

LINEAR ALGEBRA AND GEOMETRY

60 min

Instructions:

- Write your surname and name and your student number in CAPITAL LETTERS where required.
- **YOU MUST WRITE THE SOLUTIONS TO THE EXERCISE IN THE DEDICATED BOX.**
- To access your exam papers you **must** remember the **NUMBER** of the room you are sitting in and the **SYMBOL** in the top right corner of this page.
- For each multiple choice you **must** select one of the possible answers in the table in **this** page.
- Write neatly and clearly the solution of the exercise; you **must** explain your solution.
- As a draft use your own scrap paper or the one provided; only this booklet will be graded.

SURNAME, NAME: _____

STUDENT NUMBER: _____

PROFESSOR: GATTO_____

Q1	a	b	c	d	Q5	a	b	c	d
Q2	a	b	c	d	Q6	a	b	c	d
Q3	a	b	c	d	Q7	a	b	c	d
Q4	a	b	c	d	Q8	a	b	c	d

Do not write here

QUIZ

EXERCISE

TOTAL

QUIZ

Q1. Consider the lines $r : x = y = 2z$ and $s : 2x = y = z$. Find the correct statement.

- (a) $r \parallel s$.
- (b) $r \perp s$.
- (c) $r \cap s = \emptyset$.
- (d) r and s are coplanar.

Q2. Suppose that the linear system $Ax = \mathbf{b}$ has infinitely many solutions when $\mathbf{b} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$. Then it is necessarily true that.

- (a) A is invertible.
- (b) The rank of A is zero.
- (c) The augmented matrix $(A|\mathbf{b})$ is invertible.
- (d) The matrix A and the augmented matrix $(A|\mathbf{b})$ have the same rank.

Q3. Let $Q(x, y, z, t) = 2xz + y^2 - 3t^2$ be a quadratic form. Find the correct statement.

- (a) Q is positive definite.
- (b) Q is indefinite.
- (c) Q is negative definite.
- (d) None of the other options is correct.

Q4. Consider the vectors $v_1 = (1, 0, 1)$, $v_2 = (0, 1, 1)$, $v_3 = (1, 1, 2)$, and $v_4 = (2, 1, 3)$. Find the correct statement.

- (a) $\{v_1, v_2, v_3\}$ is a basis of $\mathbb{R}^3$.
- (b) $\{v_1, v_2, v_3, v_4\}$ is a basis of $\mathbb{R}^3$.
- (c) $\{v_1, v_2, v_1 \times v_2\}$ is a basis of $\mathbb{R}^3$.
- (d) $\{v_1 + v_2, v_2 + v_3, v_3 + v_4\}$ generates $\mathbb{R}^3$.

Q5. Consider the linear mapping $f \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} x + t \\ x + z \\ x + y \\ y + z \end{pmatrix}$. Find the correct statement.

- (a) f is injective.
- (b) f is not surjective.
- (c) $f^{-1} \left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} \right\}$
- (d) $\mathcal{L}\{(1, 1, 1, 1)\} = \text{Ker}(f)$.

- Q6.** Consider the circles $\mathcal{C}_1 : x^2 + y^2 + z^2 - 4 = x = 0$, $\mathcal{C}_2 : x^2 + y^2 + z^2 - 4 = y = 0$, and let r_i be the radius of $\mathcal{C}_i$. Find the correct statement.
- (a) $r_1 > r_2$.
 - (b) $r_1 = r_2$.
 - (c) $r_1 < r_2$.
 - (d) $\mathcal{C}_1$ and $\mathcal{C}_2$ lie in the same plane.
- Q7.** Let $A(1, 1, 0)$, $B(1, 0, 1)$, $C(0, 1, 1)$ be points in $\mathbb{R}^3$. Find the correct statement.
- (a) The vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are parallel.
 - (b) The points A, B and C are collinear.
 - (c) The plane containing A, B, C also contains $(0, 0, 0)$.
 - (d) The triangle of vertices A, B, C is equilateral.
- Q8.** Let A, B be $n \times n$, $n \geq 1$ matrices such that AB is not invertible. Find the correct statement.
- (a) $A^\top B^\top$ is invertible.
 - (b) $\text{rk}(AB) = n$.
 - (c) $\det(A) \det(B) = 0$.
 - (d) $\lambda = 0$ is not an eigenvalue of AB .

