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The conceptual link between measurements, evaluations, preferences and indicators, according to the representational theory

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Abstract

This paper presents a classification of the different ways by which observable or non-directly observable properties of an object or an event can be judged/described. The reference framework is the representational theory. The definition of measurement as an empirical and objective operation is the starting point. The concept of representational measurement is compared with those of evaluation, preference and indicator. Evaluation maintains the empiricity but not the objectivity of measurement: there is no unanimously acknowledged reference for the description of latent constructs. Preference is neither empirical nor objective: every subject has his/her own relation to express the judgment and this relation is not exogenously known. This article shows how all these operations can be considered as separate subsets of the concept of indicator. In the end a further operation, the dictation, is introduced. According to this scheme of classification, dictation is objective but not empirical. Finally, we trace possible research paths to be undertaken for further analysis on the argument.

Keywords: Evaluation; Dictation; Indicator; Measurement; Preference; Representational theory of measurement

1. Introduction

When we are asked to describe or evaluate the properties of an object or an event, or when we have to compare two or more objects, we may consider different basic operations. The object properties, either directly or non-directly observable, may essentially be judged or described by measurements, evaluations, preferences or indicators.

What are the substantial differences between measurements, evaluations and preferences, and the concept of indicator?

The aim of the present work is to propose a new scheme of classification. The question has a particular scientific importance and some possible repercussions involve many disciplines such as decision-making, metrology, social sciences, operations

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management, quality management, etc. Considering this latter aspect, for example, when we ask a sample of people to express opinions about the quality of a good, we carry out an operation which is in suspense among measurement, preference and evaluation. It is necessary to know what the conceptual paradigms at heart of these three operations are.

The representational theory of measurement based on the properties of binary relations is the used instrument of investigation.

Given a set of alternatives A, a binary relation R on A is a subset of the Cartesian product $A \times A$. For example, if A is the set of all people in a certain country, then the set:

$$F = \{(a, b) : a \in A, b \in A \text{ and } a \text{ is brother of } b\}$$

defines a binary relation on A, which we may call "brother of".

The properties (reflexivity, symmetry, transitivity, etc.) of a generic relation R must be defined on a specific reference set. For example, the relation "brother of" is symmetric on the set of all males in a certain country, but it is not symmetric on the set of all people in that country. Formally, rather than a binary relation, we speak of a *relational system* (A, R), that is a relation applied to a set of objects. This concept, introduced by Tarski (1954), has been the natural vehicle for the subsequent development of the representational theory of measurement (Scott and Suppes, 1958; Krantz et al., 1971, 1989, 1990).

When a subject describes objects or events (physical or abstract), he/she considers one or more comparison relations. These may be tangible or intangible, uniformly interpretable or not. For example, the relations "more beautiful than", "worthier than", "more pleasant than" are relations intangible and arbitrarily interpretable by different subjects, whereas the relations "heavier than", "longer than", "warmer than" are not. These latter are observable relations, they do not enable a free interpretation by the subjects. Comparison conditions are strictly defined. There is a direct reference to the scales of the International System of measurement.

In this paper, the analysis of the differences among the basic operations takes place considering only a specific property of an object. "*The subjects of measurement are properties. Of course, properties exist only in connection with empirical objects. Usually, one object shows various properties. In measuring one property, we neglect all the other properties the object in question may have*" (Pfanzagl, 1968).

The analysis here referred to is exclusively limited to rankings. We only consider those cases in which individuals are able to establish a priority ranking, or a hierarchy among objects.

2. Two discriminant criteria: Empiricity and objectivity

"*Measurement is the assignment of numbers to properties of objects or events in the real world by means of an objective empirical operation, in such a way as to describe them. The modern form of measurement theory is representational: numbers assigned to objects/events must represent the perceived relations between the properties of those objects/events*" (Finkelstein and Leaning, 1984).

The definition of a measurement refers to two fundamental concepts: empiricity and objectivity. They can be used to discriminate the three basic operations.

The *empiricity* arises when a judgment of a ranking "*is the result of observations and not, for example, of a thought experiment. Further, the concept of the property measured must be based on empirically determinable relations and not, say, on convention*" (Finkelstein, 2003).

There is empiricity when the relation is observable, that is the property of the object proves to be, in a precise moment, in a well-defined state characterized without ambiguity. Empiricity means that there is "*an objective rule for classifying some aspects of observable objects as manifestations of the property*" (Finkelstein, 2003).

