

A NEW APPROACH TO MULTIVARIATE  
LIFETIME DISTRIBUTIONS BASED ON THE  
EXCESS-WEALTH CONCEPT: AN  
APPLICATION IN TUMOR GROWTH

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# 1 Preface

Lifetime distributions are of great importance in the theory of stochastic modelling, renewal theory, reliability and survival analysis. As was pointed out by Kocherlakota and Wiens (1987), the ageing of a physical or biological system is known as the phenomenon by which an older system has a shorter remaining lifetime, in some stochastic sense, than a newer one. Many criteria of ageing (e.g. IFR, DMRL and NBUE notions) have been developed in the literature over many years and have been characterized by different methods. Analogous multivariate versions have also been suggested and studied. It is worth mentioning that many of these methods are included within the framework of stochastic ordering.

This work fundamentally deals with two problems: the problem of studying new multivariate ageing notions and the problem of characterizing them by means of a multivariate dispersion function. Two concepts are taken as a starting point: the multivariate quantiles and the upper-corrected orthant. The first concept, also known as standard construction, was introduced for the first time by O'Brien (1975) and it has been widely used in simulation theory as well as in multivariate stochastic orderings. The second concept was given by Fernandez-Ponce and Suarez-Llorens (2003). They provided the notion of the upper-corrected orthant associated with the standard construction and obtained the important result that the cumulated probability in this region does not depend on the distribution. These concepts, together with the work by Fernandez-Ponce et al. (1998), where the univariate ageing notions are characterized through the excess-wealth function, gave the basis for the development of this research. Obviously, numerous significant results in the reliability and stochastic order areas are considered, in order to achieve our proposals.

This work is structured in four chapters. Chapter 1 is introductory and presents the state-of-the-art in univariate and multivariate ageing notions. In particular, some works in which stochastic comparisons are used to characterize these notions are summarized. Quantiles and their generalizations, as well as the univariate excess wealth function are also considered in this chapter.

Chapter 2 aims to give a multivariate excess-wealth concept, based jointly on the work by Fernandez-Ponce et al.(1998) and Fernandez-Ponce and Suarez-Llorens (2003). It starts by introducing notions and preliminaries which will be used through the work. Then, given the importance of the upper-corrected orthant in this research, attention is focused on providing new results about this concept. The relationships between the support of a random vector, the upper-corrected orthant and the rightupper orthant at a point, are established. Finally, in the last two sections, the multivariate excess-wealth function and the multivariate excess-wealth order are studied. This function is defined in terms of the upper-corrected orthant and it is shown that it preserves the same properties which are verified by the univariate version.

The definitions of new multivariate ageing properties are the topic of Chapter 3. From

new generalizations of the hazard rate and mean residual life functions, multivariate versions of the IFR, DMRL and NBUE ageing notions are presented, together with the chain of implications between them. Following the development in Fernandez-Ponce et al. (1998), characterizations of this new property are given in terms of the multivariate excess-wealth function.

Finally, these multivariate notions allow the definition of a new ordering to compare the ageing of two random vectors. Finally, in Chapter 4, an application of a particular multivariate lifetime distribution is considered in oncology. Patient age and tumor size at spontaneous detection of the tumor, play an important role in the prevention of cancer. There is increasing interest in the early detection of chronic diseases, with the expectation that earlier diagnosis, combined with therapy, result in more cures and longer survivals. The process of tumor development can be explained in terms of patient age at the onset of the tumor (time from the patient is born until the first tumor cell appears) and the sojourn time (time from the first tumor cell appearing until the detection of the tumor). A non-deterministic exponential model that relates the sojourn time to the tumor size at spontaneous detection is suggested and studied in this chapter. In the process of estimating the parameters in this model, a constraint is used which represents an inherent multivariate ageing property of the lifetime distributions considered. The proposed model is illustrated using two real databases.