EXERCISE

Part A. (up to 4 points) Let $\alpha \in \mathbb{R}$ and consider the linear mapping $F_\alpha : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ such that

$$F_\alpha \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x + \alpha y \\ 2x + 2\alpha y \end{pmatrix}.$$

(1) Set $\alpha = 1$ and compute all the eigenvalues of F_α . (2) Set $\alpha = 1$ and compute a basis for each eigenspace of F_α .

Part B. (up to 4 points) Let $\beta \in \mathbb{R}$, $V = \mathbb{R}_2[x]$ be the vector space of polynomials with real coefficients of degree at most 2, and $W = \mathbb{R}^{2,2}$ be the vector space of real 2×2 matrices. Consider the linear mapping $G_\beta : V \longrightarrow W$ such that

$$G_\beta(p(x)) = \begin{pmatrix} p(0) & p(\beta) \\ p'(0) & \beta p(1) \end{pmatrix}.$$

(1) Set $\beta = 1$ and compute a basis of $\text{Ker}(G_\beta)$ made by elements of V . (2) Set $\beta = 1$ and compute a basis of $\text{Im}(G_\beta)$ made by elements of W .

LINEAR ALGEBRA AND GEOMETRY
60 min

Instructions:

- Write your surname and name and your student number in CAPITAL LETTERS where required.
- YOU MUST WRITE THE SOLUTIONS TO THE EXERCISE IN THE DEDICATED BOX.
- To access your exam papers you **must** remember the NUMBER of the room you are sitting in and the SYMBOL in the top right corner of this page.
- For each multiple choice you **must** select one of the possible answers in the table in **this** page.
- Write neatly and clearly the solution of the exercise; you **must** explain your solution.
- As a draft use your own scrap paper or the one provided; only this booklet will be graded.

SURNAME, NAME: _____

STUDENT NUMBER: _____

PROFESSOR: GATTO_____

Q1	a	b	c	d	Q5	a	b	c	d
Q2	a	b	c	d	Q6	a	b	c	d
Q3	a	b	c	d	Q7	a	b	c	d
Q4	a	b	c	d	Q8	a	b	c	d

Do not write here

QUIZ		EXERCISE		TOTAL	
------	-------------	----------	-------------	-------	-------------

QUIZ

- Q1.** Consider the vectors $v_1 = (1, 0, 1)$, $v_2 = (0, 1, 1)$, $v_3 = (1, 1, 2)$, and $v_4 = (2, 1, 3)$. Find the correct statement.
- (a) $\{v_1, v_2, v_3\}$ generates $\mathbb{R}^3$.
 - (b) $\{v_1, v_2, v_3, v_4\}$ does not generate $\mathbb{R}^3$.
 - (c) $\{v_1, v_2, v_1 \times v_2, v_3\}$ is a basis of $\mathbb{R}^3$.
 - (d) $\{v_1, v_3, v_4\}$ is a basis of $\mathbb{R}^3$.
- Q2.** We know that the linear system $Ax = b$ has infinitely many solutions when $b = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$. Then it is necessarily true that.
- (a) A is not invertible.
 - (b) The rank of A is zero.
 - (c) The augmented matrix $(A|b)$ is invertible.
 - (d) The matrix A and the augmented matrix $(A|b)$ do not have the same rank.
- Q3.** Let $A(2, 0, 0)$, $B(0, 2, 0)$, $C(0, 0, 2)$ be points in $\mathbb{R}^3$. Find the correct statement.
- (a) The vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are parallel.
 - (b) The points A, B, C are collinear.
 - (c) The plane containing A, B, C also contains $(3, 1, 0)$.
 - (d) The triangle of vertices A, B, C has area 1.
- Q4.** Consider the linear mapping $f(x, y, z, t) = (z, t, y, x)$. Find the correct statement.
- (a) f is not injective.
 - (b) f is not surjective.
 - (c) $f^{-1}(1, 2, 3, 4) = \{(4, 3, 1, 2)\}$
 - (d) $\mathcal{L}\{(1, 0, 1, 0)\} = \text{Ker}(f)$.
- Q5.** Let $Q(x, y, z, t) = x^2 + 2xy + y^2 + z^2 + 4t^2$ be a quadratic form. Find the correct statement.
- (a) Q is positive definite.
 - (b) Q is indefinite.
 - (c) Q is negative definite.
 - (d) None of the other options is correct.

Q6. Let A, B be $n \times n$, $n \geq 1$ matrices such that AB is not invertible. Find the correct statement.

