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Uncertainty evaluation of distributed Large-Scale-Metrology systems by a Monte Carlo approach

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ABSTRACT

Distributed systems for Large-Scale-Metrology applications generally include a set of angular and/or distance sensors, distributed around the measurement volume, and some targets to be localized, in contact with the measured object's surface. For these systems, estimating the uncertainty in target localization is far from trivial, as it may be affected by several factors: uncertainty in sensor calibration and angular/distance measurements, relative position between targets and sensors, etc. This paper proposes a novel approach based on the combined use of the Multivariate Law of Propagation of Uncertainty and Monte Carlo method. Preliminary results and experimental tests are presented and discussed.

1. Introduction

In the field of Large Scale Metrology (LSM), distributed systems are more and more diffused [1]. Industrial applications generally concern assembly and dimensional verification of large-sized mechanical components, in which levels of accuracy of several tenths of millimetre are tolerated [2]. The typical configuration of a distributed LSM system includes [3]: (i) a network of sensors, distributed around the measured object, (ii) some targets to be localized, generally in contact with the measured object's surface, or mounted on a hand-held probe, and (iii) a centralized data processing unit (DPU), which receives and processes data from sensors, in order to localize targets.

The scientific literature encompasses three possible approaches for target localization [4]: (i) *multiangulation*, using the angles subtended by targets, with respect to sensors; (ii) *multilateration*, using the distances between targets and sensors; (iii) *hybrid techniques*, based on the combined use of distances and angles. The relative positioning of sensors with respect to targets and their technical features strongly affect the system performance, both from the metrological and the operational point of view.

When designing a network of sensors for a distributed LSM system, one of the most important features to be considered is the uncertainty in target localization; in general, the more technologically advanced and expensive the sensors, the lower their uncertainty in distance/angular measurements and, hence, that in target localization. Despite this importance, the scientific literature on the subject is relatively scarce and fragmented [4,5].

The aim of this paper is to introduce a new methodology to evaluate the overall measurement uncertainty in the localization of targets by LSM systems based on distance and angular measurements. The proposed approach relies on the General Least Square (GLS) method and makes it possible to obtain the covariance matrix of the 3D coordinates of targets, using the uncertainties in the distance/angular measurements by network sensors and those of the sensors' parameters resulting from their calibration process [6].

The remainder of the paper is structured as follows. Section 2 defines a mathematical model for the 3D target localization of a general LSM system. Section 3 describes the new methodology for evaluating the uncertainty in target localization. Section 4 presents an experimental test of the methodology, using a distributed LSM system, which integrates one laser tracker and three photogrammetric cameras.

2. Target localization model

A general distributed LSM system includes a number of network sensors positioned around the measurement volume [3]. It is assumed that (i) *O-XYZ* is a *global* Cartesian coordinate system and (ii) each *i*th sensor (D_i) has a *local* coordinate system, o_i-$x_iy_iz_i$, roto-translated with respect to *O-XYZ*, reflecting its spatial position/orientation. The (six) position/orientation parameters related to each *i*th sensor (i.e., $X_{0_i}, Y_{0_i}, Z_{0_i}$ and ω_i, ϕ_i, κ_i) are treated as known parameters, since as they are measured in an initial calibration phase. The calibration process generally includes multiple measurements of calibrated artefacts, within the measurement volume [3].

Assuming that *P* is the point to be localized in the 3D space (e.g., the centre of a spherical target), associated with vector **X**, the positioning problem may be decomposed according to the following linearized model [1,4]:

$$\mathbf{MX}-\mathbf{B} \equiv \begin{bmatrix} \mathbf{M}^{dis.} \\ \mathbf{M}^{ang.} \end{bmatrix} \mathbf{X} - \begin{bmatrix} \mathbf{B}^{dis.} \\ \mathbf{B}^{ang.} \end{bmatrix} = 0, \quad (1)$$

where $\mathbf{X} = [X,\ Y,\ Z]^T$ is the position vector of *P* in the global coordinate system *O-XYZ*; $\mathbf{M}^{dis.}$, $\mathbf{M}^{ang.}$ and $\mathbf{B}^{dis.}$, $\mathbf{B}^{ang.}$ are the *design*

