**Proceedings of the ASME 2012 11th Biennial Conference On Engineering Systems Design And Analysis**
**ESDA2012**
**July 2-4, 2012, Nantes, France**

**ESDA2012-82293**

# UNCERTAINTY MODEL FOR SYSTEMS BASED ON WIRELESS SENSOR NETWORKS FOR LARGE SCALE DIMENSIONAL METROLOGY

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## ABSTRACT

Verification of dimensional compliance is becoming a crucial aspect in every kind of production, even when the size of the product to be checked is in the order of several meters. To this purpose, several tools based on different technologies, working principles, functionalities and architectures have been recently designed. Among these, a distributed flexible system based on a network of low cost infrared (IR) cameras – the Mobile Spatial coordinate Measuring System (MScMS) – has been developed. This paper proposes a model for the real time assessment of the system uncertainty referring to the measured point coordinates in the 3D space. The paper focuses on the sources of measurement uncertainty and, basing on the multivariate law of propagation of uncertainty, suggests a model for relating them to the uncertainty of a measured point.

## INTRODUCTION

Last decade has shown a significant diffusion of distributed systems for Large-Scale Dimensional Metrology (LSDM). This is the case, for example, of i-GPS$^{TM}$, HiBall$^{TM}$, photogrammetry systems, etc. [1]. Nevertheless, a definite set of procedures or best practices for uncertainty evaluation in this field of metrology is still lacking.

Furthermore, while a well-known series of standards for Coordinate Measuring Machines (CMMs) is widely diffused and studied, only recently some guidelines for the evaluation of uncertainty for LSDM instruments have been proposed. In particular, a solution for distributed measurement instruments is still under definition [1]. Actually, the references for scholars, researcher and manufacturers are the general metrology standards [2, 3] and a limited set of specific standards for optical systems or CMMs [4-6].

This paper presents a model able to provide a real time estimation of the uncertainty related to a point measurement performed by contact instruments based on photogrammetry principles. The model was adapted for MScMS-II, a prototype system designed for LSDM [7]. This model is developed also with the ambition to be helpful for the design of the measurement task, helping, for example, the user in the selection of the position of the object to be measured or in the definition of the measurement points.

The prototype system is presented in Section 2. Section 3 recalls the fundamental principles of the Multivariate Law of Propagation of Uncertainty. In Section 4 the principal uncertainty contributions are discussed and the uncertainty model is proposed. Some verification tests are shown in Section 5.

## MSCMS-II

MScMS-II is a prototype system designed for Large-Scale Dimensional Metrology. The system, relying on wireless technologies, is able to provide the coordinates of a point touched by a contact probe. Similarly to classical CMM, the system is provided with a software able to process coordinates data into generic geometric information [1].

 

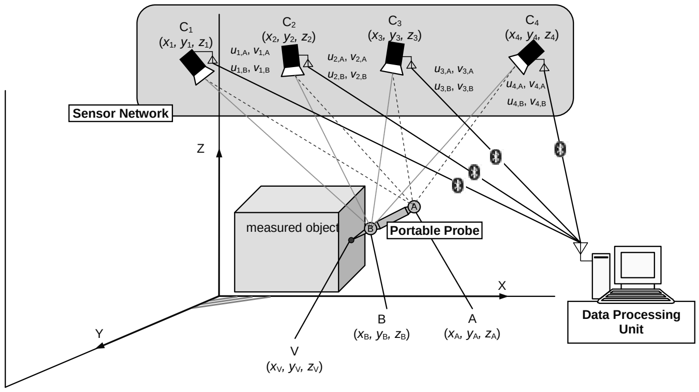


Fig. 1: Scheme of MScMS-II architecture.

The system, in its prototype version, is composed by three fundamental units (see Fig. 1):

- A network of wireless devices (see Fig.2), suitably distributed within the measurement volume in order to guarantee the best compromise between coverage and accuracy. More precisely, each device is equipped with a low-cost IR camera, characterized by an interpolated resolution of 1024×768 pixels (native resolution is 128×96 pixels), a field of view (FOV) of about 45°×30° and a working frequency of approximately 100 Hz. Each camera is attached with an IR light with wavelength $\lambda$ = 940 nm.
  The cameras are sensitive to that precise wavelength (940 $\mu$m). This avoids many of the problem occurring with visible light (such as reflections). Each network device also filters out "weak" IR sources to prevent faulty measurements.

Fig. 2: MScMS-II prototype. Detail of network device: each camera is coupled with a LED array.

