# $8^{th}$ Workshop on Algebraic Singularity Theory
## and
# $11^{th}$ Workshop on Singularity and Geometry

## Abstracts

*Updated on January, $11^{th}$*

## *MONDAY,* 16<sup>th</sup>

### Maria A. Ruas

*Determinantal varieties and singularities of matrices*

We review basic results on determinantal varieties and show how to apply methods of singularity theory of matrices to study their invariants and geometry.

### Israel Vainsencher

*Counting singular surfaces in 3-space*

This talk is about the enumerative geometry of the family of surfaces in 3-space which present singularities of some specified type. The focus will be on examples where the singular locus comprises a curve, e.g., a line, or a conic, or a twisted cubic, etc. We show that the closure in P34 of the locus of (equations of) quartic surfaces singular along the complete intersection of 2 quadrics is of dimension 18 and degree 24310.

## *TUESDAY,* 17<sup>th</sup>

### Carolina Araújo

*On a special family of Fano varieties*

The geometry of a quartic del Pezzo surface $S$ is very well understood. Embedded as a smooth complete intersection of two quadric hypersurfaces in $\mathbb{P}^4$, the surface $S$ contains exactly 16 lines. It can also be described as the blowup of $\mathbb{P}^2$ at 5 points in general linear position. In fact, there are 16 different ways to realize $S$ as such blowup: for each line $\ell \subset S$, there is one such blowup under which $\ell$ is the transform of the unique conic through the blown up points.

In this talk, we will explain how this picture generalizes to arbitrary even dimension. Given an even positive integer $n = 2m$, we consider the variety $G$ parametrizing $(m-1)$-planes in a smooth complete intersection of two quadrics in $\mathbb{P}^{n+2}$. This is a Fano variety of dimension $n$ that can also be described as a small modification of the blowup of $\mathbb{P}^n$ at $n + 3$ points in general linear position. We show that there are $2^{n+2}$ different ways to realize $G$ in this manner, one for each of the $2^{n+2}$ distinct $m$-planes contained in

the complete intersection of two quadrics, and describe these birational maps explicitly. This is a joint work with Cinzia Casagrande.

## Aron Simis

### *An analogue of the Aluffi algebra for a module*

Paolo Aluffi introduced in a new graded algebra in order to conveniently express characteristic cycles in the theory of singular varieties. This algebra is attached to a pair of ideals $J \subset I$ ideals in a given "ambient" ring $R$. At the end he envisions the possibility of an analogous notion for coherent sheaves. This is the central purpose of this work. There are two main problems at the outset: an abstract module may not admit an embedding into a free module and a central step, that of dealing with the "blowup"of a module, is not totally obvious. On the bright side, the intrusion of modules raises a few algebraic questions interesting on their own. It is to expect that this extension to modules may be transcribed in terms of coherent sheaves, thus possibly providing an answer to a question by Aluffi in this regard. Two main bodies of examples are treated in detail to illustrate how the theory works and to show the relation to finer properties of other algebras.

## Nivaldo Medeiros

### *Secants to rational normal curves and enumerative geometry*

Given a rational normal curve $C$ in a projective space $\mathbb{P}^{2r}$, let $X$ be the secant of $r-1$-planes to $C$. Then $X$ is a hypersurface, given by determinant of a Hankel matrix.

We prove that the degree of the polar map $\mathbb{P}^{2r} -- > \mathbb{P}^{2r}$ (defined by the partial derivatives of the polynomial defining $X$) is given by the degree of the Grassmannian of lines on $\mathbb{P}^{r+1}$. In particular, for $r \geq 2$, such maps are never birational, answering in the affirmative a conjecture raised by Maral Mostafazadehfard e Aron Simis.

Further, we discuss the computation of some characteristic classes associated to $X$, namely the Schwartz-MacPherson class and the Segre class of its singular locus. Finally, we indicate the connection with certain general determinantal varieties.

This is a work in progress, joint with Jefferson Nogueira and Giovanni Staglianò.

## Leonardo Câmara

### *On the moduli space of quasi-homogeneous functions*

We study the moduli space between bi-Lipschitz and analytic equivalence of reduced quasi-homogeneous functions on $(\mathbb{C}^2, 0)$. We show that any non-degenerate continuous family of (reduced) quasi-homogeneous functions with constant Henry-Parusinski invariant is analytically trivial. Further we show that there are only a finite number of distinct bi-Lipschitz classes between quasi-homogeneous functions with the same Henry-Parusinski invariant providing a maximum quota for this number. Joint work with M.A.S. Ruas.