The *objectivity* concerns the kind of results that the judgment produces, "*within the limits of error independent of the observer*" (Finkelstein, 1982). "*Experiments may be repeated by different observers and each will get the same result*" (Sydenham et al., 1989). Full objectivity means independence by subjects. The result of the operation gives only information about the measured property.

The *measurement* requires both empiricity and objectivity. It is an operation of objective description: the results of n different measurements, in the same operating conditions, are univocal and independent by subjects. It is also an empirical operation: "*Measurement has something to do with assigning numbers that correspond to or represent or "preserve" certain observed relations*" (Roberts, 1979). We suppose there is no "error" and uncertainty in an ideal measurement process. The

environmental and other influential variables are not considered in the analysis.

The *preference* is neither empirical nor objective. Preferences are, by definition, subjective and conflicting. We are not able to know exogenously, in detail, the rules that each subject applies when assigning a ranking. Different subjects interpret the relation in different ways and can establish disagreeing orderings. In this case, the uncertainty concerns deeply the kind of relation applied by each subject.

The *evaluation* is somewhere between measurement and preference. It is not objective because evaluations are individual perceptions, performed without the use of a univocal instrument of measurement. Nevertheless, it is an operation that *wants* to be empirical: the meaning of intangible relations is circumscribed by means of an exogenous process of semantic definition from the outset. Subjects are called on to conform to this process. Operatively, there is uncertainty in the interpretation that subjects give to the provided dimension of observation.

The three operations can be classified as illustrated in Fig. 1.

We make explicit the meaning of the following terms:

- *Exogenous*: the expression of a description/ordering is subordinated to a coactive, explicit and declared constraint. An outside observer imposes rules (concerning the dimension of observation, interpretation of the scales, etc.) to which subjects conform from the outset.
- *Endogenous*: the expression of a description/ordering happens according to a latent, implicit, non-declared point of view. Each subject decides to adopt the rules he considers more convenient, without declaring them.

Which is the link between these three operations and the concept of indicator? We will show that measurements, evaluations and preferences can be interpreted as special types of indicators (Franceschini et al., 2006). Each single operation may be considered as a separate subsets of the world of indicators (see Fig. 2).

If there is no doubt about the meaning of indicator, a general theory which formalize the concept of indicator from a rigorous mathematical point of view is still lacking (Fortuin, 1988; Lohman et al., 2004).

In general, the definition of *indicator* is strictly related to the notion of *representation-target*. A representation-target is the operation aimed to make a *context*, or part of it, ''tangible'' in order to perform evaluations, make comparisons, formulate predictions, take decisions, etc. Examples of contexts are: a manufacturing process (if we are dealing with production management), or a distribution/supply chain (if dealing with logistics), or a market (if dealing with business management), or a sport competition (Lins et al., 2003), etc. Given a context, one or more different representation-targets may be defined.

An *indicator* (or *set of indicators*) is a tool which operationalizes the concept of representation-target, for a specific context (Franceschini et al., 2006).

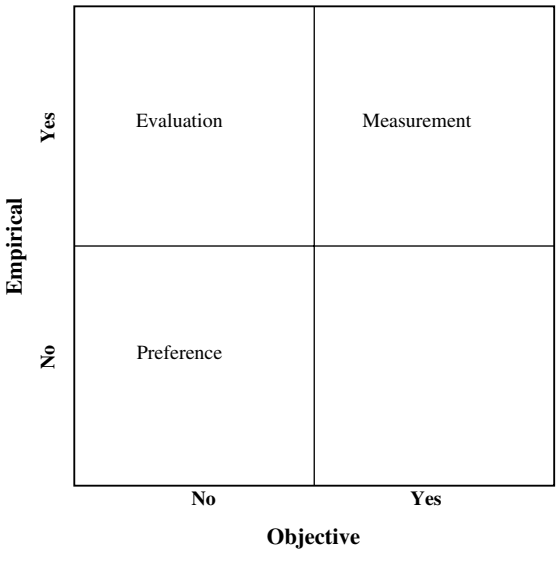

Fig. 1. Scheme of classification of the three operations: measurements, evaluations and preferences.

