

Programa de Verão 2017

Plücker Embedding of Grassmannians

Aims: The course aims to revisit basic subjects belonging to the most classical algebraic geometry from a point of view that lead to put them in a framework including more advanced topics, as integrable systems or representation theory of infinite dimensional Lie algebras. The take will be elementary. Possible prior knowledge of the Plücker embedding may be useful but not strictly necessary. The course is not specialistic but rather interdisciplinary, with a mixed blend of classical algebraic geometry, exterior algebra, algebra of formal power series and differential equations. The purpose, however, is just to learn how to write explicitly a generating function of all the Plücker quadrics cutting grassmannians in their projective embedding.

Abstract: The complex Grassmannian $G(r, n) := G(r, \mathbb{C}^n)$ is a complex smooth algebraic variety whose points parametrize r -dimensional subspaces of an n -dimensional complex vector space. The short course, to be held in Portugues (provided no non-portugues speaking is in the audience), will revise the basic definitions of Grassmannians and of their canonical embedding in a projective space, the Plücker embedding. The exposition will be classical at the very beginning, by studying the simplest example of the Grassmannian $G(2, 4)$. It will continue by showing an elegant method to write the quadratic equations of the Plücker embedding of the Grassmannians $G(r, n)$ all at once, via the natural action of their cohomologies on the exterior power of an infinite dimensional vector space. This will lead to the borders of more advanced topics, such as Plücker equations defining an infinite Grassmannian, integrable systems, vertex operators.

Background: The pre-requisites will be kept to the minimum wage (o minimo sinical). The audience will be supposed to be familiar with basics of multilinear algebra, tensor algebras, exterior algebras and formal power series. The latter will be quickly revised.

The course should consists in one hour five lectures plus about 30 minutes devoted only to younger people who wish to revise the prerequisites. The extra 30 minutes will be not part of the official schedule.

Audience. The course is advised to all my friends and to all those master and/or Ph.D. students with a taste for interdisciplinary mathematics.

SUMMARY

- 1st Lecture.** Grassmannians as collection of subspaces of fixed dimension of a given vector space and as loci of decomposable tensors in an exterior power. The Plücker embedding of $G(2, 4)$;
- 2nd Lecture.** Introduction to formal power series and Schur Polynomials;
- 3rd Lecture.** Derivations on Grassmann Algebras;
- 4th Lecture.** Generating functions of the Plücker quadrics;
- 5th Lecture.** Asymptotic expression of the generating function and the Kadomtsev - Petshiasvili (KP) Hierarchy of PDEs.

References

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- [3] L. Gatto, P. Salehyan, *Hasse-Schmidt Derivations on Grassmann Algebras*, Impa Monographs n. **4**, Springer, 2016
- [4] On the Plücker Embedding of the Grassmann Cone, [arXiv:1603.00510](https://arxiv.org/abs/1603.00510), to appear on "IMPANGA Lecture Notes, EMS, 2017.
- [5] Ph. Griffiths, J. Harris, *Principles of Algebraic Geometry*, Wiley Classics Library (1994)
- [6] V. G. Kac, A. K. Raina, N. Rozhkovskaya, *Highest Weight Representations of Infinite Dimensional Lie Algebras*, Advanced Series in Mathematical Physics, Vol. **29** Second Edition, World Scientific (2013)
- [7] M. Sato, *The KP Hierarchy and Infinite-Dimensional Grassmann Manifolds*, In Theta Functions, Bowdoin 1987, L. Ehrenpreis, R. C. Gunning Eds, Part 1, 51–66, Proc. of Symposia in Pure Math. **49** (1989)