**Saurabh Trivedi**

*Existence of maps transverse to stratifications and foliations*

We discuss existence of maps transverse to stratifications and foliations with a given derivative at a point. We then show some of the applications of existence of such maps and propose some open questions.


# *WEDNESDAY, 18$^{\text{th}}$*

**Alexandre Fernandes**

*Multiplicity of analytic hypersurface singularities under bi-Lipschitz homeomorphisms*

We give partial answers to a metric version of Zariski's multiplicity conjecture. In particular, we prove that the multiplicity of complex analytic surface (not necessarily isolated) singularities in $\mathbb{C}^3$ is a bi-Lipschitz invariant. Joint work with Edson Sampaio.


**Alcides Lins Neto**

*Componentes dos espaços de folheações do tipo "pull-back"*

Este é um trabalho conjunto com W. Costa e Silva. O objetivo é apresentar uma lista de componentes irredutíveis dos espaços de folheações holomorfas de dimensão $k$ em $\mathbb{P}^n$, $n \geq 3$, $k \geq 2$. Estas componentes estão associadas ao pull-back de folheações de dimensão um por mapas racionais genéricos não lineares $f : \mathbb{P}^n \dashrightarrow \mathbb{P}^{n-k+1}$. Estes resultados generalizam o caso de codimensão um em $\mathbb{P}^n$, $n \geq 3$, que foram obtidos anteriormente por D. Cerveau, A. Lins Neto e S. J. Edix-hoven e publicados em "Pull-back components of the space of holomorphic foliations on $\mathbb{C}p(n)$, $n \geq 3$"; Journal of algebraic geometry, 19 (2001) 695-711.


**Gilcione costa**

*On foliations by curves with singularities of positive dimension*

Let $F$ be a holomorphic foliation on $\mathbb{P}^n$ by curves such that the components of its singular locus are curves $Ci$ and points $pj$. We compute the Baum-Bott indices $BB\varphi(F, Ci)$ in terms of the main invariants of F and $Ci$. We also determine the sum of the $BB\varphi(F, Ci)$ in terms of the same invariants.When $\varphi$ corresponds to the determinant, the latter result generalizes, from special to all holomorphic foliations, a formula for the number of isolated singularities of $F$, counted with multiplicities.


**Sergey Agafonov**

*Projective invariants of linear 3-webs*

We introduce a complete set of projective differntial invariants for linear planar 3-webs and describe the webs with degenerate signature sets.

**Raphael da Costa**

Let $\mathcal{F}_k(d, n)$ denotes the space of foliations of dimension $k$ and degree $d$ on $\mathbb{P}^n$. In this talk, I shall exhibit all components $\mathcal{F}(p, q, r; \lambda, d) \subset \mathcal{F}_2(d, 3)$ associated to the affine Lie Algebra $\mathfrak{aff}(\mathbb{C})$, where $p > q > r$ are relatively prime positive integers, according to a known stability theorem. In particular, we give an answer to the problem whether there exist such components, if $p > q > r$ are fixed.

Time permitting, I will also construct components $\mathcal{F}(p_1, p_2, \ldots, p_n; \lambda, d)$ of $\mathcal{F}_2(d, n)$, where $p_1 > p_2 > \cdots > p_n$ are relatively prime positive integers, finally giving a characterization of such components.

**Keywords:** Irreducible components of the space of foliations. Components associated to the affine Lie algebra.

# *THURSDAY*, 19<sup>th</sup>

**Stanislaw Janeczko**

*Poisson-Lie algebras and singular symplectic forms associated to $A_k$ type singularities*

It is shown that there exists a natural Poisson-Lie algebra associated to a singular symplectic structure $\omega$. We construct Poisson-Lie algebras for the Martinet and Roussaire types of singularities. In the special case if the singular symplectic structure is given by the pullback from the Darboux form, $\omega = F^* \omega_0$ this Poisson-Lie algebra is a basic symplectic invariant of the singularity of the smooth mapping $F$ into symplectic space $(\mathbb{R}^{2n}, \omega_0)$. The case of $A_k$ singularities of pullbacks were considered and Poisson algebras for $\Sigma_{2,0}, \Sigma_{2,2,0}^e, \Sigma_{2,2,0}^h$ stable singularities of 2-forms were calculated. This is joint result with Takuo Fukuda.

**Severino C. Coutinho**

*On the Poincaré Problem*

I will present an algorithmic strategy to compute an upper bound for the degree of the algebraic solutions of non-degenerate polynomial differential equations in dimension two.

**Maria Michalska**

*Stability of multiplicity and degree with respect to sublevel sets*

Let $S$ be an unbounded subset of $R^n$. Consider a polynomial $f$. Let $\deg_S f$ be the smallest degree of a polynomial $h$ such that $f < h$ on $S$. We call such a number the degree of $f$ relative to $S$. Analogously, one can define a multiplicity at 0 relative to a set $S$ such that 0 lies in its closure.

