

**AVaa)** Let  $A$  be an arbitrary non-invertible real square matrix. Which of the following statements is necessarily true?

- (a)  $A$  is not diagonalizable.
- (b)  $A$  is skew-symmetric.
- (c) 0 is an eigenvalue of  $A$ ;
- (d)  $A$  has no real eigenvalues.

**GALeb)** Let the line be given  $r : \begin{cases} 2x - y - z & = & 0 \\ 2x + y + 2z - 1 & = & 0 \end{cases}$

Which of the following statements is true?

- (a)  $r$  passes through the origin;
- (b)  $r$  is contained in the plane  $4x + z - 2 = 0$  ;
- (c)  $r$  is parallel to the line parametrized by  $(10 - t, 1 - 6t, 4t)$ ;
- (d)  $r$  is contained in the plane  $yz$ .

**MATbc)** Let  $A \in \mathbb{R}^{4 \times 4}$  be a symmetric matrix with characteristic polynomial  $(t - 2)^3(t - 5)$ . Which of the following statements is true?

- (a)  $A$  is not diagonalizable, because it does not have 4 distinct eigenvalues;
- (b) The matrix  $A$  is not invertible;
- (c)  $\det(A) = 40$ ;
- (d) If  $\mathbf{1}_4$  is the  $4 \times 4$  identity matrix, then  $A - 2 \cdot \mathbf{1}_4$  is invertible.

**FQec)** Consider the quadratic form real

$$q(x, y, z) = x^2 - xy + y^2 + 2xz - 4yz - 3z^2.$$

Which statement is correct?

- (a) The conic  $q(x, y, 1)$  is an ellipse with center  $(0, 2)$ ;
- (b) The quadric  $q(x, y, z) = 0$  does not contain any lines;
- (c) The conic  $q(x, y, 1)$  is an ellipse with center  $(1, 1)$ ;
- (d) The conic  $q(x, y, 0)$  is a hyperbola.

**SLcc)** Let  $A = \begin{pmatrix} R_1(A) \\ R_2(A) \end{pmatrix} \in \mathbb{R}^{2 \times 3}$ . Which of the following statements is true?

- (a) If  $A^{-1}(\mathbf{b}) = \emptyset$ , then  $\mathbf{b} \in \mathbb{R}^2$  is a linear combination of the rows of  $A$ ;
- (b)  $R_1(A)^T \times R_2(A)^T$  is a solution of the linear system  $A\mathbf{x} = 0$ ;
- (c)  $\dim_{\mathbb{R}} \ker(A) > 1$ ;

(d)  $A$ , viewed as an element of  $\text{hom}_{\mathbb{R}}(\mathbb{R}^3, \mathbb{R}^2)$ , is surjective.

**ALed)** Let  $f(x, y, z, t) = (x - 3y + z, 4x + 3y, x - z - t)$ .

(a)  $\dim \ker(f) = 2$ ;

(b)  $f$  is injective;

(c) The subspace  $\{(u, v, v, 6u) \mid u, v \in \mathbb{R}\}$  contains the kernel  $\ker(f)$  of  $f$ ;

(d) A matrix associated with  $f$  is  $\begin{pmatrix} 1 & 4 & 1 \\ -3 & 3 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & -1 \end{pmatrix}$ .

**AVda)** Let the matrix be given

$$A = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & 4 \\ -1 & -4 & 0 \end{pmatrix}$$

(a)  $A$  is invertible;

(b)  $A$  has nonzero eigenvalues with zero imaginary part;

(c)  $A$  has at least two real eigenvalues;

(d) The characteristic polynomial of  $A$  has two imaginary roots.

**VETac)** Let  $\vec{i}, \vec{j}, \vec{k}$  be the unit vectors of  $\mathbb{R}^3$ . Let the point  $P = (1, 1, -2)$  and the vector  $\vec{u} = \vec{i} + 5\vec{j} + \vec{k}$  be given. Let  $r$  be the line passing through  $P$  with direction  $\vec{u}$ . Which of the following statements is true?