- A portable contact probe. As shown in Fig. 3 the probe is basically a rod, with two reflective markers at the extremes and a tip to “touch” the measurement points. Reflecting the IR light emitted by the IR lights, the markers on the probe can be seen at a distance up to about 4 meters from each network device.

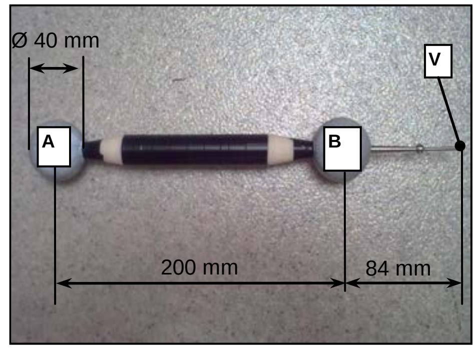


Fig. 3: MScMS-II prototype. Detail of the portable contact probe.

- A data processing unit (DPU) connected via Bluetooth to each network device. The DPU is in charge of most of the data processing tasks, such as probe localization, geometrical error correction, outlier filtering, shape reconstruction, etc.

The system is calibrated trough a procedure proposed by Svoboda et al. in 2005 [8] which practically consists in arbitrarily moving a single reflective marker in front of the camera of each network device. The calibration procedure provides position and orientation (external parameters) of each device as well as other important technical parameters (such as focal length, coordinates of image center, and parameters for the lens distortion model). In its current layout, the system is made by six network devices, deployed in a 5.0×6.0×3.0 m working environment.
With the information provided by the calibration, the system is able to localize the probe in a 3D space merging the tracking information deriving from each network device [1].

Fig. 4: MScMS-II prototype. Sensor network layout: black circles highlight the spatial position of the network device.

The working principles are those of close-range photogrammetry [9].

 

## MULTIVARIATE LAW OF PROPAGATION OF UNCERTAINTY

This section shortly recalls the principle of the Multivariate Law of Propagation of Uncertainty as specified in the "Guide to the expression of uncertainty in measurement" [2]. When a measurand $Y$ is determined as a function of $N$ other quantities $X_1, X_2, ..., X_N$ :

$$Y = f\left(X_1, X_2, ..., X_N\right), \quad (1)$$

the Law of Propagation of Uncertainty (MLPU) is a useful approach to estimate the uncertainty corresponding to the measurand (JCGM 100:2008 2008).

Eq. 1 can be rewritten as a vector:

$$\boldsymbol{Y} = \boldsymbol{f}\left(\boldsymbol{X}\right), \quad (2)$$

when $\boldsymbol{Y} = \begin{bmatrix} Y_1 & \cdots & Y_M \end{bmatrix}^T$ and $\boldsymbol{X} = \begin{bmatrix} X_1 & \cdots & X_N \end{bmatrix}^T$.

Eq.2 can be linearized expanding $\boldsymbol{f}\left(\boldsymbol{X}\right)$ into a first order Taylor series around the average values $\bar{\boldsymbol{x}}$ of the estimates vector $\boldsymbol{X}$. The MLPU states that an estimate of covariance matrix $\boldsymbol{\Sigma_Y} \in \mathbf{R}^{M,M}$ associated with $\boldsymbol{Y}$ can be obtained as [2]:

$$\boldsymbol{\Sigma_Y} = \boldsymbol{J}\boldsymbol{\Sigma_X}\boldsymbol{J}^T, \quad (3)$$

where $\boldsymbol{J} \in \mathbf{R}^{M,N}$ is the Jacobian of $\boldsymbol{f}\left(\boldsymbol{X}\right)$:

$$J_{i,j} = \left.\frac{\partial Y_i}{\partial X_j}\right|_{\bar{x}} \qquad i = 1, ..., M \qquad j = 1, ..., N, \quad (4)$$

and $\boldsymbol{\Sigma_X} \in \mathbf{R}^{N,N}$ is the covariance matrix of $\boldsymbol{X}$ .

## UNCERTAINTY EVALUATION IN MSCMS-II MEASUREMENTS

Few approaches for the assessment of the uncertainty related to the 3D measurement using triangulation techniques are available in scientific literature [4, 10]. Among the possible approaches, the one presented herein has the advantages of facing the problem from a theoretical point of view, focusing on all the various contributions of uncertainty.