- (a) $AB^\top$ is invertible.
- (b) $\text{rk}(AB) > n$.
- (c) $\lambda = 0$ is an eigenvalue of $(AB)^\top(AB)$.
- (d) A and B are invertible.

Q7. Consider the planes $\alpha : x + 2y + z = 5$ and $\beta : 2x - y + z = 1$. Find the correct statement.

- (a) $\alpha \parallel \beta$.
- (b) $\alpha \perp \beta$.
- (c) $\alpha \cap \beta \neq \emptyset$.
- (d) $\alpha \cup \beta$ is a plane.

Q8. Consider the circles $\mathcal{C}_1 : x^2 + y^2 + z^2 - 4 = z - 1 = 0$, $\mathcal{C}_2 : x^2 + y^2 + z^2 - 4 = x + y = 0$, and let r_i be the radius of $\mathcal{C}_i$. Find the correct statement.

- (a) $r_1 > r_2$.
- (b) $r_1 = r_2$.
- (c) $r_1 < r_2$.
- (d) $\mathcal{C}_1$ and $\mathcal{C}_2$ lie in the same plane.

EXERCISE

Part A. (up to 4 points) Let $\alpha \in \mathbb{R}$ and consider the linear mapping $F_\alpha : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ such that

$$F_\alpha \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x + \alpha y \\ 2x + 2\alpha y \end{pmatrix}.$$

(1) Set $\alpha = 1$ and compute all the eigenvalues of F_α . (2) Set $\alpha = 1$ and compute a basis for each eigenspace of F_α .

B

Part B. (up to 4 points) Let $\beta \in \mathbb{R}$, $V = \mathbb{R}_2[x]$ be the vector space of polynomials with real coefficients of degree at most 2, and $W = \mathbb{R}^{2,2}$ be the vector space of real 2×2 matrices. Consider the linear mapping $G_\beta : V \longrightarrow W$ such that

$$G_\beta(p(x)) = \begin{pmatrix} p(0) & p(\beta) \\ p'(0) & \beta p(1) \end{pmatrix}.$$

(1) Set $\beta = 1$ and compute a basis of $\text{Ker}(G_\beta)$ made by elements of V . (2) Set $\beta = 1$ and compute a basis of $\text{Im}(G_\beta)$ made by elements of W .

LINEAR ALGEBRA AND GEOMETRY

60 min

Instructions:

- Write your surname and name and your student number in CAPITAL LETTERS where required.
- **YOU MUST WRITE THE SOLUTIONS TO THE EXERCISE IN THE DEDICATED BOX.**
- To access your exam papers you **must** remember the *NUMBER* of the room you are sitting in and the *SYMBOL* in the top right corner of this page.
- For each multiple choice you **must** select one of the possible answers in the table in **this** page.
- Write neatly and clearly the solution of the exercise; you **must** explain your solution.
- As a draft use your own scrap paper or the one provided; only this booklet will be graded.

SURNAME, NAME: _____

STUDENT NUMBER: _____

PROFESSOR: GATTO _____

Q1	a	b	c	d	Q5	a	b	c	d
Q2	a	b	c	d	Q6	a	b	c	d
Q3	a	b	c	d	Q7	a	b	c	d
Q4	a	b	c	d	Q8	a	b	c	d

Do not write here

QUIZ

EXERCISE

TOTAL

QUIZ

- Q1.** Consider the linear mapping $f(x, y, z, t) = (t, x, z, y)$. Find the correct statement.
- (a) f is not injective.
 - (b) $\mathcal{L}\{(1, 1, 1, 1)\} = \text{Ker}(f)$.
 - (c) f is surjective.
 - (d) $f^{-1}(1, 2, 3, 4) = \{(1/4, 1/2, 1/3, 1)\}$.
- Q2.** Consider the vectors $v_1 = (2, 1, 0)$, $v_2 = (1, 2, 0)$, $v_3 = (3, 3, 0)$, and $v_4 = (0, 0, 1)$. Find the correct statement.
- (a) $\{v_1, v_2, v_4\}$ is a basis of $\mathbb{R}^3$.
 - (b) $\{v_1, v_2, v_3\}$ generates $\mathbb{R}^3$.
 - (c) $\{v_1, v_2, v_1 + v_2\}$ is a basis of $\mathbb{R}^3$.
 - (d) $\{v_1, v_2, v_3\}$ is linearly independent.
- Q3.** We know that the linear system $Bx = \mathbf{b}$ has infinitely many solutions when $\mathbf{b} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$. Then it is necessarily true that.
- (a) B is invertible.
 - (b) The rank of B is not zero.
 - (c) The augmented matrix $(B|\mathbf{b})$ is invertible.
 - (d) The matrix B and the augmented matrix $(B|\mathbf{b})$ do not have the same rank.
- Q4.** Let A, B be $n \times n$, $n \geq 1$ matrices such that AB is not invertible. Find the correct statement.
- (a) $\text{rk}(A) = n$ and $\text{rk}(B) = n$.
 - (b) either A is not invertible or B is not invertible.
 - (c) BA is necessarily invertible.
 - (d) $\det(AB) \neq 0$.
- Q5.** Let $Q(x, y, z, t) = x^2 + y^2 + z^2 + 2zt + t^2$ be a quadratic form. Find the correct statement.
- (a) Q is positive definite.
 - (b) Q is indefinite.
 - (c) Q is negative definite.
 - (d) None of the other options is correct.