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matrices (or Jacobian matrices) and the *reduced measured observations* respectively, both roto-translated in order to refer to the global coordinate system *O-XYZ*. The matrices related to distance sensors are labelled with superscript "*dis.*", while those related to angular sensors with superscript "*ang.*". **M** and **B** contain several parameters relating to each *i*th sensor, such as the distances and/or angles subtended by *P* (i.e., d_i, θ_i and φ_i) and the sensor position/orientation (i.e., $X_{0_i}, Y_{0_i}, Z_{0_i}$ and ω_i, ϕ_i, κ_i). Since the "true" values of the above parameters are never known exactly, they can be replaced with appropriate estimates: $\hat{d}_i, \hat{\theta}_i$ and $\hat{\varphi}_i$, resulting from distance and angular measurements, and $\hat{X}_{0_i}, \hat{Y}_{0_i}, \hat{Z}_{0_i}, \hat{\omega}_i, \hat{\phi}_i, \hat{\kappa}_i$, resulting from an initial calibration process. For details on the construction of **M** and **B**, see [1,4].

The unknown coordinates of point *P* are determined solving the system in Eq. (1), which is generally *overdefined* (i.e. there are more equations than unknown parameters).

It is worth remarking that the equations of the system may differently contribute to the uncertainty in the localization of *P*. Specifically, the three main factors affecting this uncertainty are:

1. uncertainty in the *distance* and *angular measurements* ($\hat{d}_i, \hat{\theta}_i, \hat{\varphi}_i$) by sensors, which generally depends on their metrological characteristics;
2. *relative position* between point *P* and each *i*th sensor; e.g., for angular sensors, the uncertainty in the localization of *P* increases proportionally to the distance between *P* and sensors;
3. uncertainty in the *sensor position/orientation* parameters ($\hat{X}_{0_i}, \hat{Y}_{0_i}, \hat{Z}_{0_i}, \hat{\omega}_i, \hat{\phi}_i, \hat{\kappa}_i$), resulting from the calibration process.

For this reason, it is appropriate to solve the system in Eq. (1) giving greater weight to the contributions from the network sensors that produce less uncertainty and vice versa. To this purpose, the most practical approach is that of the *Generalized Least Squares* (GLS) method [3,6], in which a weight matrix (**W**) taking into account the uncertainty produced by the equations of the system, is defined as:

$$\mathbf{W} = (\mathbf{D}^T \boldsymbol{\Sigma}_\mathbf{S} \mathbf{D})^{-1}, \tag{2}$$

where **D** is the matrix (or Jacobian matrix) of the partial derivatives of the elements in the first member of Eq. (1) with respect to the each sensor's measured observations (i.e., distance and angles), and $\boldsymbol{\Sigma}_\mathbf{S}$ is the covariance matrix of these measured observations [3].

Assuming that sensors work independently from each other and there is no correlation between the angular/distance measurements related to different sensors, $\boldsymbol{\Sigma}_\mathbf{S}$ is a diagonal matrix containing the variances related to the sensor measurements. These variances can be obtained in several ways: (i) from manuals or technical documents relating to the network sensors in use, (ii) estimated through ad hoc experimental tests, or (iii) estimated using data from previous calibration processes. We remark that these values should reflect the network sensors' uncertainty in realistic working conditions, e.g., in the presence of vibrations, light/temperature variations and other typical disturbance factors.