Referring to MScMS-II, the major contributions to the overall uncertainty can be identified in [1]:

1) Uncertainty in the localization of the two targets on the probe ($\left(x_A, y_A, z_A\right)$ and $\left(x_B, y_B, z_B\right)$), which can be attributed to the localization algorithm. In details, this contribution to uncertainty can be charged to other factors:
   a) uncertainty due to the point projection in the image plane of each camera,
   b) uncertainty due to the camera calibration parameters,
   c) uncertainty due to camera synchronization, which can be considered negligible in static conditions,
   d) uncertainty due to the triangulation algorithm for 3D point reconstruction,
   e) uncertainty due to the parameters defining the corrections to camera lens distortion.
2) Uncertainty related to the calibration of probe geometric parameters.

The following Sections discuss and analyze the way all this contributions combine with each other.

## UNCERTAINTY OF 2D POINT COORDINATES

As sketched in Fig.5, a generic three-dimensional point is seen by each camera as a two-dimensional point on its view plane. In fact, the target is a physical object of spherical geometry, whose position is identified in its center.

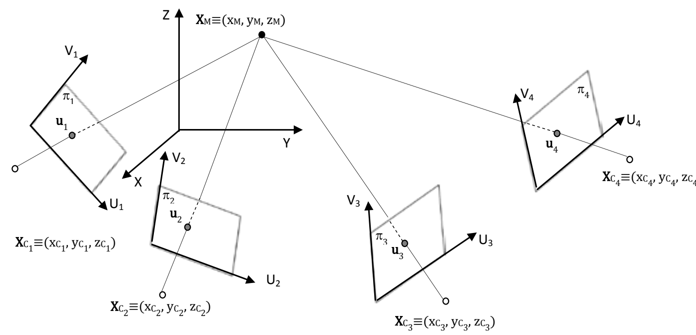


Fig. 5: Schematic representation of photogrammetry working principle. The 3D position of a generic point $X_M$ can be reconstructed according its projections on each camera view plane ( $\pi_j$ ).

The operation of projection of the center of the target is not trivial and may be influenced by several aspects such as:

- camera focus;
- size of the target and image processing algorithms;
- vibration or environmental disturbances;
- technical features of the cameras (resolution, focal length, FOV, sensitivity, lens distortion);
- visibility of the marker (which can sometimes be partially covered).

Some studies were conducted in order to *a priori* quantify the contribution of some of these factors to the uncertainty in the measurement of 2D point coordinates [10] [11]. Nevertheless these studies considered one factor at a time neglecting eventual correlations among the factors.

Alternatively, in the case of MSCMS-II, an empirical estimation of uncertainty in the measurement of 2D point coordinates ( $\boldsymbol{\Sigma}_{ij} \in \mathbf{R}^{2\times2}$ for a given camera $i$ and a specific point $j$ in the 3D space) can be directly obtained from the system. Working at a frequency of the order of hundreds of Hertz, MSCMS-II allows to replicate measurements several times within fractions of a second. In detail in a third of a second, the system provides about 30 replications of the measurement. Under the hypothesis of the absence of systematic errors, starting from the replication is possible, for each camera, to estimate the matrix $\boldsymbol{\Sigma}_{ij} \in \mathbf{R}^{2\times2}$ .

This approach is preferable because it allows binding the estimation to the actual conditions of use. On the other hand, it can be applied only during the measurement phase, so that it cannot be used for any *a priori* optimization of the measurement task.

## UNCERTAINTY OF 3D POINT COORDINATES

The Collinearity Equations are the basis of photogrammetry. These equations originate from the perspective projection of a 3D point onto a camera viewing plane. According to Fig. 5, the Collinearity Equations for a generic point $j$ and a given camera $i$ are [10, 11]:

$$\begin{aligned} u_{ij} &= u_{0i} - c_{ui}\frac{r_{11i}\left(x_j - x_{Ci}\right) + r_{12i}\left(y_j - y_{Ci}\right) + r_{13i}\left(z_j - z_{Ci}\right)}{r_{31i}\left(x_j - x_{Ci}\right) + r_{32i}\left(y_j - y_{Ci}\right) + r_{33i}\left(z_j - z_{Ci}\right)} - \delta u_{ij} \\ v_{ij} &= v_{0i} - c_{vi}\frac{r_{21i}\left(x_j - x_{Ci}\right) + r_{22i}\left(y_j - y_{Ci}\right) + r_{23i}\left(z_j - z_{Ci}\right)}{r_{31i}\left(x_j - x_{Ci}\right) + r_{32i}\left(y_j - y_{Ci}\right) + r_{33i}\left(z_j - z_{Ci}\right)} - \delta v_{ij} \end{aligned} \quad (5)$$

where:

- $\boldsymbol{x}_j \equiv \left(x_j, y_j, z_j\right)$ are the coordinates of point $\boldsymbol{x}_j$ to be localized,
- $u_{0i}$ and $v_{0i}$ are the principal point coordinates,
- $c_{ui}$ and $c_{vi}$ are the principal distance (focal length) $c_i$, scaled by different factors respectively in the $u$ and $v$ directions,
- $\delta u_{ij}$ and $\delta v_{ij}$ are the total lens distortions in the $u$ and $v$ directions,
- $r_{11}$, … , $r_{33}$ are the elements of the rotation matrix $\boldsymbol{R}_i$ defining camera orientation. $\boldsymbol{R}_i$ is obtained by considering three sequential rotations corresponding to the orientation angles of camera optical axis (about the $X$ $Y$ $Z$ -axes).

Having at disposal more than one camera with known parameters (location and orientation as well as other technical parameters) it is possible to revert Eqs. 5 to infer the 3D position of the point.

The uncertainty of 3D point coordinates can be derived from the linearization of the collinearity equations. For each camera *i* and each point *j*, Eqs. 5 can be reorganized into the following form:

$$\begin{aligned} \left(u_{ij} - u_{0i}\right) + c_{ui}\frac{r_{11i}\left(x_j - x_{Ci}\right) + r_{12i}\left(y_j - y_{Ci}\right) + r_{13i}\left(z_j - z_{Ci}\right)}{r_{31i}\left(x_j - x_{Ci}\right) + r_{32i}\left(y_j - y_{Ci}\right) + r_{33i}\left(z_j - z_{Ci}\right)} + \delta u_{ij} &= f_{1ij} \\ \left(v_{ij} - v_{0i}\right) + c_{vi}\frac{r_{21j}\left(x_j - x_{Ci}\right) + r_{22j}\left(y_j - y_{Ci}\right) + r_{23j}\left(z_j - z_{Ci}\right)}{r_{31i}\left(x_j - x_{Ci}\right) + r_{32i}\left(y_j - y_{Ci}\right) + r_{33i}\left(z_j - z_{Ci}\right)} + \delta v_{ij} &= f_{2ij} \end{aligned} \quad (6)$$

where (observations) $f_{1ij}$ and $f_{2ij}$ are the differences between the measured and the computed (or projected) 2D coordinates. Ideally these terms should be exactly equal to zero, however, being an over-determined system of equations, these terms may slightly differ from zero. In fact, in the measurement phase, the system of equations has three unknowns for each point to be measured and 2*i* equations. Already two or more cameras results in a redundancy of information that makes the system unsolvable except through a best compromise solution.

Eqs. 6 can be synthetically rewritten as:

$$F(\dot{\boldsymbol{\delta}}_i, \hat{\boldsymbol{\delta}}_i, \ddot{\boldsymbol{\delta}}_j) = \begin{bmatrix} f_{1ij} \\ f_{2ij} \end{bmatrix}, \quad (7)$$

where:

- $F(\dot{\boldsymbol{\delta}}_i, \hat{\boldsymbol{\delta}}_i, \ddot{\boldsymbol{\delta}}_j) \equiv \begin{bmatrix} F_1(\dot{\boldsymbol{\delta}}_i, \hat{\boldsymbol{\delta}}_i, \ddot{\boldsymbol{\delta}}_j) \\ F_2(\dot{\boldsymbol{\delta}}_i, \hat{\boldsymbol{\delta}}_i, \ddot{\boldsymbol{\delta}}_j) \end{bmatrix}$,
- $\dot{\boldsymbol{\delta}}_i$ is the vector of unknowns corresponding to the corrections to the approximations for the 6 external camera parameters (position and orientation),
- $\hat{\boldsymbol{\delta}}_i$ is the vector of unknowns corresponding to the corrections to the approximations for the 5 internal camera parameters (focal lengths $c_{ui}$ and $c_{vi}$, principal point $u_{0i}$ and $v_{0i}$, skewness coefficient) and the parameters of the lens distortions model (i.e. the coefficient of both radial (usually less than 3 parameters) and tangential (2 parameters) distortion correction polynomial)
- $\ddot{\boldsymbol{\delta}}_j$ is the vector of unknowns corresponding to the corrections to the approximations for the 3 point coordinates in the 3D space ( $\boldsymbol{x}_j \equiv \left(x_j, y_j, z_j\right)$, 3 parameters),