(a) The line  $r$  is orthogonal to the plane  $x - y + 4z = 0$ .

(b) The line  $r$  is orthogonal to the direction of the vector  $3\vec{i} - \vec{j} + 2\vec{k}$ .

(c) The point  $(2, -1, 3)$  belongs to the line  $r$ .

(d) The point  $(1, 1, 1)$  belongs to the line  $r$ .

**GALbc)** In space let the sphere be given  $\mathcal{S}$  with equation

$$4x^2 + 4y^2 + 4z^2 + 2x + 4y + 2z = 0.$$

Which of the following statements is true?

(a) The center of  $\mathcal{S}$  is  $(-1/4, -1/2, -1/4)$ .

(b) The center of  $\mathcal{S}$  has distance 1 from the point  $(0, 0, -2)$ .

(c) The plane passing through the origin and tangent to the sphere is  $x - 2y + z = 0$ ;

(d)  $(0, 0, -1) \in \mathcal{S}$ .

**MATca)** Let  $P \in \mathbb{R}^{3 \times 3}$  be an orthogonal matrix such that  $\det(P) = 1$ . Which of the following statements is true?

- (a)  $P$  has three real eigenvalues;
- (b) The linear system  $P\mathbf{x} = \mathbf{b}$  has no solutions;
- (c) There exists  $\mathbf{u} \in \mathbb{R}^3 \setminus \{\mathbf{0}\}$  such that  $P\mathbf{u} = \mathbf{u}$ ;
- (d) The linear system  $P\mathbf{x} = \mathbf{b}$  has infinitely many solutions.

**FQbc)** Data the quadratic form

$$f(x, y) = (x, y)A \begin{pmatrix} x \\ y \end{pmatrix} = x^2 - 3xy + 8y^2$$

identify the correct statement:

- (a) The symmetric matrix  $A$  is positive definite;
- (b) The matrix  $A$  has eigenvalues of opposite sign;
- (c) there exists  $(a, b) \in \mathbb{R}^2$  per cui one abbia  $f(a, b) < 0$ ;
- (d) The equation  $f(x, y) = -1$  has at least a real solution.

**SLaa)** Let  $A \in \mathbb{R}^{m \times n}$  and  $\mathbf{b} \in \mathbb{R}^m$ , and suppose that the system  $AX = \mathbf{b}$  has a unique solution. Which of the following statements is true?

- (a) The matrix  $A$  is invertible;
- (b) The rank of  $A$  is  $m$ ;
- (c) The matrix  $A$  has at least as many rows as columns, i.e.  $m \geq n$ ;
- (d) The rank of  $A$  is less than  $n$ .

**CaaAc)** Let

$$C = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

Which of the following statements is correct?

- (a) 1 is an eigenvalue of  $C$ ;
- (b) The sum of due eigenvalues of  $A$  is 14;
- (c) The vector  $(\vec{i} + 2\vec{j} + 3\vec{k}) \times (7\vec{i} + 8\vec{j} + 9\vec{k})$  is an eigenvector of  $C$ ;
- (d) The characteristic polynomial of  $A$  is  $T^3 - T^2 + 5T - 9$ .

**ALdb)** Let  $f : \mathbb{R}^2 \rightarrow \mathbb{R}^3$  definita da  $f(x, y) = (x + 2y, x + y, 2x + 3y)$ . Which of the following statements is correct?

- (a)  $f$  is surjective.
- (b)  $\dim \text{Im}(f) = 2$ .

- (c)  $f$  is an isomorphism.
- (d)  $\dim \ker(f) = 1$ .

**MATfd)** Let  $A \in \mathbb{R}^{5 \times 5}$  be a symmetric matrix with characteristic polynomial  $P_A(t) := t^2(t - 5)^3$ . Which of the following statements is true?

- (a)  $A$  is not diagonalizable because it has a triple eigenvalue;
- (b)  $\det(A) = 1$ ;
- (c) There does not exist a matrix  $P \in \mathbb{R}^{5 \times 5}$  such that  $P^T A P$  is diagonal;
- (d) The kernel of  $A$  has dimension 2.