Consider a real polynomial mapping $(g_1, \ldots, g_k) : R^n \to R^k$ and its sublevel set $S_c$, where $c \in R^k$, given by inequalities $g_1 < c_1, \ldots, g_k < c_k$. We show that there exists

a semialgebraic set $V_g \subset R^k$ of positive codimension such that if $c, C$ are contained in the same connected component of $R^k \setminus V_g$, then the relative degrees coincide i.e. $\deg_{S_c} \equiv \deg_{S_C}$. Analogous property is true for the relative multiplicity.

We will discuss the relation of $V_g$ with bifurcation values at infinity of $g$, the moment problems and Positivstellensaetze. This is joint work with V. Grandjean.

**Letterio Gatto**

*The cohomology of the Grassmannian represents*
*Lie Algebras of Vector Spaces Endomorphisms*

Based on some joint work in progress with P. Salehyan, the talk places itself in that fuzzy region of mathematics populated by results that should be either well known or are known as folklore, but for which seems hard to find an explicit adherent statement in the literature. The original motivation comes from our attempt to read the so-called vertex representation of the Lie algebra $gl_\infty$ (endomorphisms of an infinite dimensional vector space enjoying suitable finiteness conditions), originally due to Date, Jimbo, Kashiwara and Miwa (DJKM), within a Schubert calculus setting.

Forget now the fancy and/or difficult words and let $V$ be an $n$-dimensional $\mathbb{Q}$-vector space and $\mathrm{End}(V)$ its Lie algebra of endomorphisms. A natural $\mathrm{End}(V)$-module structure of $H^*(G(r,n), \mathbb{Q})$, the singular cohomology of the complex Grassmannian of $r$-planes in $\mathbb{C}^n$, will be described for all $1 \leq r < n$. While the extremal case $r = n = \infty$ is taken into account by the aforementioned DJKM vertex representation (see e.g. [2, 3]), the case $r = 1$ is just the obvious one: $V \cong H^*(G(1,n), \mathbb{Q})$ is an $\mathrm{End}(V)$-module. We guess it is natural to wonder what it stays in between. That's why in the talk we shall sketchily discuss the case $1 \leq r < \infty$, using the fact that the Chern classes of the universal quotient bundle over $G(r,n)$ operate as (higher order) derivations on the exterior algebra of $V$ (as explained e.g. in [1]).

# Referências

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[3] E. Date, M. Jimbo, M. Kashiwara, T. Miwa, "Operators approach to the Kadomtsev-Petviashvili equation, Transformation groups for soliton equations III". *J. Phys. Soc. Japan* **50** (1981) 3806–3812

**Josney Novacoski**

*Some recent advances towards resolution of singularities in positive characteristic*

In this talk we will describe the Zariski's method to resolution of singularities of algebraic varieties. This method consists in proving first a local version (local uniformization) and use the compactness of the Zariski topology on the space of valuations to reduce the problem to patching together finitely many of these local solutions. We describe some recent advances in positive characteristic. One example is the recent work of Cossart and Piltant, which shows that every variety of dimension three over a field of positive characteristic admits resolution of singularities.

# *FRIDAY, 19*[th]

**Rogério Mol**

*Local polar invariants and the Poincaré problem in the dicritical case*

We study local polar invariants of planar complex analytic foliations, which leads to the characterization of second type foliations and of generalized curve foliations, as well as a description of the GSV-index. This local study is applied to the Poincaré problem for foliations on the complex projective plane, establishing, in the dicritical case — in fact, in the general case —, sufficient conditions for the existence of a bound depending on the degree of the foliation which controls the degree of an invariant algebraic curve. These are established in terms of the structure of the sets of local separatrices at singularities of the foliation over the invariant algebraic curve. Our general method in particular recovers known bounds for the non-dicritical case. (Joint work with Yohann Genzmer).

**Gabriel Calsamiglia**

*Que modelos analíticos locais aparecem em folheações algébricas?*

Apresentaremos exemplos e teoremas sobre modelos locais que podem ser realizados (inclusive fixando a superfície algébrica ambiente) e mostraremos que existe uma infinidade de modelos locais que não podem ser realizados em $\mathbb{P}^2$.

**Fernando H. Iglesias**

*Polares Newton Não-degeneradas.*

Nós caracterizamos os tipos topológicos de curvas planas irredutíveis cujo elemento genérico tem a sua polar Newton Não-degenerada, também damos descrição explícita destes tipos topológicos.