The pedestrian-pedestrian synchronization coefficient, S_{pp}, is expressed as:

$$S_{pp}(u) = \frac{1}{2}\left\{1 + erf\left[a\left(u - \frac{u_{sync} + u_c}{2}\right)\right]\right\}, \tag{7}$$

where $a = 3.14$, $u_c = 0.3$ ped/m^2 is the upper limit for unconstrained free walking [12] and $u_{sync} = 1.8$ ped/m^2 is assumed to correspond to complete synchronization.

As far as the frequency content is concerned, F_{ps} varies in time with the same frequency f_s as the structure lateral frequency, while the other two terms vary with the step lateral frequency f_{pl}, which is calculated as a function of the walking velocity v [9], using the experimental data in [19].

The force model allows some important features of the synchronous lateral excitation phenomenon to be taken into account: the existence of two kinds of synchronization; the presence of different frequency components in the overall force; triggering of the lock-in phenomena and the resulting self-limited oscillations.

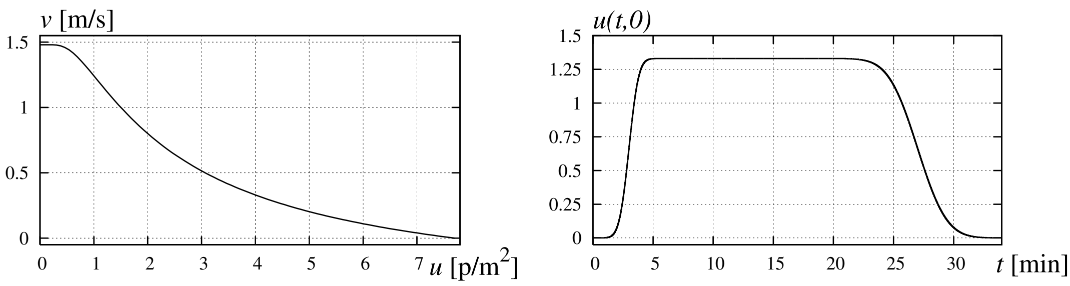
3. Application

This model has been tested by computational simulation of a crowd event on a cable-stayed footbridge, the T-bridge (Toda Park Bridge, Toda City, Japan), which is widely described in literature [3,4,17,20]. The T-bridge links a boat race stadium to a bus terminal: when big boat races finish, up to 20000 people leave the stadium to reach the bus terminal, resulting in very congested conditions along the footbridge span $L = 178$ m.

The closure equation has been adapted for the case of Asia and rush-hour traffic, that is, $u_M = 7.7$ ped/m^2, $v_M = 1.48$ m/s and $\gamma = 0.273u_M$ [8] (Fig. 2a). The imposed initial and boundary conditions on u have been established in order to simulate about 14000 people leaving the stadium and crossing the bridge, which is initially empty, with a maximum crowd density of 1.33 ped/m^2 (Fig. 2b).

Fig. 3 reports the time-space distributions of some main variables obtained through the computational simulation: the crowd density u, the deck lateral acceleration $\ddot{z}$ and the force components per unit length. In order to retain only the large time-scale fluctuation, figures 3b-f graph the envelope of the variable maxima.

The overall evolution in time of u is mainly due to the imposed boundary condition at the inlet. In other words, the crowd dynamics is not affected by non-linear traffic phenomena due to a crowd density above the capacity value (i.e. the value of u for which the flow $q = uv$ reaches its maximum value) or to the effects of excessive lateral acceleration of the deck, that is, $\ddot{z} \geq \ddot{z}_M$. As far as the deck response is concerned, the steady-state response is shorter in time than the density one: the deck lateral acceleration



(a) (b)

Figure 2. Closure equation (a) and boundary condition at the inlet (b)

reaches the steady-state ($t \approx 15$ min) more slowly than u and decreases abruptly just after the maximum amplitude has been reached ($t \approx 25$ min), when u still has its steady-state value. An analysis of the results shows that these transient structural responses have different causes.