**GALda)** Let the lines be given

$$P(t) = (1 + t, 2 - t, 1 + 2t) \quad \text{e} \quad x + y - 4 = 2x - y + z - 5 = 0$$

- (a)  $r_1$  and  $r_2$  are intersecting;
- (b)  $r_1$  and  $r_2$  are orthogonal;
- (c)  $r_1$  and  $r_2$  are coplanar;
- (d)  $r_1$  and  $r_2$  are skew.

**FQab)** Let the quadratic form be given  $q(x, y) = 2x^2 - 4xy + 2y^2$ . Which of the following statements is true?

- (a)  $q(1, -3) < 0$ ;
- (b) The conic  $q(x, y) - 2 = 0$  is a circle;
- (c) The conic  $q(x, y) = 0$  is a conic degenerate of tipo parabólico;
- (d) The conic  $q(x, y) - 2 = 0$  has no real points.

**AVcb)** Let  $3x - y + 2z = 0$  be the eigenspace corresponding to the eigenvalue 12 of a symmetric matrix  $A$ .

- (a)  $\text{Spec}(A) = \{12\}$ ;
- (b) The algebraic multiplicity of 12 is 1;
- (c) The eigenspace corresponding to the second eigenvalue of  $A$  is the direction  $[3\mathbf{e}_1 - \mathbf{e}_2 + 2\mathbf{e}_3]$ ;
- (d) There do not exist an orthogonal basis of  $\mathbb{R}^3$  consisting of eigenvectors of  $A$ .

**SLcd)** Let  $A = \begin{pmatrix} R_1(A) \\ R_2(A) \end{pmatrix} \in \mathbb{R}^{2 \times 3}$ . Which of the following statements is true?

- (a) If  $A^{-1}(\mathbf{b}) \neq \emptyset$ , then  $\mathbf{b} \in \mathbb{R}^2$  is a linear combination of the rows of  $A$ ;
- (b) If  $\text{rk}(A) = 2$  and  $\mathbf{u} \in A^{-1}(\mathbf{b})$ , then  $A^{-1}(\mathbf{b}) = \mathbf{u} + [R_1(A)^T \times R_2(A)^T]$ ;

- (c)  $\dim_{\mathbb{R}} \ker(A) > 1$ ;
- (d)  $A$ , viewed as an element of  $\text{hom}_{\mathbb{R}}(\mathbb{R}^3, \mathbb{R}^2)$ , is surjective.

**ALbb)** Let the matrix be given

$$A := \begin{pmatrix} 3 & 4 & 5 \\ 3 & 4 & 5 \\ 21/5 & 28/5 & 7 \end{pmatrix}$$

Which of the following statements is true?

- (a)  $\text{rk}(A) = 2$ .
- (b) The matrix  $A$  is invertible;
- (c)  $A$  is diagonalizable;
- (d) The kernel of the linear map associated with  $A$  has dimension 1.

**CaAa)** Let  $C = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$  Which of the following statements is correct?

- (a) 1 is an eigenvalue of  $A$ ;
- (b) The kernel of  $A$  has dimension 2
- (c) The matrix  $A$  is invertible;
- (d) The characteristic polynomial  $P_A(T)$  of  $A$  is divisible by  $T$ .

**MATda)** Let  $\mathcal{A} := (A_{11}, A_{12}, A_{21}, A_{22})$  be a quadruple of matrices such that

$$A_{11} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad A_{12} = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}, \quad A_{21} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad A_{22} = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}.$$

Then:

- (a)  $\mathcal{A}$  is a basis of  $\mathbb{R}^{2 \times 2}$  with respect to which the coordinate vector of  $\begin{pmatrix} 5 & 2 \\ -1 & -3 \end{pmatrix}$  is  $(3, 2, 2, -3)^T$ , where  $^T$  denotes transpose;
- (b)  $\mathcal{A}$  is not a basis of  $\mathbb{R}^{2 \times 2}$ ;
- (c) The coordinate vector of  $\begin{pmatrix} 2 & 1 \\ 0 & 0 \end{pmatrix}$  is  $(1, 1, 0, 0)^T$ ;
- (d) The coordinate vector of  $\begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$  with respect to the basis  $\mathcal{A}$  is  $(0, 0, 1, 1)^T$ .