Since **D** depends on the coordinates of *P* [1,3], the definition of the elements of this matrix requires the (at least, rough) localization of *P*. One option is to use the *Ordinary Least Squares* method (OLS) to solve the system in Eq. (1), as [1]:

$$\hat{X} = (\mathbf{M}^T \mathbf{M})^{-1} \mathbf{M}^T \mathbf{B}. \tag{3}$$

A fine estimate of **X** is then obtained by applying the GLS method to Eq. (1) [4]:

$$\hat{X} = (\mathbf{M}^T \mathbf{W} \mathbf{M})^{-1} \mathbf{M}^T \mathbf{W} \mathbf{B}. \tag{4}$$

This approach can be classified as *cooperative fusion* as it may fuse, in the same localization problem, data from sensors of different type (e.g., sensors performing angular and distance measurements) and metrological characteristics (e.g., sophisticated sensors and low-end ones, characterized by different measurement uncertainties) [1].

3. Uncertainty evaluation

Returning to the aforedescribed target localization problem, the covariance matrix relating to the solution (**X**) can be estimated analytically, by applying the Multivariate Law of Propagation of Uncertainty (MLPU) [7], as:

$$\hat{\boldsymbol{\Sigma}}_X = (\mathbf{M}^T \widehat{\mathbf{W}} \mathbf{M})^{-1}. \tag{5}$$

The uncertainty related to estimates of the individual Cartesian coordinates ($\hat{X}$) can be obtained considering the diagonal elements of $\hat{\boldsymbol{\Sigma}}_X$:

$$U_X = k \cdot \sqrt{\hat{\boldsymbol{\Sigma}}_{X,11}}, \quad U_Y = k \cdot \sqrt{\hat{\boldsymbol{\Sigma}}_{X,22}}, \quad U_Z = k \cdot \sqrt{\hat{\boldsymbol{\Sigma}}_{X,33}}, \tag{6}$$

where *k* is the coverage factor, which in most of cases is fixed at *k* = 2 (which means that, assuming a normal distribution of each measured component of the $\hat{X}$ vector, the corresponding coverage probability is equal to 95%) [7,8].

Although being simple and compact, the suggested model has an important limitation: it does not take into account the uncertainty related to the sensor position/orientation parameters. Two possible approaches for overcoming this limitation are:

- reviewing and expanding the mathematical model for target localization, weighing the equations of the systems on the basis of the position/orientation parameters. From a practical point of view, the matrix **W** should be modified so as to consider the uncertainty contributions due to the above parameters;
- integrating the model for target localization through numerical simulations of the uncertainty related to the above parameters.

Even if the first approach is formally correct and prescribed by the scientific literature [1,7], it has some limitations. The major ones are related to the linearization of the model and the assumption of independence between observations and position/orientation parameters. Moreover, it is based exclusively on the normal distribution of the variables involved and does not allow the use of other distributions. Also, the implementation of the first approach is anything but trivial from a mathematical viewpoint.

We also remark that the system in Eq. (4) is defined for the localization of a single target. The model should be significantly revised in the case of multiple targets arranged on a rigid body, such as in the case of 6-DOF probes [3]. This would further complicate the analytical construction of the matrix **W**, as required by the first approach.

In the present paper, we focus on the second approach, and develop a Monte Carlo simulation, which uses the distributions of the observations of each single sensor and the distribution of the relevant sensor parameters as input elements.

A set of *m* replications of the localization of *P* have been performed using Eq. (4), and randomly varying the sensor parameters and the measurement observations within suitable fields of uncertainty. Specifically, for each replication, the position/orientation of sensors, estimated during the calibration, is deliberately distorted, to take account of the uncertainties resulting from the calibration process. The distortion of the position/orientation parameters can be formally expressed as (*j* = 1, . . ., 6):

$$\hat{\hat{\xi}}_{j_i} = \hat{\xi}_{j_i} + \epsilon_{\xi_{j_i}} \quad \text{with } \epsilon_{\xi_{j_i}} \sim N(0, \sigma^2_{\xi_{j_i}}), \tag{7}$$

which means thatthe parameters related to the "nominal" position/orientation of each *i*th sensor (included in vector $\boldsymbol{\xi}_i = [X_{0_i}, Y_{0_i}, Z_{0_i}, \omega_i, \phi_i, \kappa_i]^T$ and obtained through the calibration process) are distorted by adding zero-mean normally distributed errors, with known variances ($\sigma^2_{X_{0_i}}, \sigma^2_{Y_{0_i}}, \sigma^2_{Z_{0_i}}, \sigma^2_{\omega_i}, \sigma^2_{\phi_i}, \sigma^2_{\kappa_i}$). This hypothesis is reasonable in the absence of systematic error causes.