  Being $\boldsymbol{\delta}$ the union of the three vectors

$$\boldsymbol{\delta} = \begin{bmatrix} \dot{\boldsymbol{\delta}}_i \\ \hat{\boldsymbol{\delta}}_i \\ \ddot{\boldsymbol{\delta}}_j \end{bmatrix}, \quad (8)$$

after the linearization of Eq. 7, the general stochastic model of Eqs. 6 is

$$\boldsymbol{B}\,\boldsymbol{\delta} + \boldsymbol{\varepsilon}_{ij} = \boldsymbol{f}_{ij}, \quad (9)$$

where:

- $\boldsymbol{f}_{ij}$ is the vector of observations, $\boldsymbol{f}_{ij} = \left[\, f_{1ij} \quad f_{2ij} \,\right]^T$,
- $\boldsymbol{\varepsilon}_{ij} \sim N\left(0; \sigma_{ij}\right)$ is the vector of the stochastic errors,
- $\boldsymbol{B} \in \mathbf{R}^{2\times19}$ is the Jacobian of the two functions in Eq. 7 with respect to each of the 19 components of $\boldsymbol{\delta}$ .

Neglecting the correlations between $\dot{\boldsymbol{\delta}}_i, \hat{\boldsymbol{\delta}}_i$ and $\ddot{\boldsymbol{\delta}}_j$, an alternative formulation of the linearization of Eq. 7 is the one proposed by [10]:

$$\dot{\boldsymbol{B}}_{ij}\dot{\boldsymbol{\delta}}_i + \hat{\boldsymbol{B}}_{ij}\hat{\boldsymbol{\delta}}_i + \ddot{\boldsymbol{B}}_{ij}\ddot{\boldsymbol{\delta}}_j + \boldsymbol{\varepsilon}_{ij} = \boldsymbol{f}_{ij}, \quad (10)$$

where $\dot{\boldsymbol{B}}_{ij} \in \mathbf{R}^{2\times6}$, $\hat{\boldsymbol{B}}_{ij} \in \mathbf{R}^{2\times10}$ and $\ddot{\boldsymbol{B}}_{ij} \in \mathbf{R}^{2\times3}$ are respectively the Jacobians of $F(\dot{\boldsymbol{\delta}}_i, \hat{\boldsymbol{\delta}}_i, \ddot{\boldsymbol{\delta}}_j)$ with respect to $\dot{\boldsymbol{\delta}}_i$, $\hat{\boldsymbol{\delta}}_i$ and $\ddot{\boldsymbol{\delta}}_j$.

The model variables were grouped into the three different vectors ($\dot{\boldsymbol{\delta}}_i$, $\hat{\boldsymbol{\delta}}_i$ and $\ddot{\boldsymbol{\delta}}_j$) expressly designed to justify the hypothesis of no correlation. In other words, there is no physical reason suggest a correlation between the internal parameters and the position of the devices as well as the position of measured points.

Given that $W_{ij} \in \mathbf{R}^{2\times 2}$ is the inverse of the covariance matrix associated with the image (2D) coordinates of the $j$-th point on the $i$-th camera, left multiplying Eq. 10 for $\ddot{B}_{ij}^T W_{ij}$ it is possible to obtain the least squares normal equations associated with one image of one point [10]:

$$\begin{bmatrix} \ddot{B}_{ij}^T W_{ij} \dot{B}_{ij} & \ddot{B}_{ij}^T W_{ij} \hat{B}_{ij} & \ddot{B}_{ij}^T W_{ij} \ddot{B}_{ij} \end{bmatrix} \begin{bmatrix} \dot{\delta}_i \\ \hat{\delta}_i \\ \ddot{\delta}_j \end{bmatrix} = \ddot{B}_{ij}^T W_{ij} f_{ij}, \quad (11)$$