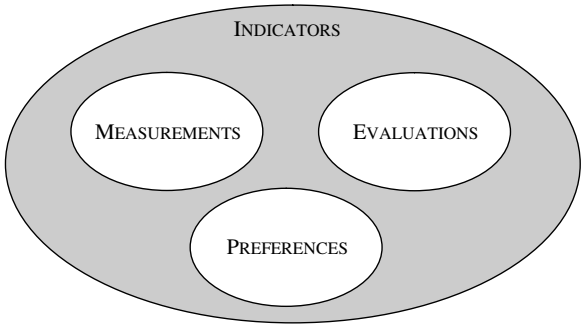

Fig. 2. Measurements, evaluations and preferences interpreted as a subset of indicators.

For example, if the context is the "logistic process" of a company and the representation-target is "the classification of suppliers", the "delivery time" and the "lead time" can be two of the possible related indicators. In general, it can be shown that, given a representation-target, a set of associated indicators is not algorithmically generable (Roy and Bouyssou, 1993).

3. The representational definition of measurement

The assumption that the relations are observable is at heart of the representational point of view. In a measurement, subjects are called on to "judge" an observable relation, on which there are no doubts about meaning and interpretation. Relations like "longer than", "heavier than", "warmer than", etc. are some possible examples.

The representational theory of measurement is based on four fundamental pillars (Finkelstein, 2003):

(1) *An empirical relational system.* Consider some quality (for example the length of an object) and let q_i represent an individual manifestation of the quality Q, so that we can define a set of all possible manifestations as $Q = \{q_1 \dots\}$. Let there be a family of empirical relations R_i on Q, $R = \{R_1, \dots, R_n\}$. Then the quality can be represented by an empirical relational system $\mathcal{L} = \langle Q; R\rangle$.

(2) *A numerical relational system.* Let N represent a class of numbers $N = \{n_1 \dots\}$. Let there be a family of relations $P = \{P_1, \dots P_n\}$ defined on N. Then $\mathcal{N} = \langle N ; P\rangle$ represents a numerical relational system.

(3) *A representation condition.* Measurement is defined as an objective empirical operation such that $\mathcal{L} = \langle Q; R\rangle$ is mapped homomorphically into (onto) $\mathcal{N} = \langle N ; P\rangle$ by M and F. Specifically, M is the function mapping Q to N, so that $n_h = M(q_h)$ ($M: Q \to N$). F is the function mapping one-to-one the relations of R on P ($F: R \to P$).
The above homomorphism is the representation condition.
Firstly it implies that if q_h is related to q_m by an empirical relation R_k, that is $R_k(q_n, q_m)$, P_k is the numerical relation corresponding to R_k, $n_h = M(q_h)$ is the image of q_h in N under M then $R_k(q_h, q_m)$ implies and is implied by $P_k(n_h, n_m)$.
Measurement is a homomorphism – not an isomorphism – because M is not a one-to-one function. It maps separate but indistinguishable property manifestations into the same number. $S = \langle \mathcal{L}, \mathcal{N}, M, F\rangle$ constitutes a scale of measurement for $\mathcal{L}$. The definition of a representational measurement is illustrated in Fig. 3.
"*In measurement we start with an observed or empirical relational system and we seek a mapping to a numerical relational system which "preserves" all the relations and operations observed in the empirical one*" (Roberts, 1979).
"*Whatever inferences can be made in the numerical relational system apply to the empirical one*" (Dawes and Smith, 1985).

(4) *Uniqueness condition.* The representation condition may be valid for more than one mapping function M. There are admissible transformations from one scale to another scale without invalidating the representation condition. The uniqueness condition defines the class of transformations for which the representation condition is valid. For ordinal scales, all monotone increasing functions are admissible transformations (Finkelstein, 2003).

The hardness measurement of minerals is a typical example of ordinal measurement according to this definition. The Mohs scale orders minerals from diamond to talc, on the basis of which scratches which. The ability to scratch (i.e., to etch, to cut in surface) is the empirical relation and the ordering is the formal binary relation (Finkelstein, 1982).

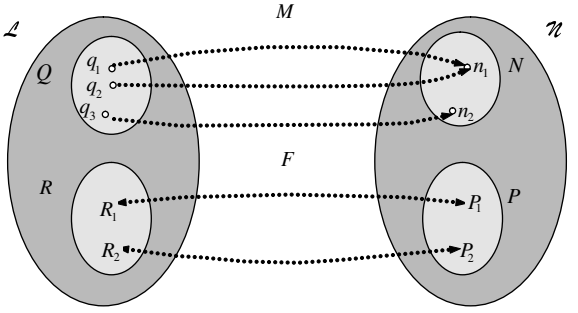

Fig. 3. Homomorphism of $\mathcal{L}$ onto $\mathcal{N}$. Two elements in Q may be mapped into the same number in N.

