# Polynomial Rings are $gl_\infty$-modules


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The talk aims to dig up the Schubert Calculus roots of the so-called bosonic vertex representation of the Lie algebra $gl_\infty$ of all infinite square matrices having all entries zero but finitely many, originally due to Date, Jimbo, Kashiwara and Miwa (DJKM), obtained in the context of the infinite dimensional integrable systems [2, 3]. To this purpose, we start considering $V := \mathbb{Q}[T]$, an infinite-dimensional $\mathbb{Q}$-vector space together with $E_{ij} \in End_{\mathbb{Q}}V$, defined by $E_{ij}(T^k) = T^i\delta_j^k$. Let $gl_\infty$ be the $\mathbb{Q}$-linear span of the $E_{ij}$. Due to the isomorphism of $\bigwedge^r \mathbb{Q}[T]$ with the polynomial ring $B_r := \mathbb{Q}[e_1, \ldots, e_r]$, the latter inherits a natural structure of $gl_\infty$-module, which will be explicitly described by computing the action of the generating function $\mathcal{E}(z, w) := \sum_{i,j\geq 0} E_{ij}z^iw^{-j} : B_r \to B_r[[z, w^{-1}]$. The computations heavily rely on Schubert Calculus techniques applied not on one, but on all Grassmannians at once [1]. The method so supplies the expression of a certain operator $\Gamma_r(z, w) : B_r \to B_r[z, w^{-1}]$, whose asymptotic espression for $r \to \infty$ is exactly the one appearing in the DJKM representation.

## References


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