In other specific situations, a more appropriate distribution must be considered [9].

The variances of the above parameters can be estimated experimentally during the calibration process of each ith sensor, under the assumption of independence between sensors.

Similarly, the uncertainty related to distance/angular measurements of sensors can be considered by deliberately distorting the "nominal" values (i.e., $\hat{d}_i, \theta_i, \varphi_i$, obtained in the measurement phase): $d_i = \hat{d}_i + \epsilon_{d_i}$ with $\epsilon_{d_i} \sim N(0, \sigma^2_{d_i})$, $\theta_i = \theta_i + \epsilon_{\theta_i}$ with $\epsilon_{\theta_i} \sim N(0, \sigma^2_{\theta_i})$, $\varphi_i = \varphi_i + \epsilon_{\varphi_i}$ with $\epsilon_{\varphi_i} \sim N(0, \sigma^2_{\varphi_i})$.

In the absence of systematic error causes and spatial/directional effects, it is reasonable to assume that (i) for each ith network sensor, the variances are isotropic (i.e. they do not depend on the position and orientation of the sensor in the measurement space), and (ii) sensors of the same type are characterized by the same variances.

Σ_X matrix can be estimated *numerically* using the results of the m replications, and then the uncertainties of the P coordinates can be obtained using Eq. (6).

Summarizing, according to the proposed approach, the localization of point P and the relevant uncertainties can be found through the following four-step procedure:

(i) rough localization of P, using the OLS method (see Eq. (3)),
(ii) construction of **W** matrix,
(iii) fine localization of P, using GLS method (using Eq. (4)),
(iv) computation of Σ_X matrix using Monte Carlo simulation.

4. Application of the methodology

4.1. Experimental set-up

The methodology has been applied to a LSM system consisting of four sensors (see Figs. 1 and 2): (i) three Hitachi – Gigabit Ethernet photogrammetric IR cameras (pixel resolution: 1360 × 1024, frame rate: 30 fps) [10] equipped with a 38.1 mm reflective spherical markers, and (ii) a laser tracker (LT) API Radian™ [11] equipped with a spherical retro-reflector of the same diameter.

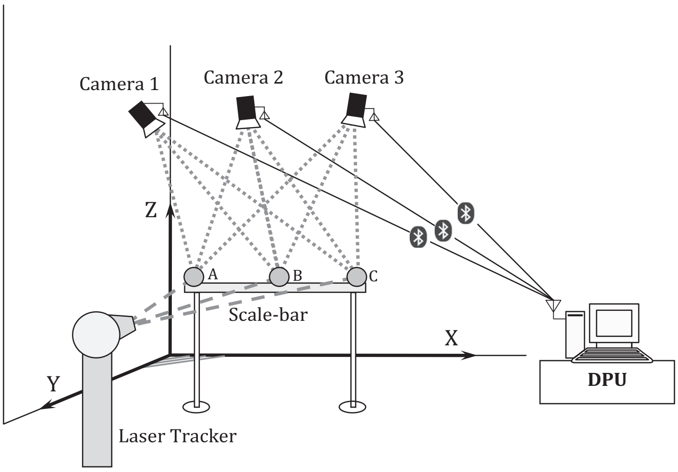

Fig. 1. Scheme of the experimental set-up (A, B and C are the reference points of the scale-bar).

Each camera (C1 to C3) has been calibrated in order to be able to provide azimuth (θ_1 to θ_3) and elevation (φ_1 to φ_3) angular measurements of the target point P, with respect to its local coordinate system [10]. The LT is equipped with an ADM or a laser interferometer and two angular encoders, calibrated in order to provide respectively one distance (d_{LT}) measurement and two – azimuth (θ_{LT}) and elevation (φ_{LT}) – angular measurements of point P, with respect to the same local coordinate system [11].