Q6. Let $A(0, 2, 0)$, $B(2, 0, 0)$, $C(0, 0, 2)$ be points in $\mathbb{R}^3$. Find the correct statement.

- (a) The vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are parallel.
- (b) The triangle of vertices A, B, C is equilateral.
- (c) The points A, B, C are collinear.
- (d) The plane containing A, B, C also contains $(1, -1, 0)$.

Q7. Consider the lines $r : x = y = 2z$ and $s : 2x = y = z$. Find the correct statement.

- (a) $r \parallel s$.
- (b) $r \perp s$.
- (c) $r \cap s \neq \emptyset$.
- (d) r and s are not coplanar.

Q8. Consider the circles $\mathcal{C}_1 : x^2 + y^2 + z^2 - 9 = x + y = 0$, $\mathcal{C}_2 : x^2 + y^2 + z^2 - 9 = z - 2 = 0$, and let r_i be the radius of $\mathcal{C}_i$. Find the correct statement.

- (a) $r_1 > r_2$.
- (b) $r_1 = r_2$.
- (c) $r_1 < r_2$.
- (d) $\mathcal{C}_1$ and $\mathcal{C}_2$ lie in the same plane.

EXERCISE

Part A. (up to 4 points) Let $\alpha \in \mathbb{R}$ and consider the linear mapping $F_\alpha : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ such that

$$F_\alpha \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x + \alpha y \\ 2x + 2\alpha y \end{pmatrix}.$$

(1) Set $\alpha = 1$ and compute all the eigenvalues of F_α . (2) Set $\alpha = 1$ and compute a basis for each eigenspace of F_α .

Part B. (up to 4 points) Let $\beta \in \mathbb{R}$, $V = \mathbb{R}_2[x]$ be the vector space of polynomials with real coefficients of degree at most 2, and $W = \mathbb{R}^{2,2}$ be the vector space of real 2×2 matrices. Consider the linear mapping $G_\beta : V \longrightarrow W$ such that

$$G_\beta(p(x)) = \begin{pmatrix} p(0) & p(\beta) \\ p'(0) & \beta p(1) \end{pmatrix}.$$

(1) Set $\beta = 1$ and compute a basis of $\text{Ker}(G_\beta)$ made by elements of V . (2) Set $\beta = 1$ and compute a basis of $\text{Im}(G_\beta)$ made by elements of W .

LINEAR ALGEBRA AND GEOMETRY

60 min

Instructions:

- Write your surname and name and your student number in CAPITAL LETTERS where required.
- YOU MUST WRITE THE SOLUTIONS TO THE EXERCISE IN THE DEDICATED BOX.
- To access your exam papers you **must** remember the NUMBER of the room you are sitting in and the SYMBOL in the top right corner of this page.
- For each multiple choice you **must** select one of the possible answers in the table in **this** page.
- Write neatly and clearly the solution of the exercise; you **must** explain your solution.
- As a draft use your own scrap paper or the one provided; only this booklet will be graded.

SURNAME, NAME: _____

STUDENT NUMBER: _____

PROFESSOR: GATTO_____

Q1	a	b	c	d	Q5	a	b	c	d
Q2	a	b	c	d	Q6	a	b	c	d
Q3	a	b	c	d	Q7	a	b	c	d
Q4	a	b	c	d	Q8	a	b	c	d

Do not write here

QUIZ		EXERCISE		TOTAL	
------	-------------	----------	-------------	-------	-------------

QUIZ

Q1. Let A, B be $n \times n, n \geq 1$ matrices such that AB is not invertible. Find the correct statement.