Eq. 11 can be reversed as:

$$\begin{aligned} \ddot{\delta}_j &= (\ddot{B}_{ij}^T W_{ij} \ddot{B}_{ij})^{-1} (\ddot{B}_{ij}^T W_{ij} f_{ij} - \ddot{B}_{ij}^T W_{ij} \dot{B}_{ij} \dot{\delta}_i - \ddot{B}_{ij}^T W_{ij} \hat{B}_{ij} \hat{\delta}_i) = \\ &= \ddot{N}_j^{-1} (\ddot{t}_{ij} - \bar{N}_{ij}^T \dot{\delta}_i - \tilde{N}_{ij}^T \hat{\delta}_i), \end{aligned} \quad (12)$$

where for each image $i$ and each point $j$:

$$\begin{aligned} \bar{N}_{ij} &= \dot{B}_{ij}^T W_{ij} \ddot{B}_{ij} & \bar{N}_{ij} &\in \mathbf{R}^{6x3}, \\ \tilde{N}_{ij} &= \hat{B}_{ij}^T W_{ij} \ddot{B}_{ij} & \tilde{N}_{ij} &\in \mathbf{R}^{10x3}, \\ \ddot{N}_j &= \ddot{B}_{ij}^T W_{ij} \ddot{B}_{ij} & \ddot{N}_j &\in \mathbf{R}^{3x3}, \\ \ddot{t}_{ij} &= \ddot{B}_{ij}^T W_{ij} f_{ij} & \ddot{t}_{ij} &\in \mathbf{R}^{3x1}. \end{aligned} \quad (13)$$

Under the hypothesis of absence of systematic errors, it is reasonable to expect that the uncertainty in each point image is uncorrelated with that of any other image. Systematic errors, if present and detected, can generally be corrected so as to make acceptable the assumptions introduced above. Some examples of these corrections can be found in [12] and [1]. Eq. 12 extended to a set of $i = n$ cameras and $j = 1$ point, becomes:

$$\ddot{\delta} = \ddot{N}^{-1} \left( \ddot{t} - \bar{N}^T \dot{\delta} - \tilde{N}^T \hat{\delta} \right), \quad (14)$$

where $\ddot{\delta} = \begin{bmatrix} \ddot{\delta}_{1j} & \ddot{\delta}_{2j} & \cdots & \ddot{\delta}_{nj} \end{bmatrix}^T$, $\dot{\delta} = \begin{bmatrix} \dot{\delta}_{1j} & \dot{\delta}_{2j} & \cdots & \dot{\delta}_{nj} \end{bmatrix}^T$, $\hat{\delta} = \begin{bmatrix} \hat{\delta}_{1j} & \hat{\delta}_{2j} & \cdots & \hat{\delta}_{nj} \end{bmatrix}^T$ and:

$$\begin{aligned} \bar{N} &= \dot{B}^T W \ddot{B} & \bar{N} &\in \mathbf{R}^{6nx3n}, \\ \tilde{N} &= \hat{B}^T W \ddot{B} & \tilde{N} &\in \mathbf{R}^{10nx3n}, \\ \ddot{N} &= \ddot{B}^T W \ddot{B} & \ddot{N} &\in \mathbf{R}^{3nx3n}, \\ \ddot{t} &= \ddot{B}^T W f & \ddot{t} &\in \mathbf{R}^{3nxn}. \end{aligned} \quad (15)$$

The total weights matrix $W$ is the inverse of the total covariance matrix $\Sigma \in \mathbf{R}^{2n\times 2n}$. Under the hypothesis of no correlation between different images, $\Sigma \in \mathbf{R}^{2n\times 2n}$ is a 2n x 2n block diagonal matrix. Similarly, $\dot{B}$, $\hat{B}$, $\ddot{B}$ are respectively 2n x 6n, 2n x 10n, 2n x 3n block diagonal matrices. In detail

$$\dot{B} = \begin{bmatrix} \dot{B}_{1j} & 0 & 0 & 0 \\ 0 & \dot{B}_{2j} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \dot{B}_{nj} \end{bmatrix}, \quad (16)$$

$$\hat{B} = \begin{bmatrix} \hat{B}_{1j} & 0 & 0 & 0 \\ 0 & \hat{B}_{2j} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \hat{B}_{nj} \end{bmatrix}, \quad (17)$$

$$\ddot{B} = \begin{bmatrix} \ddot{B}_{1j} & 0 & 0 & 0 \\ 0 & \ddot{B}_{2j} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \ddot{B}_{nj} \end{bmatrix}. \quad (18)$$

Similar considerations hold for $\ddot{t}$:

$$\ddot{t} = \begin{bmatrix} \ddot{t}_{1j} & 0 & 0 & 0 \\ 0 & \ddot{t}_{2j} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \ddot{t}_{nj} \end{bmatrix}. \quad (19)$$

