

Preface

This booklet is intended to serve as a ready-to-use reference for the minicourse given by the author during the XXII *École de Algebra*, in Salvador de Bahia (July 2012). The purpose of these lecture notes is twofold. On one hand they aim to introduce and advertise a natural, flexible and elegant purely combinatorial–algebraic approach to the well-known classical theory of linear ODEs with constant coefficients (Chapter 3), and to introduce generalised Wronskians associated to a fundamental system of solutions (Chapter 4). Elementary applications will be shown, e.g. to the computation of the exponential of a square matrix without reducing to the Jordan normal form (Chapter 5). On the other hand it wishes to bring to the fore a number of relationships with other branches of mathematics. Examples include the theory of symmetric functions (Example 2.1.3), the theory of universal decomposition algebras associated to a polynomial (Example 3.2.8 and Remark 6.1.8), derivations of the exterior algebra of a free module (Chapter 6), D -modules (Example 3.2.3), Schubert calculus for the complex Grassmannian (Section 6.2, boson–fermion correspondence in the representation theory of infinite–dimensional Lie algebras (like the Virasoro algebra) seen as an infinite–dimensional analogue of Poincaré’s duality for the complex Grassmannians. The present exposition is totally inspired by the paper [15] and must be considered an expanded version of it.

The level of the exposition is elementary, given that more than seventy percent of the material can be followed with a basic understanding of polynomial algebras and the Leibniz rule for the product of two differentiable functions. More advanced topics, like Schubert Calculus or the bosonic representation of the oscillator algebra have been only sketched in the last two chapters. A deeper knowledge of those subjects is not necessary for the purposes of the minicourse, as they have been treated just to provide further examples to certify the surprising ubiquity of the Jacobi-Trudy formula in mathematics.

The present lecture notes have been written on a short notice, so they will certainly contain misprints and omissions and possibly some mistakes. Corrections

and/or integrations can be found in the author's web page at the url

<http://calvino.polito.it/~gatto/public/XXIIEA/bahia.htm>

ACKNOWLEDGMENTS. The author wishes to express his warm gratitude to the Scientific Committee of the XXII Algebra School for having offered him the opportunity to give a minicourse on this subject, as well as to the Organizing Committee, especially to Thierry Petit Lobão, for careful assistance. It is also a pleasure to thank Parham Salehyan which made possible a longer stay in Brasil.

Very special thanks are due to Ina Scherbak, the ideal coauthor of these notes, who generously shared her insight and helped me with many advises. I also thank Caterina Cumino for many discussions and my friend Simon Chiossi not only for enlightening discussions but especially for his constant encouragement. For the last sketchy chapter of these notes I am deeply indebted to Maxim Kazarian, from whom I first learned about the boson–fermion correspondence.

This work has been partially sponsored by the italian GNSAGA¹-INDAM², the PRIN³ “Geometria sulle Varietà Algebriche” (coordinated by A. Verra), by *Filters srl* (Scalenghe, TO) and the coffee brand *Curt'eNiro* (Pianezza, TO).

Estas notas de aula são dedicadas ao Aron Simis, por ocasião do septuagésimo aniversário dele, desejando-lhe mais outros setenta anos de feliz atividade matemática.

Sangano, 23 de Maio 2012.

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