3.1. Some clarifications about the concept of measurement

The *homomorphism* is a faithful representation of empirical relations by symbolic relations. The condition of homomorphism, namely of an assignment of symbols (in general numbers) to objects\events according to the degree of presence of a certain property, is considered by the followers of the representational theory as a necessary and sufficient condition to define a measurement.

Some authors (Dawes and Smith, 1985) assert that not all possible rules of assignment yield right measurements. The assignment of numbers is a representational measurement only if three requirements are satisfied. "*The assignment of numbers must*:"

(a) *be orderly*;
(b) *represent meaningful attributes*;
(c) *yield meaningful predictions*.

According to the authors, the presence of these three conditions defines the "*automatic consistency check*", which realizes the difference between a representational measurement and non-representational one: "*when mineral (a) scratches mineral (b), then (a) is represented above (b) in the order*" (Dawes and Smith, 1985).

If the consistency check fails, we cannot speak about representational measurements.

Measurements such as, for example, hardness, mass, length, etc. present a consistency check. This check is performed by an appropriate measurement system realizing the homomorphism from the empirical relational system to the numerical one.

The measurement system guarantees the two fundamental components of a measurement: "*assignment and empirical determination*" (Mari, 1997). It circumscribes without ambiguity the empirical relations (harder than, heavier than, longer than, etc.), and performs the assignment of numbers to the objects according to the rule of the corresponding homomorphism.

The presence of a conventional, non-ambiguous, empirical reference is the reason at the base of the objectivity of measurement results.

4. Evaluations

In general, the qualities of an object/event can be classified as physical or non-physical, observable or non-directly observable. The utility, the performance of a process, the customer satisfaction, the attitude, etc. are examples of non-physical and non-directly observable magnitudes. Attempts to measure these latter cause many problems.

The first problem is "*the difficulty of establishing an adequate objective concept, or theoretical construct, of these qualities based on empirical operations*" (Finkelstein, 1982).

The description of these attributes or latent constructs is not objective. It may origin a free interpretation of the meaning by the subjects. In these situations, we cannot speak about real measurement, but just about evaluations or attributions of values to individual judgments.

Descriptions about non-tangible qualities are possible only by means of subjective judgments expressed on adequate scales (see Fig. 4).

Fig. 4 illustrates the concept of evaluation of a non-physical magnitude. The subject ideally compares the object (first scale pan) with the reference terms on the scale (second scale pan). The evaluation consists in identifying the judgment on the scale which balances the two scale pans.

If there is no definition of exogenous rules to which evaluators conform from the outset, the first fundamental component of a measurement (empiricity) runs short. Consider, for example, the aesthetical beauty of an object: in this case "*there is not an objective rule for classifying some aspect of observable objects as manifestations of the beauty. Similarly, there are no objective empirical relations such as indistinguishability or precedence, in respect of beauty. The basis for the measurement of beauty is thus absent from the outset*" (Finkelstein, 1982).

The aim of an evaluation is that of "building" and "imposing" to evaluators, in some way, this empiricity component by defining rules to which subjects should conform from the outset.

This way, Pawson (1997) asserts that: "*First, evaluation deals with the real, that is we evaluate things and empirical relations about things. Secondly, evaluation should follow a realist methodology. Thirdly, evaluation, perhaps above all, needs to be realistic*".

4.1. The evaluation of non-tangible qualities: The exogenous imposition of rules for the subjects

Let A be a set of objects and R the relation "heavier than". That is, for any pair of object x, y in A we define:

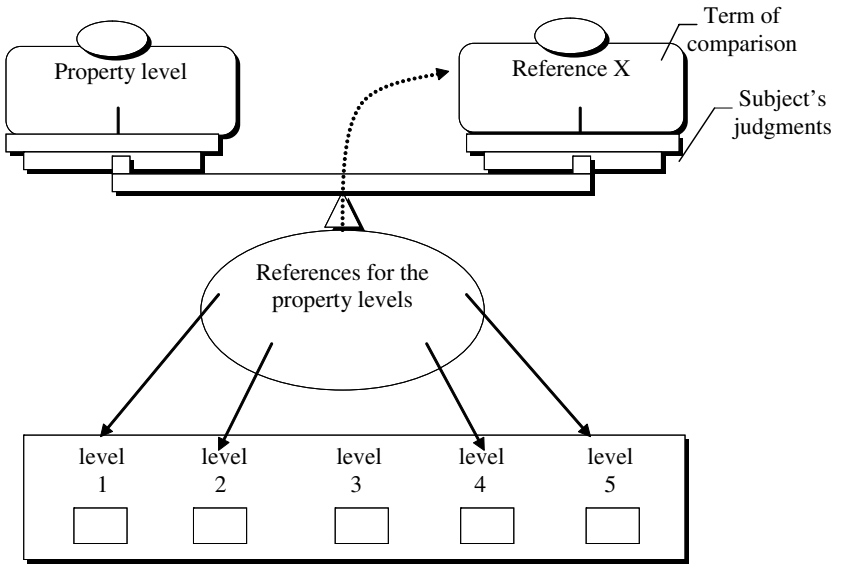

Fig. 4. Scheme of evaluation of a non-physical magnitude (Franceschini, 2001).

xRy *x* is heavier than *y*

"*Note that R can be defined either by a balance or by psychophysical experiments using an observer to compare the weights. The two procedures yield similar empirical relational systems with the same object set. The interpretation of the relation "heavier than", however, is physical in the former system and psychological in the latter*" (Coombs et al., 1970).

In the second case, we should speak of evaluations.

The evaluation is typically a normative process. It is based on exogenous rules driving subjects in the attribution of values to intangible and interpretable qualities. Essentially, there are two kinds of rules:

- clear and precise definition of non-physical attributes (abstract constructs);
- definition of operating evaluation scales.

The first fundamental step of an evaluation process consists in the definition of a *reference axis*. The evaluation process wants to make the initial latent construct observable and less interpretable, giving to it "*empirical substance*", by specific empirical rules. The second phase consists in providing suitable evaluation scales.

An evaluation requires that there is "uniformity" for subjects in accepting the rules provided. This uniformity does not exist in preference judgments, where everyone is free to choose the interpretation with which the judgment takes place.

4.2. Evaluation: A subjective homomorphism

We said that evaluation is a partially empirical and subjective operation. We are going to justify this position by means of the representational theory. Psychological tests or questionnaires for the evaluation of product or service quality are examples of evaluation processes.

Paraphrasing measurement definition, the evaluation becomes the assignment of numbers, or labels to properties or events of the real world by means of an empirical subjective operation, in such a way as to describe them.

In the evaluation context, the mapped relation *R* is not empirical. The relation becomes empirical by means of a set of semantic (what the construct means) and operating rules (evaluation scale). We exogenously impose the dimension or relation that subjects have to observe.

What is perceived as "having the property more than", in the empirical world, has an immediate translation in the ranking performed by the subject. In ordinal evaluation, there is a homomorphism from the empirical world onto the symbolic (numeric) one, that is to say, there is a faithful representation of empirical objects and their relations in the numerical world. The representational form is, therefore, maintained. From this point of view, the homomorphism is not able to formalize the difference between the operation of evaluation and measurement (Mari, 2003).

The question is that we deal with a subjective operation. The ranking performed by a subject will not coincide with that of another subject. It is not univocal. Different subjects can observe different degrees of the property in the same object.

4.3. Problems and questions still open

Some fundamental problems arise from ordinal evaluations.