**GALfd)** In space let the sphere be given  $\mathcal{S}$  and the plane  $\Pi$  with equations, respectively:

$$\mathcal{S} := 4x^2 + 4y^2 + 4z^2 - 2x + 4y + 2z = 0, \quad \Pi : x - 2y - z = 0$$

Which of the following statements is true?

- (a) The center of  $\mathcal{S}$  is  $(1, -2, 11)$ .
- (b) The circle  $\mathcal{S} \cap \Pi$  has radius 0;
- (c) The center of  $\mathcal{S}$  has distance 1 from the point  $(0, 0, -2)$ .
- (d) The distance of the plane  $x - 2y + z = 0$  paxiss through the center of  $\mathcal{S}$ .

**FQaa)** Let the quadratic form be given  $q(x, y) = 2x^2 + 2xy - 3y^2$ . Which of the following statements is true?

- (a)  $q(x, y) = 0$  is a nondegenerate conic;
- (b)  $q(1, -3) < 0$ .
- (c) The conic  $q(x, y) - 2 = 0$  is a parabola;
- (d) The conic  $q(x, y) - 2 = 0$  is un'ellipif.

**AVbc)** Let  $A$  be a real matrix and  $P_A(t) = t^3 - 1$  its characteristic polynomial. Which of the following statements is true?

- (a)  $A$  is a  $4 \times 4$  square matrix;
- (b)  $A$  is symmetric;
- (c) If  $P(t)$  is the characteristic polynomial of  $A$  then  $A$  has two distinct eigenspaces;
- (d)  $A$  is diagonalizable over the complex numbers.

**SLba)** A linear system  $AX = B$  of 3 equations and 3 unknowns with  $\det(A) \neq 0$ :

- (a) has infinitely many solutions because it is nonhomogeneous;
- (b) if solvable, has one free unknown;
- (c) always has two free unknowns;
- (d) has a unique solution.

**VETcb73)** Let  $p_1, p_2, p_3, q \in \mathbb{R}_{\leq 2}[X]$  be given by:

$$p_1 := 1 + X; \quad p_2 := 2 + X; \quad p_3 := 1 + X + X^2; \quad q := 3 + 5X + 3X^2$$

Determine the correct statement.

- (a)  $p_3$  is a linear combination of  $p_1$  and  $p_2$ ;
- (b)  $p_1, p_2, p_3$  and  $q$  are linearly independent;
- (c)  $\mathcal{B} := (p_1, p_2, p_3)$  is a basis of  $\mathbb{R}_{\leq 2}[X]$  and the coordinate column of  $q$  with respect to  $\mathcal{B}$  is:  $(q)_{\mathcal{B}} = \begin{pmatrix} 4 \\ -2 \\ 3 \end{pmatrix}$

- (d)  $\mathcal{B} := (p_1, p_2, p_3)$  is a basis of  $\mathbb{R}_{\leq 2}[X]$  and the coordinate column of  $q$  with respect to  $\mathcal{B}$  is:  $(q)_{\mathcal{B}} = \begin{pmatrix} 4 \\ 3 \\ -2 \end{pmatrix}$

**ALec)** Let  $f(x, y, z, t) = (x - 2y + z, 2x + 3y, x - z - t)$ .

- (a) The subspace  $\{(u, v, 2v - u, w) \mid u, v \in \mathbb{R}\}$  contains the kernel  $\ker(f)$  of  $f$ ;  
 (b)  $\dim \ker(f) = 2$ ;  
 (c)  $f$  is injective;  
 (d) A matrix associated with  $f$  is  $\begin{pmatrix} 1 & 2 & 1 \\ -2 & 3 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & -1 \end{pmatrix}$ .

