ON THE ALGEBRAIC CONSTRUCTION OF UNIVERSAL GRASSMANN MANIFOLD (CONTRIBUTED TALK)

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ABSTRACT. I will report on some joint work with P. Salehyan [2, 3] and I. Scherbak [4]. The complex Grassmannian $G(r,n)$ parametrizing r-dimensional subspaces of $\mathbb{C}^n$ can be looked at as the locus of the non zero decomposable tensors of $\bigwedge^r \mathbb{C}^n$ modulo the natural action of $\mathbb{C}^*$. It can be expressed as the zero locus variety of quadratic polynomials in a projective space (Plücker embedding). Let $\bigwedge^\infty \mathbb{C}[X]$ be the projective limit (in the category of *graded* algebra) of the inverse system $\bigwedge^r \mathbb{C}[X]$ with respect to natural projections $\bigwedge^s \mathbb{C}[X] \to \bigwedge^r \mathbb{C}[X]$ defined for all $s \geq r$. An infinite complex Grassmannian $G(\infty, \mathbb{C}^\infty)$ will be realized as the locus of elements of $\bigwedge^\infty \mathbb{C}[X]$ satisfying the asymptotic expressions of the Plücker quadrics for finite dimensional Grassmannians. The latter can be in fact written in an explicit compact way and their limit for $(r,n) \to \infty$ give precisely the Hirota bilinear expression of the celebrated system of infinitely many PDEs known as KP hierachy. Although many observations mentioned in the talk belong to the folklore of the representation theory of the Heisenberg oscillator algebra, the novelty is the relationship of the latter with Schubert calculus. The Plücker equations of all the Grassmannians at once can be easily deduced through a distinguished *Hasse-Schmidt derivation* on a exterior algebra, a fruitful notion that, as a major surprise, does not require any prerequisite going beyond a little graduate multi-linear algebra.

REFERENCES

[1] L. Gatto, *Schubert Calculus via Hasse–Schmidt Derivations*, Asian J. Math., **9**, No. 3, (2005), 315–322

[2] *Hasse-Schmidt Derivations on a Grassmann Algebra*, Springer IMPA Monographs, n. **4**, 2016;

[3] L. Gatto, P. Salehyan, *On Plücker Equations Characterizing Grassmann Cones*, arXiv:1603.00510

[4] L. Gatto, I. Scherbak, *Remarks on the Cayley-Hamilton Theorem*, arXiv:1510.030 22v1

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