# A Fall Meeting in Algebraic Geometry 



Titles and Abstract

# Theta divisors and the Geometry of the tautological model 

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#### Abstract

Let $E$ be a stable vector bundle of rank $r$ and slope $2 g-1$ on a smooth irreducible complex projective curve $C$ of genus $g \geq 3$. In this talk we show a relation between theta divisor $\Theta_{E}$ and the geometry of the tautological model $P_{E}$ of $E$. In particular, we prove that for $r>g-1$, if $C$ is a Petri curve and $E$ is general in its moduli space then $\Theta_{E}$ defines an irreducible component of the variety parametrizing $(g-2)$-linear spaces which are $g$-secant to the tautological model $P_{E}$.


## References

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# On the existence of Ulrich bundles on some surfaces 

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#### Abstract

An Ulrich bundle on a variety embedded in the projective space is a vector bundle that admits a linear resolution as a sheaf on the projective space.

Ulrich bundles have many interesting properties: e.g. they are semistable and have no intermediate cohomology. Moreover, the existence of Ulrich bundles of low rank on $X$ is related to the problem of finding, whenever possible, linear determinantal or linear pfaffian descriptions of the Chow form of $X$.

Ulrich bundles on curves can be easily described. This is no longer true for Ulrich bundles on a surface, though an almost easy characterization is still possible.

In the talk we focus our attention on this latter case. In particular we deal with surfaces $S$ such that $p_{g}(S)=0, q(S) \leq 1$ and the hyperplane linear system is non-special. In this case, we prove the existence of certain Ulrich bundles of rank 2 on them, giving some results on the size of families of Ulrich bundles and dealing with their stability properties.


# Weierstrass points that occur at most once 

## Coppens Marc ${ }^{1}$

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Abstract. Let $a \geq 3$ and $b \geq a+2$ with $b \neq n a+(a-1)$ for some natural number $n$. We prove that if a smooth curve $C$ has a Weierstrass point $P$ with Weierstrass semigroup equal to $\langle a, b\rangle$ (the semigroup generated by $a$ and $b$; those are the so-called $C_{a, b}$ curves) then this point $P$ is unique. This is proved in two steps: the linear system $g_{a}^{1}$ on $C$ is unique and each other point $Q$ satisfying $a Q \in|a P|$ has Weierstrass semigroup different from $\langle a, b\rangle$.

In case of small genus $g$ curves $C$ and Weierstrass points $P$ with Weierstrass semigroup containing $\langle a, b\rangle$ we obtain a lower bound on $g$ such that all other points $Q$ with $a Q \in|a P|$ have another Weierstrass semigroup. We prove this lower bound is sharp in some sense. Also in many cases we obtain $C$ has a unique $g_{a}^{1}$, therefore obtaining a lot of Weierstrass semigroups that can occur at most once on a curve.

# Degenerations of Hilbert schemes of points 

Martin G. Gulbrandsen ${ }^{1}$<br>${ }^{1}$ Department of Mathematics and Natural Sciences<br>University of Stavanger


#### Abstract

In an attempt at exploring degenerations of hyperKähler manifolds, and with the Kulikov-Pinkham-Persson degenerations of K3 surfaces as guide, I will explain a GIT construction of degenerations of Hilbert schemes, building on ideas due to $\mathrm{Li}-\mathrm{Wu}$ on admissible subschemes in expanded degenerations. As application we produce examples of degenerations of $\operatorname{Hilb}^{n}(\mathrm{~K} 3)$ which are good minimal dlt-models, with dual complex of the degenerate fibre being simplicial subdivisions of an n-simplex. This is joint work with Lars Halle, Klaus Hulek and Ziyu Zhang.


## References

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# Abel-Jacobi faces in the symmetric product of a curve 

Angelo Felice Lopez ${ }^{1}$

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Abstract. Let $C$ be a smooth irreducible curve of genus $g$. We study the cones of pseudoeffective cycles on the symmetric product $C_{d}$ of $C$. We will show that, using the contractibility properties of Abel-Jacobi morphism $C_{d} \rightarrow \operatorname{Pic}^{d}(C)$, we can produce several examples of perfect extremal faces, extremal rays and edges of the cone $\operatorname{PsEff}_{n}\left(C_{d}\right)$.

# The arc space of the Grassmannian 

Antonio Nigro ${ }^{1}$<br>${ }^{1}$ Universidade Federal Fluminense, Brazil

Abstract. Arc spaces (infinite jets) have been featured repeatedly in recent years in algebraic geometry as a tool to study invariants of varieties and of singularities. In this talk we shall focus on the Grassmannian and on its Schubert varieties. We will discuss a stratification of the arc space of the Grassmannian, indexed by plane partitions, whose geometric and combinatorial structure leads to the computation of invariants, mainly the log canonical threshold of pairs involving Schubert varieties. Joint work with Roi Docampo.