**GALaa)** Consider the plane  $\Pi : 6x + 3y + 2z = 0$ , the line  $r : x = 2y = 3z$  and the point  $P(6, 3, 2)$ . Indicate the correct statement.

- (a)  $P \in r \cap \Pi$ ;  
 (b)  $P \in r$  and  $r$  is contained in  $\Pi$ ;  
 (c)  $\Pi \supset r$  and  $P \notin r$ ;  
 (d)  $r \perp \Pi$  and  $P \in r$

**MATcb)** Let  $P \in \mathbb{R}^{3 \times 3}$  be an orthogonal matrix. Which of the following statements is true?

- (a)  $P$  has three real eigenvalues;  
 (b) The linear system  $P\mathbf{x} = \mathbf{b}$  has the unique solution  $\mathbf{x} = P^T \mathbf{b}$ ;  
 (c) There does not exist  $\mathbf{u} \in \mathbb{R}^3 \setminus \{\mathbf{0}\}$  such that  $P\mathbf{u} = \mathbf{u}$ ;  
 (d) The linear system  $P\mathbf{x} = \mathbf{b}$  has no solutions.

**FQeb)** Consider the quadratic form real  $q(x, y, z) = x^2 - xy + y^2 + 2xz - 4yz - 3z^2$ . Which statement is correct?

- (a) The quadric  $q(x, y, z) = 0$  non contains delle lines;  
 (b) The quadric  $q(x, y, z) = 0$  is an cone of vertice  $(0, 1, 0)$ ;  
 (c) The conic  $q(x, y, 1)$  is ana ellipif with center  $(0, 2)$ ;  
 (d) The conic  $q(x, y, 0)$  is ana parabola.

**AVab)** Let  $A$  be an arbitrary invertible real square matrix. Which of the following statements is necessarily true?

- (a)  $A$  is diagonalizable.  
 (b)  $A$  is symmetric.

- (c) The determinant of  $A$  is zero;
- (d)  $A$  has no zero eigenvalues.

**SLca)** Let  $A = \begin{pmatrix} R_1(A) \\ R_2(A) \end{pmatrix} \in \mathbb{R}^{2 \times 3}$ . Which of the following statements is true?

- (a)  $A^{-1}(\mathbf{b}) \neq \emptyset$  for every  $\mathbf{b} \in \mathbb{R}^2$  if and only if  $\text{rk}(A) = 2$ ;
- (b) The linear system  $A\mathbf{x} = \mathbf{b}$  has solutions for every  $\mathbf{b} \in \mathbb{R}^3$ ;
- (c)  $\dim_{\mathbb{R}} \ker(A) = 2$ ;
- (d) If the system  $A\mathbf{x} = \mathbf{b}$  is solvable, the solution is unique.

**ALca)** Let  $V_1$  and  $V_2$  be vector subspaces of  $\mathbb{R}^4$  of dimension 2. Which of the following statements is true?

- (a)  $V_1$  and  $V_2$  have at least a vector in comune;
- (b) The subspaces  $V_1$  and  $V_2$  could have empty intersection;
- (c)  $\dim(V_1 + V_2) = 2$ ;
- (d)  $\dim(V_1 + V_2) = 4$ ;

**CaaAd)** Let

$$C = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

Which of the following statements is correct?

- (a) 1 is an eigenvalue of  $C$ ;
- (b) The sum of two eigenvalues of  $A$  is 14;
- (c) The vector  $(4\vec{i} + 5\vec{j} + 6\vec{k}) \times (7\vec{i} + 8\vec{j} + 9\vec{k})$  is an eigenvector of  $C$ ;
- (d) The characteristic polynomial of  $A$  has no real root.

**MATaa)** Consider the matrix  $B := \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & 2 \\ 1 & 1 & 5 \\ 2 & 2 & 10 \end{pmatrix}$

- (a) The rank of  $B$  is 3;
- (b) L'immagine dell'linear map  $\mathbb{R}^3 \rightarrow \mathbb{R}^4$  associated a  $B$  has dimension 3;
- (c) The first, the second and the third row are linearly independent;
- (d) Two columns distinct any of  $B$  form a basis dello column space.