A. Dimension of representation "compatible" with the way of thinking of subjects.
   The priority aim of an evaluation consists in yielding empirical predictions, for example, about the purchasing intentions of customers, the undertaking of an investment, etc. Therefore, it is necessary that the provided dimension of representation is compatible with the way of thinking of the subjects, otherwise the evaluations will be neither significant nor useful.
   Setting up a good evaluation scale means having to know the dimension of representation which is most important for the subjects. "*Clearly, not all rating scales are compatible with intuitive thought, nor does compatibility imply that rating scales are isomorphic with such thought*" (Dawes and Smith, 1985).
B. The design of a scale reflecting the real capacity of discrimination of subjects.
   This problem mostly arises from rating scales with enumerated categories, where the subject expresses himself on a verbal category and not on a linear continuum. In fact, problems of interpretation of the categories meaning can subsist. A verbal label, "very satisfying", may not have the same meaning for both a very demanding subject and one who is easy to please. This poses a real problem to codify information, being usually unknown the interpretation of the scale adopted by each subject (Franceschini, 2001).
   The response category can be interpreted by subjects as too wide or too narrow. When the category is too wide, the subject is forced to classify as equal objects that he perceives as slightly different. In this case, there are no representational measurements, because a faithful representation of observed relations in numerical relations does not occur: "*it is essential that the relations among the objects of the world be properly reflected by the relations among the numbers assigned to them*" (Coombs et al., 1970).
   The opposite problem can also appear. The response categories could be too detailed and the subject is confused in giving an answer. He may find himself in more than one response category.
   The adequate definition of the width and of the meaning of each scale category brings evaluations to the stage of homomorphisms from an empirical relational system to a numerical one, that is to say, representational evaluations. This result remains an actionable target only after "*numerous interactions with evaluator subjects*" (Finkelstein, 1982).
C. The aggregation of evaluations expressed by many subjects.
   This problem is connected with any subjective operation. The aggregation of individual rankings in a social global ranking is object of study of many disciplines: social and behavioural sciences, operational research and economics (Fishburn, 1973; Keeney and Raiffa, 1976; Roy, 1996).

5. Preference

Preference is the act by which, in presence of two or more possible objects, one of them can be chosen over the other, because it is considered more pleasant, more convenient, more conform to own tastes, interests, ideals, etc.

"*Preference is necessarily relative to a subject. A preference is always somebody's preference. A preference, moreover, is relative not only to a subject but also to a certain moment or occasion or situation in the life of a subjects. Not only may have different people with different preferences, but one and the same man may revise his preferences in the course of his life...the concept of preference is related to the notion of betterness*" (Wright, 1963).

"*Preferences, to a greater or lesser extent, govern decisions...into our axiomatic system an individual's preference relation on a set of alternatives enters as a primitive or a basic notion. This means that we shall not attempt to define preferences in terms of other concepts...preferences between decision alternatives might be characterized in terms of several factors relating to the alternatives*" (Fishburn, 1970).

When a subject says that he/she prefers alternative a to b, he/she makes a relation between a and

b which seems perfectly mouldable with the mathematic notion of a binary relation.

Suppose A is a collection of alternatives among which you are choosing, and suppose

$$P = \{(a,b) \in A \times A : \text{ you (strictly) prefer } a \text{ to } b\}$$

Then P is called (strict) preference relation.

If A is a set of alternatives, aPb holds if and only if you prefer (strictly) a to b, it is possible to assign a real number $u(a)$ to each $a \in A$, such that for all $a, b \in A$

$$aPb \Longleftrightarrow u(a) \succ u(b)$$

The function u is often called ordinal utility function. This assignment allows the relation "preferred to" of the single subject to be observed.

Nevertheless, as Roberts asserts, "*often, "preferred to" doesn't define a relation*" (Roberts, 1979, pp. 272–273). With these remarks, Roberts wants to underline the absolute peculiarity of this relation that enjoys neither the property of consistency nor that of transitivity.

Consider the following example: a subject is called on to vote among three candidates A, B and C. If the subject prefers alternative A to alternative B and B to C, he/she will not necessarily prefer alternative A to C, as the transitivity property requires. It can occur that C is preferred to A. In the background concerning the problems connected with the aggregation of preferences, the situation just described is noted as Condorcet Paradox (Condorcet, 1785).

The subject assigning preferences could have an implicit model of preferences such that it cannot be mapped on any numerical structure (it may contain non-transitivity chains).

The lack of transitivity often arises for intangible relations like "preferred to", "more beautiful than", "more elegant than", etc. relations arbitrarily interpretable, because not directly observable.

5.1. The impossibility of the representational form for preferences

What differences emerge comparing the definition of preference with that of measurement?

In general, we are faced with a preference assignment every time the subject is called on to perform a ranking among things without a "measurement system" or a set of predefined rules, such as in the evaluation process. The preference ranking among alternatives is the result of an endogenous activity of a subject who chooses, in an arbitrary way, how to represent the relation.

Preference becomes "*an arbitrary measurement conceived as a decision-making activity*" (Sartori, 1991).

Examples of arbitrarily interpretable relations, if no rules are predefined, are: "worthier than", "better than", "more beautiful than", etc.