So far, the covariance of $\ddot{\delta}_j$ (i.e. the 3D point coordinates) can be calculated by applying the MLPU to Eq. 14. $\hat{\Sigma}$ and $\dot{\Sigma}$, i.e. the covariance matrices respectively related to $\hat{\delta}_i$ and $\dot{\delta}_i$ can be obtained by the calibration procedure. $\Sigma$ (and as a consequence $W$), i.e. the covariance matrix related to the 2D coordinates provided by each camera, may be given *a priori*, or obtained a posteriori during the measurement task. In fact, each measurement is normally the result of a number of replications which can be used to infer this parameter (for instance, the number of replications is equal to 30 for MScMS-II).
Under the hypothesis introduced, the MLPU applied to Eq. 14 gives:

$$\ddot{\Sigma} = \ddot{N}^{-1} + \ddot{N}^{-1} \bar{N}^T \dot{\Sigma} \bar{N} \ddot{N}^{-1} + \ddot{N}^{-1} \tilde{N}^T \hat{\Sigma} \tilde{N} \ddot{N}^{-1}. \quad (20)$$

$\ddot{\Sigma} \in \mathbf{R}^{3x3}$ is the covariance matrix associated with the 3D coordinates of the measured point. Notice how $\bar{N}$ directly depends on $\dot{B}_{ij}$ which is strictly related to the positions (and orientations) of the camera. This suggests that opportunely modifying these parameters (i.e. repositioning the camera in the measuring space), it is possible to optimize the values of $\ddot{\Sigma}$, i.e. the measurement uncertainty.

## UNCERTAINTY OF PROBE TIP COORDINATES

The geometry of the mobile probe is a design parameter. Thus, the coordinate of the tip $x_V = (x_V, y_V, z_V)$ can be found as a function of the positions ($x_A$ and $x_B$) of the two targets on the mobile probe:

$$x_V = x_A + \frac{x_B - x_A}{|x_B - x_A|} |x_V - x_A|, \quad (21)$$

where $|x_B - x_A|$ and $|x_V - x_A|$ are respectively the distances between device A and B (fixed *a priori* as design parameters), and the distance between device A and the probe tip (V). Under the hypothesis of independence between $x_A$, $x_B$, $|x_B - x_A|$ and $|x_V - x_A|$, the MLPU can be applied to Eq. 21 in order to obtain an estimate of the covariance of the probe tip coordinates $\Sigma_V \in \mathbf{R}^{3,3}$ as:

$$\Sigma_V = J_V \Sigma_\omega J_V^T, \quad (22)$$

where $J_V \in \mathbf{R}^{3,8}$ is the matrix of the partial derivatives of Eq. 21, with respect to the coordinates of the two targets of the probe, and the parameters $|x_B - x_A|$ and $|x_V - x_A|$. $\Sigma_\omega \in \mathbf{R}^{8,8}$ is the estimate of the covariance matrix of parameters in Eq. 21:

$$\Sigma_\omega = \begin{bmatrix} \Sigma_A & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \Sigma_B & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \sigma^2_{|x_V - x_A|} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \sigma^2_{|x_B - x_A|} \end{bmatrix}, \quad (23)$$

where $\Sigma_A, \Sigma_B$ (obtained with Eq. 20) are the estimated covariance matrices of point A and point B while $\sigma^2_{|x_V - x_A|}$ and $\sigma^2_{|x_B - x_A|}$ are the variances corresponding to $|x_V - x_A|$ and $|x_B - x_A|$ measurements respectively.

Given $\Sigma_V \in \mathbf{R}^{3,3}$, a synthetic indicator of the expanded overall uncertainty is the 3D radial uncertainty [2]:

$$U_V = k \cdot \sqrt{\Sigma_{V,1,1} + \Sigma_{V,2,2} + \Sigma_{V,3,3}}, \quad (24)$$

where $k$ is a properly chosen coverage factor $k$ (usually fixed equal to 2).

## TEST

To give evidence of the approach proposed, the following test was designed.

A total of 225 points were acquired on a horizontal plane using MScMS-II. Fig. 6 shows the measured points.