**GALac)** Consider the plane  $\Pi : x + y + z - 3 = 0$ , the line  $r : x - y = y - z = 0$  and the point  $P(1, 1, 1)$ . Indicate the correct statement.

- (a)  $P \notin \Pi$ ;

- (b)  $P \in \Pi$  and  $r$  is orthogonal  $\Pi$ ;
- (c)  $\Pi \supset r$  and  $P \notin r$ ;
- (d)  $r$  is parallel a  $\Pi$  and  $P \in r$ .

**FQed)** Consider the quadratic form real

$$q(x, y, z) = x^2 - xy + y^2 + 2xz - 4yz - 3z^2.$$

Which statement is correct?

- (a) The conic  $q(x, y, 1)$  is ana hyperbola with center  $(-2, 2)$ ;
- (b) The quadric  $q(x, y, z) = 0$  non contains delle lines;
- (c) The conic  $q(x, y, 1)$  is ana ellipif with center  $(1, 1)$ ;
- (d) The quadric  $q(x, y, z) = 0$  is an cone of vertice l'origin;

**AVac)** Let  $A \in \mathbb{R}^{n \times n}$  be an arbitrary real square matrix with zero determinant. Which of the following statements is necessarily true?

- (a) The system  $A\mathbf{x} = 0$  has infinitely many nonzero solutions;
- (b)  $A$  has no zero eigenvalues.
- (c) The matrix  $A$  is invertible;
- (d)  $A$  has no eigenvectors.

**SLad)** Let  $A \in \mathbb{R}^{m \times n}$  and  $\mathbf{b} \in \mathbb{R}^m$ , and suppose that the system  $A\mathbf{x} = \mathbf{0}$  has at least two distinct solutions. Which of the following statements is true?

- (a) The matrix  $A$  is invertible;
- (b) The linear system  $A\mathbf{x} = \mathbf{b}$  has solutions if  $\text{rk}(A) = \text{rk}(A|\mathbf{b})$ ;
- (c) The linear system  $A\mathbf{x} = \mathbf{b}$  has infinitely many solutions;
- (d) The matrix  $A$  has at least as many columns as rows, i.e.  $m \leq n$ .

**VEtaa)** Let  $\vec{i}, \vec{j}, \vec{k}$  be the unit vectors of  $\mathbb{R}^3$ . Let  $P = (2, 1, 3)$  and  $\vec{u} = 2\vec{i} - \vec{j} + 3\vec{k}$ . Let  $r$  be the line passing through  $P$  with direction  $\vec{u}$ . Which of the following statements is true?

- (a) The line  $r$  is orthogonal to the plane  $x - y + 3z = 0$ .
- (b) The line  $r$  is orthogonal to the vector  $\vec{i} + 5\vec{j} + \vec{k}$ .
- (c) The point  $(2, 2, 3)$  belongs to the line  $r$ .
- (d) The point  $(2, -1, 3)$  belongs to the line  $r$ .

**ALdc)** Let  $f : \mathbb{R}^2 \rightarrow \mathbb{R}^3$  definita da  $f(x, y) = (x + y, 2x + 2y, 3x + 3y)$ . Which of the following statements is correct?

- (a)  $f$  is surjective.
- (b)  $f$  is injective;

- (c)  $\dim \operatorname{Im}(f) = 1$ .
- (d)  $\dim \ker(f) = 2$ ;

**MATbb)** Let  $A \in \mathbb{R}^{4 \times 4}$  be a symmetric matrix with characteristic polynomial  $P_A(t) := (t - 2)(t - 5)^3$ . Which of the following statements is true?

- (a)  $A$  is not diagonalizable, because it has a triple eigenvalue;
- (b) The matrix  $A$  is not invertible;
- (c) There exists a matrix  $P \in \mathbb{R}^{4 \times 4}$  such that  $P^{-1}AP$  is diagonal and  $\det(P) = 1$ ;
- (d) If  $\mathbf{1}_4$  is the  $4 \times 4$  identity matrix, then  $A - 2 \cdot \mathbf{1}_4$  is invertible.