The assignment of preferences is neither empirical, nor objective. It does not deal with an empirical operation because the subject chooses arbitrarily the relation considered remarkable for the ranking. The above choice is completely endogenous, different from subject to subject. We are not able to understand exogenously what is the dimension of representation selected and followed by the subject and the interpretation he/she has given to it.

There is not a transfer of observable relations into numerical relations. We cannot speak of a homomorphism as defined by the representational point of view. There is no mapping onto a symbolic relational system "*preserving both relations* (*and operations*) *observed in the empirical relational system*" (Roberts, 1979).

Paraphrasing a Stevens' expression, preference can be defined as "*a measurement according to any rule*" (Stevens, 1951). Subjects may have chosen to observe one of the infinite possible relations on the objects in order to perform a preference ranking.

It is obvious that this operation is completely subjective: "*the measurement value is not so much a property of the thing measured as something which expresses an appreciation of the measurer towards the thing itself. What counts, does not count because it counts in itself, but because it is judged to count by someone*" (Mari, 1997).

When subjects report their own opinions about not adequately detailed constructs, an attribution of values to individual preference judgments takes place.

Constructs like the utility of a service, the aesthetic of a product, the guidablity of a vehicle, must be specified and detailed to transform preferences into evaluations. Otherwise, each subject will interpret the construct as he/she considers more convenient. The reason for this is a "*semantic ambiguity*" in the constructs (Dawes and Smith, 1985).

The presence of this ambiguity is the main difference between preference and evaluation. In an evaluation process we will try to circumscribe this semantic ambiguity.

The consequence is that a preference has not the empiricity and objectivity requirements of a measurement. However, the representational theory considers that representational measurement of preference are possible. It is important to concisely define this position. When subjects assign a ranking to the elements of a certain set, they make their own preferences explicit and consequently their own relation "preferred to" on that set is made observable. According to the representational theory, this assignment is a real representational measurement. It is viewed as a homomorphic representation of the relation "preferred to" from the empirical world of the objects into the numerical or symbolic world (even if the relation is not explicit in the empirical domain).

Roberts (1979) presents the case of preferences among classical music composers. The author points out that it is possible to speak of representational measurements of individual preferences when these satisfy the axioms of Cantor's theorem. Given a set of objects and defined a relation of preference P on it, the relational system (A, P) has to satisfy the following conditions:

asymmetry: if $aPb \Rightarrow \sim bPa$,

$\forall a, b \in A$ (natural property of preferences)

negative transitivity: if $\sim aRb \;\&\; \sim bRc \Rightarrow \sim aRc$,

$\forall a, b, c \in A$

Axioms of Cantor's theorem are necessary and sufficient conditions to have ordinal representational measurements.

However, enacting Roberts' position, "*some information can be obtained on the evaluating subject, about their way of seeing things, but surely not on the elements of the set, i.e. the empirical world*" (Mari, 2003). Roberts does not dwell on the analysis of the meaning of the relation but he simply analyses and interprets the results of the representation. The author seems to neglect that the relation "preferred to" is not empirical because it is interpretable and therefore observable in an arbitrary way.

The problem is that we do not know what is the meaning of the relation "preferred to", in terms of empirical relations. Therefore, an empirical relational system for the preference cannot be identified.

6. The representational approach for indicators

Referring again to the representational theory of measurement, an indicator I can be considered as a "map" from an empirical system (the "real world") into a representational system (usually, a numerical system). However, the mapping between the empirical and symbol relations, unlike measurement, is not required:

$$I : q_j \in Q \rightarrow I(q_j) \in E$$

where E is the set of representation elements in the representational system $\mathscr{E} = \langle E; T \rangle$ (T is a set of relations on E), Q is a set of manifestations of the empirical system $\mathscr{L}$, q_j is a manifestation of Q, and $I(q_j)$ is the representation of q_j onto the representational system $\mathscr{E}$.

In general, if the representational system is a numerical system, an indicator is defined as a real values function on the set of empirical system manifestations.

Reminding that an empirical system is said relational if there exists a set of empirical relations among empirical manifestations, the identification of relations is conditioned both by the context and the way we are able to interpret it. The context is filtered by how we perceive and model it. For indicators, the mapping of the empirical system into a representational one may introduce new relations or modify the existing ones.