# A tropical approach to the Brill-Noether theory of curves on $\mathbb{P}^{1} \times \mathbb{P}^{1}$ 

Marta Panizzut ${ }^{1}$
${ }^{1}$ TU Berlin


#### Abstract

Baker and Norine introduced the terminology of linear systems on graphs in analogy with the one on algebraic curves. Their groundbreaking work has led to the development of a Brill-Noether theory on tropical curves, which provides new combinatorial insights in the study of linear systems on algebraic curves. The interplay between the tropical and the classical theory is given by specialization of linear systems from curves to dual graphs of degenerations.

In this talk, I will address questions on the Brill-Noether theory of generic smooth curves on $\mathbb{P}^{1} \times \mathbb{P}^{1}$ by specializing to complete bipartite graphs.

This is a joint work with with Filip Cools, Michele D'Adderio and David Jensen.


# Kaledin class and formality of the moduli space of sheaves on K3s. 

Elena Martinengo ${ }^{1}$<br>${ }^{1}$ Dipartimento di Matematica, Universitá di Torino


#### Abstract

In 2007 Kaledin proved a geometric theorem of formality in familes using as major tool a class that he associated to a differential graded algebra (or in general to an A-infinity algebra). In this talk I will explain how this "Kaledin" class is defined, in which part of the Hochschild cohomology it lives and why its vanishing controls the formality. As an application of this theory I will sketch a proof of the conjecture formulated by Kaledin and Lehn in 2007 about the singularities of the moduli space of sheaves on a K3 surface. This last part is a work in progress with Manfred Lehn.


# Euclidean projective geometry, reciprocal polar varieties, and focal loci 

Ragni Piene ${ }^{1}$<br>${ }^{1}$ Department of Mathematics, University of Oslo, Norway


#### Abstract

We can regard an affine space as projective space minus a hyperplane at infinity, together with a notion of perpendicularity in that hyperplane. This gives a "Euclidean" structure on the affine space, including a Euclidean normal bundle of a given variety. This is used to define the reciprocal polar varieties, the end point map, and the focal loci. The degrees of the reciprocal polar varieties can be expressed in terms of the degrees of the ordinary polar varieties, also in the case that the variety is singular. There has been recent interest in computing, or bounding, these degrees. I will give several examples, especially in the case of curves, surfaces, and toric varieties.


# A differential equation on elliptic curves and an enumerative problem 

Gian Pietro Pirola ${ }^{1}$,

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Abstract. Let $E$ be a complex elliptic curve. We study the equation

$$
\begin{equation*}
\mathrm{dg}=\mathrm{f}^{2} \mathrm{dz} \tag{1}
\end{equation*}
$$

where $f, g: E \rightarrow \mathbb{C}$ are meromorphic functions on $E$. We discuss low degree the solutions of (11). The main motivation is the following enumerative problem: find the number of odd ramification coverings $h: C \rightarrow \mathbb{P}^{1}$ of degee $2 g+1$ where $C$ is the general curve of genus $g$. This last problem is a collaboration project with G. Farkas, R. Moschetti and J.C.Naranjo.

# The DT/PT correspondence at a smooth curve 

Andrea Ricolfi ${ }^{1}$

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#### Abstract

The "DT/PT correspondence" is an example of wallcrossing formula relating two different theories enumerating curves on Calabi-Yau threefolds, namely Donaldson-Thomas (DT) theory and Pandharipande-Thomas theory. The correspondence was conjectured in [2], and proved in [1, 3]. The enumerative invariants appearing in the formula are defined for each homology class $\beta \in H_{2}(Y, \mathbb{Z})$ on the ambient threefold $Y$. We will show a "local" version of this wall-crossing formula, involving DT and PT invariants computing the "contribution" of a single smooth curve $C \subset Y$ to the global invariants.


## References

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# Degrees of spaces of holomorphic foliations in $\mathbb{C P}^{n}$ 

Israel Vainsencher ${ }^{1}$<br>${ }^{1}$ Dept. Math. UFMG

Abstract. A holomorphic foliation of codimension 1 and degree $k$ in $\mathbb{C P}^{n}$ is defined by a 1 -form $\omega=A_{0} d x_{0}+\cdots+A_{n} d x_{n}$, up to scalar multiple, where the $A_{i}$ denote homogeneous polynomial of degree $k+1$, satisfying (i) $i_{R} \omega:=\sum A_{i} x_{i}=0$ (projectivity), and (ii) $\omega \wedge d \omega=0$ (Frobenius integrability).

Condition (i) (resp. (ii)) yields linear (resp. quadratic) equations for the space of foliations $\mathbb{F}(k, n)$, a closed subscheme of $\mathbb{P}^{N}=$ $\mathbb{P}\left(H^{0}\left(\Omega_{\mathbb{P}^{n}}^{1}(k+2)\right)\right)$. Since Jouanolou [3] and Cerveau\&Lins Neto [1], the quest for describing the irreducible components of $\mathbb{F}(k, n)$, has received a lot of attention. We survey results concerning the determination of the degrees of some families of components obtained in collaboration with Fernando Cukierman, Jorge Vitório Pereira [2], Daniel Leite 4] and Artur Rossini [5],

## References

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