DIMENSION OF FAMILIES OF DETERMINANTS SCHEMES

Rosa M. Miró-Roig

Abstract

This talk describes joint work with Jan O. Kleppe. A scheme $X \subset P^{n+c}$ of codimension c is called *standard determinantal* if its homogeneous saturated ideal can be generated by the maximal minors of a homogeneous $t \times (t+c-1)$ matrix and X is said to be *good determinantal* if it is standard determinantal and a generic complete intersection. Given integers $a_0, a_1, ..., a_{t+c-2}$ and $b_1, ..., b_t$ we denote by $W(\underline{b}; \underline{a}) \subset \mathrm{Hilb}^p(P^{n+c})$ (resp. $W_s(\underline{b}; \underline{a})$) the locus of good (resp. standard) determinantal schemes $X \subset P^{n+c}$ of codimension c defined by the maximal minors of a $t \times (t+c-1)$ matrix $(f_{ij})^{i=1,...,t}_{j=0,...,t+c-2}$ where $f_{ij} \in k[x_0, x_1, ..., x_{n+c}]$ is a homogeneous polynomial of degree $a_j - b_i$.

In the talk I will address the following three fundamental problems :

(1) To determine the dimension of $W(\underline{b}; \underline{a})$ (resp. $W_s(\underline{b}; \underline{a})$) in terms of a_j and b_i;
(2) To determine whether the closure of $W(\underline{b}; \underline{a})$ is an irreducible component of $\mathrm{Hilb}^p(P^{n+c})$; and
(3) To determine when $\mathrm{Hilb}^p(P^{n+c})$ is generically smooth along $W(\underline{b}; \underline{a})$.

Concerning question (1) I will give an upper bound for the dimension of $W(\underline{b}; \underline{a})$ (resp. $W_s(\underline{b}; \underline{a})$) which works for all integers $a_0, a_1, ..., a_{t+c-2}$ and $b_1, ..., b_t$, and we conjecture that this bound is sharp. The conjecture is proved for $2 \leq c \leq 5$, and for $c \geq 6$ under some restriction on $a_0, a_1, ..., a_{t+c-2}$ and $b_1, ..., b_t$. For questions (2) and (3) we have an affirmative answer for $2 \leq c \leq 4$ and $n \geq 2$, and for $c \geq 5$ under certain numerical assumptions.