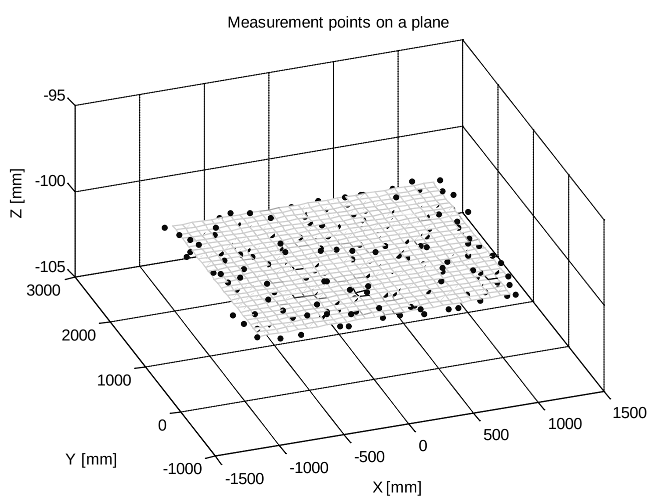


Fig.6: Measured points. The mesh plot represents the least square approximation of the plane.

For each measured point, the relative uncertainty in terms of covariance matrix was calculated by means of the approach described in previous sections. Fig. 7 synthetically shows the 3D radial uncertainty (with $k = 2$) associated with each point of the plane using a grayscale plot: the darker is the point the lower is the related radial uncertainty. At first sight, there is no evidence of a macroscopic trend in the color distribution (and thus in the related 3D radial uncertainty).

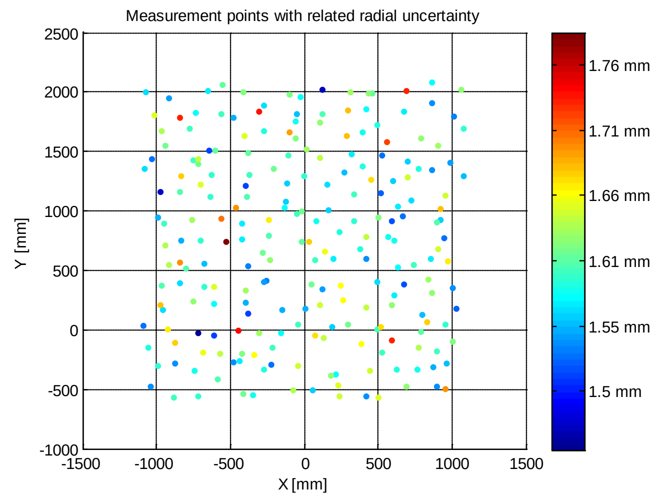


Fig. 7: Measured points with related 3D radial uncertainty. The value of radial uncertainty ($k = 2$) is given by the color of the plotted point.

In order to analyze these results, the fact that the points were taken in a limited part of the working volume of the instrument has to be considered. In fact in this portion of the working volume, the 3D radial uncertainty does not change significantly ($1.5\, mm < U_V < 1.8\, mm$).

To have an idea of the potential of the model, just consider the alternative procedures to produce similar results. The classic approach to achieve an uncertainty assessment related to a specific measurement activity is to repeat the measurements in order to evaluate the variability and bias the results [2]. Evidently this approach may result time consuming, so that the proposed model can constitute a viable alternative.

## CONCLUSIONS

Every measurement has no meaning if it is not associated with an uncertainty band [2].

This paper proposes a methodology for the assessment of the uncertainty associated with each measurement made with MScMS-II, a system based on wireless sensor networks for large scale dimensional metrology.

The method is based on MLPU and offers several advantages:

- It can result helpful for the design of the measurement tasks, helping, for example, the user in the selection of the position of the object to be measured, in the definition of the measurement points or even in the positioning of the network devices.
- During the analysis and processing of measurement results, the method can be interesting in the definition of weights associated with the measurements: greater importance can be attributed to those points with lower uncertainty when reconstructing surfaces or known geometries.

Even if the approach is promising, further analysis can be interesting to quantify the advantages, in terms of improvement of accuracy, the method can guarantee.

To date, the "weak point" of the study reported in the paper is the model validation which is intentionally left to future works.

The problem of a validation study resides in the fact that the model depends on many factors, such as the position of the devices, the position of the measured points as well as the internal camera parameters. In order to achieve a general result of validation all these factors have to be concurrently considered. Probably the most reasonable approach would be to compare the results provided by the model with the results in terms of mean and standard deviation obtainable through measurements repetition.

Future developments of this research can be also addressed in the direction of the study of methods for the uncertainty assessment when, instead of the collinearity equations, alternative methods (such as the projection matrices) are used [9, 10].

## ACKNOWLEDGMENTS

Financial support to this work has been provided by Italian Ministry of Education, University and Research, as part of the project PRIN2008 (prot. 200853ZT3Z).

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