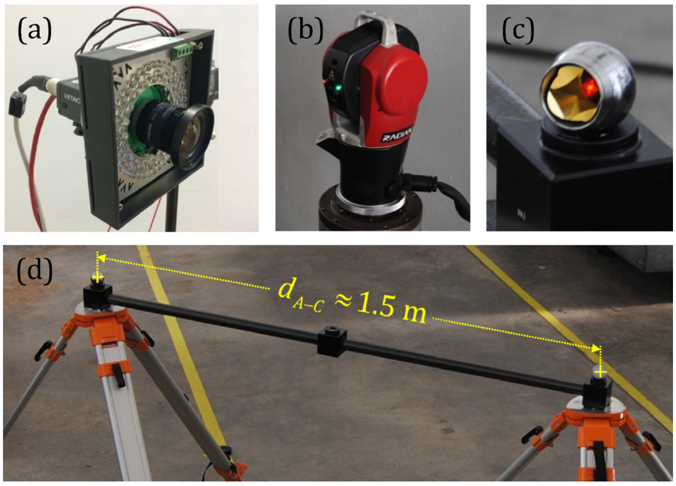

Fig. 2. Photos of (a) one of the IR cameras, (b) the LT, (c) the LT target and (d) the calibrated scale-bar used for the experiments.

The proposed cooperative fusion approach is able to estimate the 3D position of each measured point, based on the 9 distance/angular measurements available (i.e., 2 angular measurements for each of the 3 photogrammetric cameras, and two angular measurements plus one distance measurement for the LT).

The standard-deviation values introduced for distorting the nominal distance/angular measurement values have been defined considering the relevant technical specifications of sensors and other information from the scientific literature [10–12]. The resulting values are $\sigma_{\theta_C} = \sigma_{\varphi_C} = 3.5 \times 10^{-4}$ rad for the three cameras, which have been considered as nominally identical, $\sigma^2_{d_{LT}} = 2.5 \cdot d_{LT} \times 10^{-3}$ mm and $\sigma_{\theta_{LT}} = \sigma_{\varphi_{LT}} = 1.4 \times 10^{-5}$ rad for the LT.

The parameters describing the position/orientation of sensors, obtained from their calibration process, are reported in Table 1. The relevant standard deviations are $\sigma_{X_0} = \sigma_{Y_0} = \sigma_{Z_0} = 0.1$ mm and $\sigma_\omega = \sigma_\phi = \sigma_\kappa = 3.5 \times 10^{-4}$ rad for the three cameras and the LT.

Table 1
Parameters concerning the position/orientation of sensors.

	C1	C2	C3	LT
X_0 [mm]	0.000	0.000	−0.235	9887.628
Y_0 [mm]	0.000	397.863	−401.156	254.327
Z_0 [mm]	0.000	0.000	0.091	−453.275
ω [rad]	0.000	0.008	0.003	−0.003
ϕ [rad]	0.000	0.031	−0.018	0.013
κ [rad]	0.000	−0.153	0.134	3.125

Next, we have considered the positions (A, B and C) of the centre of spherical targets, in three reference positions on a calibrated scale-bar (see Fig. 2).

According to the experimental se-up in Fig. 1, the LT has been positioned at about 5.5 m from the barycentre of the measured points and the cameras have been placed all around the scale-bar in the opposite position of the LT at about 4.5 m each from the barycentre of the measured points. It is a common practise in industrial applications to position the LT at a relatively large distance with respect to the targets, so as to "cover" them, with no need of additional repositionings [4].