- (a) $\lambda = 1$ is the only eigenvalue of AB .
- (b) $B^T A$ is invertible.
- (c) $\text{rk}(AB) = n + 1$.
- (d) $\text{rk}(AB) < n$.

Q2. Let $Q(x, y, z, t) = x^2 + xy + y^2 + z^2 + zt + t^2$ be a quadratic form. Find the correct statement.

- (a) Q is positive definite.
- (b) Q is indefinite.
- (c) Q is negative definite.
- (d) None of the other options is correct.

Q3. Let $A(1, 0, 1), B(0, 1, 1), C(1, 1, 0)$ be points in $\mathbb{R}^3$. Find the correct statement.

- (a) The vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are parallel.
- (b) The plane containing A, B, C also contains $(1, 1, 1)$.
- (c) The points A, B, C are collinear.
- (d) The triangle of vertices A, B, C has area less than 1.

Q4. Consider the planes $\alpha : x - y + z = 2$ and $\beta : x - z = 1$. Find the correct statement.

- (a) $\alpha \parallel \beta$.
- (b) $\alpha \perp \beta$.
- (c) $\alpha \cap \beta = \emptyset$.
- (d) $\alpha \cup \beta$ is a plane.

Q5. We know that the linear system $B\mathbf{x} = \mathbf{b}$ has infinitely many solutions when $\mathbf{b} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$. Then it is necessarily true that.

- (a) B is invertible.
- (b) The rank of B is zero.
- (c) The augmented matrix $(B|\mathbf{b})$ is not a square invertible matrix.
- (d) The matrix B and the augmented matrix $(A|\mathbf{b})$ do not have the same rank.

Q6. Consider the vectors $v_1 = (1, 2, 0)$, $v_2 = (2, 1, 0)$, $v_3 = (3, 3, 0)$, and $v_4 = (0, 0, 2)$. Find the correct statement.

- (a) $\{v_1, v_2, v_3 \times v_3\}$ is a basis of $\mathbb{R}^3$.
- (b) $\{v_1, v_2, v_4\}$ generates $\mathbb{R}^3$.
- (c) $\{v_1, v_2, v_3\}$ generates $\mathbb{R}^3$.
- (d) $\{v_1, v_3, v_1 + v_3\}$ is a basis of $\mathbb{R}^3$.

Q7. Consider the circles $\mathcal{C}_1 : x^2 + y^2 + z^2 - 4 = x - y = 0$, $\mathcal{C}_2 : x^2 + y^2 + z^2 - 4 = x - 2 = 0$, and let r_i be the radius of $\mathcal{C}_i$. Find the correct statement.

- (a) $r_1 > r_2$.
- (b) $r_1 = r_2$.
- (c) $r_1 < r_2$.
- (d) $\mathcal{C}_1$ and $\mathcal{C}_2$ lie in the same plane.

Q8. Consider the linear mapping $f(x, y, z, t) = (z, y, t, x)$. Find the correct statement.

- (a) f is bijective.
- (b) $f^{-1}(1, 0, 1, 0) = \{(1, 0, 1, 0)\}$
- (c) $\mathcal{L}\{(1, 2, 3, 4)\} = \text{Ker}(f)$.
- (d) f is not injective.

EXERCISE

Part A. (up to 4 points) Let $\alpha \in \mathbb{R}$ and consider the linear mapping $F_\alpha : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ such that

$$F_\alpha \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x + \alpha y \\ 2x + 2\alpha y \end{pmatrix}.$$

(1) Set $\alpha = 1$ and compute all the eigenvalues of F_α . (2) Set $\alpha = 1$ and compute a basis for each eigenspace of F_α .

Part B. (up to 4 points) Let $\beta \in \mathbb{R}$, $V = \mathbb{R}_2[x]$ be the vector space of polynomials with real coefficients of degree at most 2, and $W = \mathbb{R}^{2,2}$ be the vector space of real 2×2 matrices. Consider the linear mapping $G_\beta : V \longrightarrow W$ such that

$$G_\beta(p(x)) = \begin{pmatrix} p(0) & p(\beta) \\ p'(0) & \beta p(1) \end{pmatrix}.$$

(1) Set $\beta = 1$ and compute a basis of $\text{Ker}(G_\beta)$ made by elements of V . (2) Set $\beta = 1$ and compute a basis of $\text{Im}(G_\beta)$ made by elements of W .