**GALbb)** In space let the sphere be given  $\mathcal{S}$  with equation

$$4x^2 + 4y^2 + 4z^2 + 2x + 4y + 2z = 0.$$

Which of the following statements is true?

- (a) The center of  $\mathcal{S}$  is  $(-1, -2, -1)$ .
- (b) The center of  $\mathcal{S}$  has distance 1 from the point  $(0, 0, -2)$ .
- (c) The plane passing through l'origin and tangent to the sphere is  $x + 2y + z = 0$ ;
- (d)  $(0, 0, -2) \in \mathcal{S}$ .

**FQba)** Consider the quadratic form

$$f(x, y) = (x, y)A \begin{pmatrix} x \\ y \end{pmatrix} = x^2 - 3xy + 8y^2$$

- (a)  $f(x, y) = 0$  if and only if  $(x, y) = (0, 0)$ ;
- (b)  $f(a, b) < 0$  per qualthat pair  $(a, b) \in \mathbb{R}$ ;
- (c) The conic  $f(x, y) = -1$  has at least one point with real coordinates;
- (d) The prodotto degli eigenvalues of the matrix  $A$  is negative.

**AVcd)** Let  $[(3, 1, 2)^T]$  be the eigenspace corresponding to the eigenvalue 5 of a symmetric matrix  $3 \times 3$ .

- (a) The algebraic multiplicity of 5 is 2;
- (b)  $\operatorname{Spec}(A) = \{5, 1\}$ ;
- (c) There do not exists an orthogonal basis of  $\mathbb{R}^3$  consisting of eigenvectors of  $A$ .
- (d) The vectors  $\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$  and  $\begin{pmatrix} 1 \\ -3 \\ 0 \end{pmatrix}$  are linearly independent eigenvectors of  $A$ .

**SLbc)** A linear system  $AX = B$  of 3 equations and 3 unknowns with  $\det(A) = 0$ :

- (a) has infinitely many solutions if and only if  $B$  belongs to the image of  $A : \mathbb{R}^3 \rightarrow \mathbb{R}^3$  viewed as a linear map;
- (b) if solvable, has one free unknown;
- (c) always has two free unknowns;
- (d) has one and only one solution.

**ALab)** Consider i vectors  $\mathbf{v}_1 = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$ ,  $\mathbf{v}_2 = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$  and  $\mathbf{v}_3 = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ . Indicate the correct statement.

- (a) The vectors  $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$  are linearly independent;
- (b)  $\lambda_1 \mathbf{v}_1 + \lambda_2 \mathbf{v}_2 = \mathbf{v}_3$  per qualthat  $\lambda_1, \lambda_2 \in \mathbb{R}$ ;
- (c)  $\mathbf{v}_1 + \mathbf{v}_2 = \lambda \mathbf{v}_3$  per qualthat  $\lambda \in \mathbb{R}$ ;
- (d)  $[\mathbf{v}_1, \mathbf{v}_2] = \mathbb{R}^2$ .

**CaaAb)** Let  $\mathcal{C} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$  Which of the following statements is correct?

- (a) The vector  $(\vec{i} + 2\vec{j} + 3\vec{k}) \times (4\vec{i} + 5\vec{j} + 6\vec{k})$  is an eigenvector of  $\mathcal{C}$ ;
- (b) 5 is an eigenvalue of  $\mathcal{C}$ ;
- (c) The prodotto degli eigenvalues of  $\mathcal{C}$  is 40;
- (d) The characteristic polynomial of  $A$  has no integer roots.

**MATcc)** Let  $P \in \mathbb{R}^{3 \times 3}$  be an orthogonal matrix such that  $\det(P) = -1$ . Which of the following statements is true?