4.2. Experimental results

The position measurements of the three reference points on the scale-bar have been replicated 50 times. The mean values of the distances between the reference points (A–B, B–C, and A–C) and the relevant standard deviations (repeatability) are reported in Table 2, and compared with those (uncertainty) estimated applying the proposed methodology (see Section 3) [8]. Results are then compared with the nominal values obtained in a calibration procedure based on 10 replications for each distance,

Table 2
Distances measured on the scale-bar and relevant standard deviations obtained respectively (i) in the scale-bar calibration, (ii) using 50 replicated measurements and (iii) using the Monte Carlo approach for uncertainty evaluation (values reported in mm).

	d_{A-B}	$\sigma_{d_{A-B}}$	d_{B-C}	$\sigma_{d_{B-C}}$	d_{A-C}	$\sigma_{d_{A-C}}$
Nominal	728.294	0.003	727.703	0.003	1455.996	0.003
Repl. (50)	728.296	0.109	727.698	0.110	1455.993	0.107
M. C. (1000)	728.294	0.112	727.705	0.116	1455.999	0.115

performed using a DEA SCIROCCO 251310 Coordinate Measuring Machine (CMM).

The results obtained by the Monte Carlo method have been obtained with an ad hoc Matlab® routine implementing the procedure described in Section 3, performing 1000 replications, with an CPU time lower than 3 s using a 3.50 GHz Intel i7-4770k processor.

Table 1 shows that the results obtained with the system, considering the relevant uncertainty, are compatible with the nominal distances at a 95% confidence level. The standard deviation obtained with the 50 replicated measurements are, in the same order of magnitude, but slightly smaller than the standard deviations obtained with the Monte Carlo approach. This is in line with the fact that the standard deviations obtained through the former approach include only the repeatability contributions, while those obtained with the latter one contain also the uncertainty contributions relating to the sensors' position/orientation, as well as the contributions related to their distance/angular measurements and their relative position with respect to targets. This is also a practical proof of the fact that small uncertainties of the sensor position/orientation parameters (such as in the present experimental case) marginally contribute to the overall uncertainty in the position of the target. The approximations introduced in Eqs. (4) and (5) can therefore be considered as relatively reasonable.

5. Conclusions

The paper proposed a methodology for the estimation of the measurement uncertainty based on the Monte Carlo approach when using distributed systems for LSM applications.

The proposed methodology has been automated through an ad hoc routine, developed in Matlab®, which is able to estimate the 3D position of a measured point and the relevant uncertainty, using (i) the measurements provided by each sensor (and the relevant uncertainties) and (ii) the sensors' position/orientation parameters (and the relevant uncertainties), as input elements.

The mathematical model for target localization is based on the GLS method and is well suited to multi-sensor networks, i.e., networks in which distributed sensors of different nature coexist. Through some experiments, the plausibility of the results of the methodology has been checked for a specific distributed LSM system consisting of three photogrammetric IR cameras and a laser tracker. These experiments confirmed the feasibility and practicality of the proposed methodology.

The main advantages of the proposed approach are that: (i) it can be applied to any kind of distributed LSM system equipped with distance and/or angular sensors; (ii) it considers the uncertainty contribution related to the distance/angular measurements and that related to the position/orientation parameters of each sensor; (iii) other contributions, even related to systematic effects (vibrations, light/temperature variations, etc.), can be included in the simulation; (iv) the method does not require normal distribution of input variables, hence it can be implemented using the most appropriate distributions, case by case; (v) the method can be used both in the design phase, in order to estimate the metrological performance of a distributed multi-sensor LSM system before realizing it, and the measurement phase, in order to evaluate the current measurement uncertainty; (vi) the computational time is in the order of some seconds, for typical LSM applications.

The experimental case study presented in the paper showed that, when the uncertainties related to the sensors' parameters are small, their contribution to the overall uncertainty in the target localization is marginal. Hence, these contribution are negligible in the definition of the localization model with the application of the GLS method.

Future research will focus on the sensitivity analysis of the method, considering different sensor configurations/densities, and introducing other sources of uncertainty (e.g., temperature variation, gravity influence on the measured object, etc.). Furthermore, the proposed model will be extended to 6-DOF probes including additional equations relating to the rigid-body constraints between probe targets.

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