The $\ddot{z}$ crisis at $t \approx 25$ min can be explained by looking at the distributions of the force components. When $\ddot{z}$ exceeds the threshold of motion perception $\ddot{z}_c$, some pedestrians synchronize with the structure (eqn. (5)), so that F_{ps} is not null and has a space distribution that matches the deck deformed shape. As a consequence, the number of pedestrians syncronized to each other decreases, causing a decay of F_{pp}. Otherwise, if $\ddot{z} \leq \ddot{z}_c$, F_{pp} follows the same trend as u. F_s follows from the other two components: it has an increase when both F_{ps} and F_{pp} are null, that is, when $u \leq u_c$ and $\ddot{z} \leq \ddot{z}_c$. Fig. 3f clearly shows that the resulting total force F is mainly due to the F_{pp} component, since the magnitude of F_{ps} is small in the case-study. The total force amplitude decay in the time interval 15-25 min can explain the corresponding crisis in the deck response at $t \approx 25$ min.

The obtained results also show a good agreement with the measurements reported in [3], as far as the crowd condition along the deck and the structural response are concerned, both from a qualitative and quantitative point of view.

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References

[1] S. Živanović, A. Pavic, and P. Reynolds. Vibration serviceability of footbridges under human-induced excitation: a literature review. *J. of Sound and Vibration*, 279:1–74, 2005.

[2] P. Dallard, T. Fitzpatrick, A. Flint, S. Le Bourva, A. Low, R. M. Ridsdill, and M. Willford. The London Millennium Footbridge. *The Structural Engineer*, 79(22):17–33, 2001.

[3] Y. Fujino, B. M. Pacheco, S. Nakamura, and P. Warnitchai. Synchronization of human walking observed during lateral vibration of a congested pedestrian bridge. *Earthquake Engineering and Structural Dynamics*, 22:741–758, 1993.

[4] S. Nakamura and T. Kawasaki. Lateral vibration of footbridges by synchronous walking. *Journal of Constructional Steel Research*, 62:1148–1160, 2006.

[5] F. Venuti, L. Bruno, and N. Bellomo. Crowd-structure interaction: dynamics modelling and computational simulation. In *Proceedings Footbridge 2005*, Venezia, 2005.