- (a)  $P$  has three real eigenvalues;
- (b) The linear system  $P\mathbf{x} = \mathbf{b}$  has no solutions;
- (c) There exists  $\mathbf{u} \in \mathbb{R}^3 \setminus \{\mathbf{0}\}$  such that  $P\mathbf{u} = -\mathbf{u}$ ;
- (d) The linear system  $P\mathbf{x} = \mathbf{b}$  has infinitely many solutions.

**GALed)** Let the line be given  $r : \begin{cases} 2x - y - z & = & 0 \\ 2x + y + 2z - 1 & = & 0 \end{cases}$ . Which of the following statements is true?

- (a)  $r$  is contained in the plane  $4x + z - 1 = 0$ ;
- (b)  $r$  paxiss through l'origin;
- (c)  $r$  is parallel to the line  $\mathbf{r}(t) = (10 - t, 1 - 3t, 4t)$ ;
- (d)  $r$  is contained in the plane  $yz$ .

**FQbb)** Data the quadratic form

$$f(x, y) = (x, y)A \begin{pmatrix} x \\ y \end{pmatrix} = x^2 - 3xy + 8y^2$$

identify the correct statement:

- (a) If  $xy \neq 0$ , then  $f(x, y) > 0$ ;
- (b) The determinant of the matrix  $A$  is negative;
- (c) there exists  $(a, b) \in \mathbb{R}^2$  tale that  $f(a, b) < 0$ ;
- (d) The equation  $f(x, y) = -1$  has at least a real solution.

**AVad)** Let  $A \in \mathbb{R}^{n \times n}$  be an arbitrary real square matrix with nonzero determinant. Which of the following statements is necessarily true?

- (a) The system  $AX = 0$  has infinitely many nonzero solutions;
- (b)  $A$  has no zero eigenvalues.
- (c) The matrix  $A$  is not invertible;
- (d)  $A$  has real eigenvalues.

**SLcb)** Let  $A = \begin{pmatrix} R_1(A) \\ R_2(A) \end{pmatrix} \in \mathbb{R}^{2 \times 3}$ . Which of the following statements is true?

- (a)  $A^{-1}(\mathbf{b}) \neq \emptyset$  for every  $\mathbf{b} \in \mathbb{R}^2$  if and only if  $\text{rk}(A) = 2$ ;
- (b) The linear system  $A\mathbf{x} = \mathbf{b}$  has solutions for every  $\mathbf{b} \in \mathbb{R}^3$ ;
- (c)  $\dim_{\mathbb{R}} \ker(A) = 2$ ;
- (d) If the system  $A\mathbf{x} = \mathbf{b}$  is solvable, the solution is unique.

**VETab)** Let  $\vec{i}, \vec{j}, \vec{k}$  be the unit vectors of  $\mathbb{R}^3$ . Let  $P = (1, 1, -2)$  be a point and let  $\vec{u} = \vec{i} + 5\vec{j} + \vec{k}$  be a vector. Let  $r$  be the line passing through  $P$  and having direction  $\vec{u}$ . Which of the following statements is true?

- (a) The line  $r$  is orthogonal to the direction of the vector  $2\vec{i} - \vec{j} + 3\vec{k}$ .
- (b) The line  $r$  is orthogonal to the plane  $x - y + 3z = 0$ .
- (c) The point  $(2, -1, 3)$  belongs to the line  $r$ .
- (d) The point  $(2, 2, 3)$  belongs to the line  $r$ .

**ALaa)** Consider i vectors  $\mathbf{v}_1 = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ ,  $\mathbf{v}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$  and  $\mathbf{v}_3 = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$ . Indicate the correct statement.

- (a) The vectors  $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$  are linearly independent;
- (b)  $\mathbf{v}_1 + \mathbf{v}_2 - 2\mathbf{v}_3 = \mathbf{0}$ , per qualthat  $\lambda \in \mathbb{R}$ ;
- (c) The vectors  $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$  span  $\mathbb{R}^3$ ;
- (d)  $\lambda_1\mathbf{v}_1 + \lambda_2\mathbf{v}_2 = \mathbf{v}_3$  per qualthat  $\lambda_1, \lambda_2 \in \mathbb{R}$ .