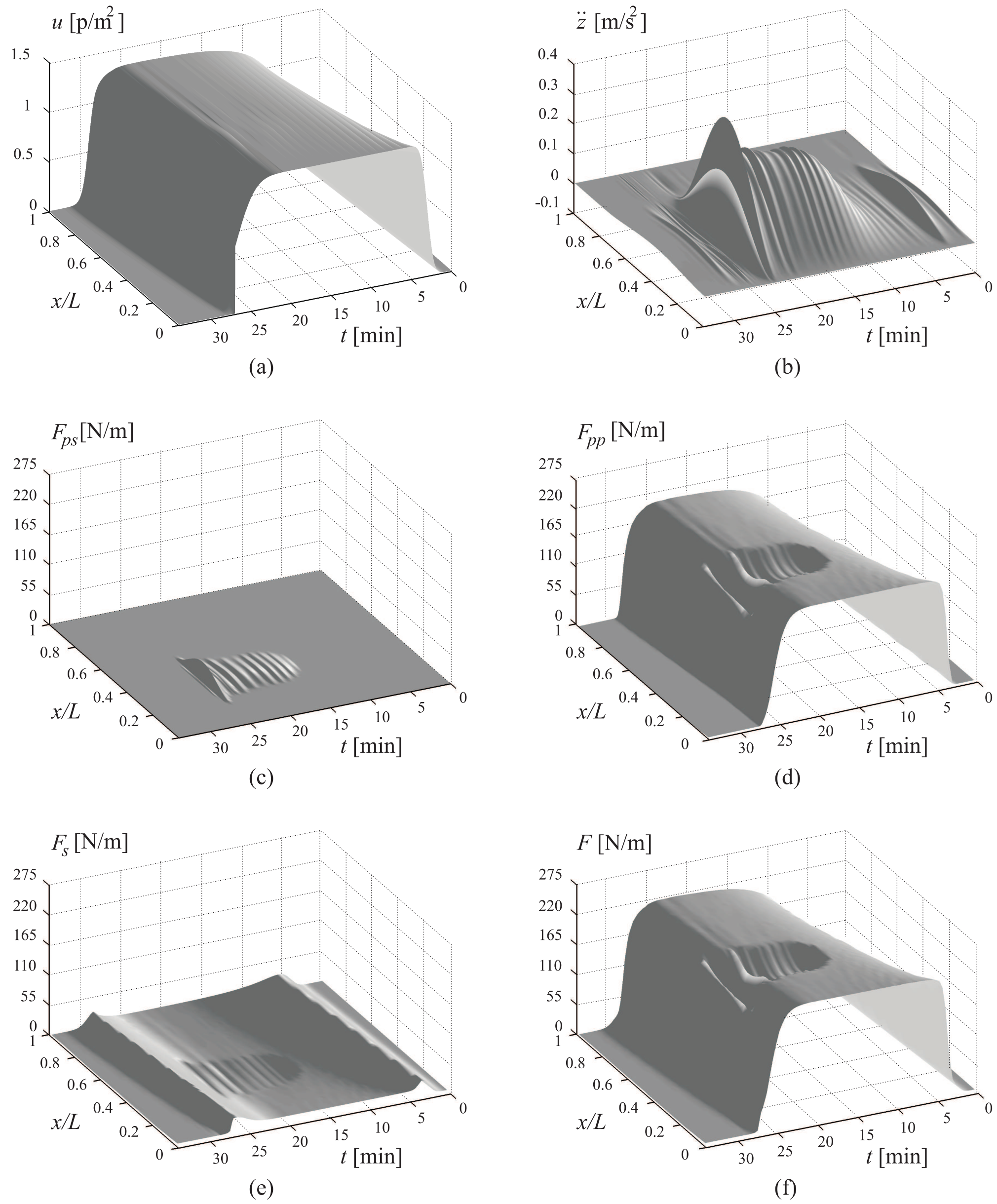

Figure 3. Space-time distributions of the main variables

[6] F. Venuti, L. Bruno, and N. Bellomo. Crowd dynamics on a moving platform: Mathematical modelling and application to lively footbridges. *Mathematical and Computer Modelling*, 45:252–269, 2007.

[7] J. Bodgi, S. Erlicher, and P. Argoul. Lateral vibration of footbridges under crowd-loading: Continuous crowd modeling approach. *Key Engineering Materials*, 347:685–690, 2007.

[8] F. Venuti and L. Bruno. An interpretative model of the pedestrian fundamental relation. *C.R. Mecanique*, 335:194–200, 2007.

[9] F. Venuti and L. Bruno. Pedestrian lateral action on lively footbridges: a new load model. *SEI Structural Engineering International*, 17(3), 2007.

[10] K. C. Park, C. A. Felippa, and C. Farhat. *Partitioned analysis of coupled mechanical systems*. University of Colorado, March 1999. Report no. CU-CAS-99-06.

[11] N. Bellomo and V. Coscia. First order models and closure of mass conservation equations in the mathematical theory of vehicular traffic flow. *C.R. Mecanique*, 333:843–851, 2005.

[12] D. Oeding. Verrkhersbelastung und Dimensionierung von Gehwegen und anderen Anlagen des Fußgangerverkhers. *Strassenbau and Strassenverkherstechnik*, 22:36–40, 1963.

[13] J. J. Fruin. *Pedestrian planning and design*. Elevator World Inc., 1987.

[14] S. Buchmueller and U. Weidmann. *Parameters of pedestrians, pedestrian traffic and walking facilities*. ETH Zürich, October 2006. Ivt Report no. 132.

[15] M. Delitala and A. Tosin. Mathematical modeling of vehicular traffic: a discrete kinetic approach. *Math. Mod. Meth. Appl. Sci.*, 17:901–932, 2007.

[16] ISO International Standardization Organization, Geneva, Switzerland. *Bases for Design of Structures - Serviceability of Buildings Against Vibrations*, 1992. ISO 10137.

[17] S. Nakamura. Field measurement of lateral vibration on a pedestrian suspension bridge. *The Structural Engineer*, 81(22):22–26, 2003.

[18] A. D. Pizzimenti. *Analisi sperimentale dei meccanismi di eccitazione laterale delle passerelle ad opera dei pedoni*. PhD thesis, Universitá degli Studi di Catania, Dottorato in Ingegneria delle Strutture XVII ciclo, 2005.

[19] J. E. Bertram and A. Ruina. Multiple walking speed-frequency relations are predicted by constrained optimization. *J. theor. Biol.*, 209:445–453, 2001.

[20] S. Nakamura and Y. Fujino. Lateral vibration on a pedestrian cable-stayed bridge. *Structural Engineering International*, 12(4):295–300, 2002.