In accordance with this approach, three elements have to be considered: the *model* (i.e., the conceptualization of the real world), the *representation-target*, and the *rules* to determine the related set of indicators together with their associated relations. The representation does hold if these three elements are delineated (see Fig. 5).

For example, if we want to identify the winner of a competitive tender:

- the model is given by "how we evaluate competitors' credentials",
- the representation-target is "finding a winner",
- the indicators and the associated relations originate from the rules we establish for obtaining a final score.

On the basis of this representational approach, measurements may be interpreted as a subset of indicators. The basic difference between measurements and indicators is the way the relations of the empirical systems are mapped. Indicators do not require an isomorphism between empirical and representational relations. This means that, while a measurement is certainly an indicator, the vice versa is not true.

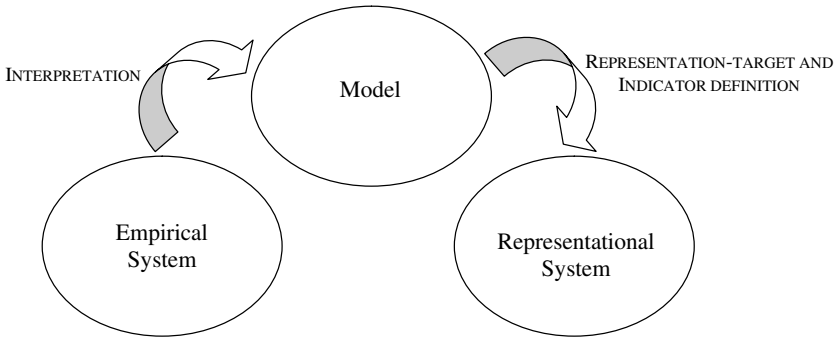

Fig. 5. Scheme of the representational approach of an empirical system through the concepts of model, representation-target, and indicators.

According to this general definition of indicator, given a representation-target, the related indicator (or set of indicators) is not univocally defined. In general, due to the fact that indicators do not require an isomorphism between empirical and representational relations, the uniqueness condition is not always verified.

Such as measurements, also for evaluations the isomorphism between the empirical system onto the representational system is defined, in some way. Hence, also evaluations can be considered as a subset of indicators.

Interpreting indicators by a map from the empirical system to a representational one, also preferences can be considered as special types of indicators.

Measurements, evaluations and preferences are three separate subset of indicators (see Fig. 2).

In those cases in which the mapping of relations is not defined, two situations can occur:

- The relations in the representational relational system are subjectively imposed by whom performs the mapping: this is the case of preferences.
- The relations are conventionally defined in order to be objectively utilized by anybody who performs the mapping. This is the case, for example, of the practice of "triage" in the hospitals or the use of air quality indexes (Franceschini et al., 2005). The basic properties of the empirical system: "equal" and "not equal", are converted on the "greater than" relation in the representational system. In this situation we cannot speak neither of measurements, or evaluations, or preferences.

7. The dictation in the light of the representational theory

Referring to Fig. 1, a quadrant still remains empty. This is the occurrence of "Yes" for "Objective" and "Not" for "Empirical". We can define this situation as a new operation called *dictation*.

We have a dictation when the mapping between the empirical system and the representational one is defined in order to give a predefined result, independently by the occurrence of manifestations in the empirical space. This operation reflects the intervention of a *dictator* who "dictates" a priori the result.

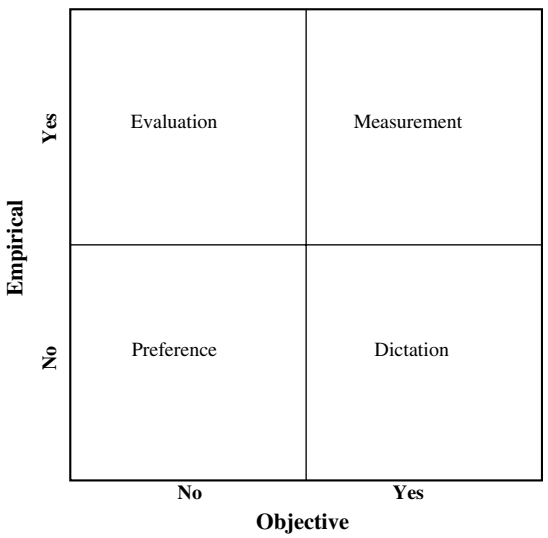

Fig. 6. Scheme of classification of measurements, evaluations, preferences and dictations.

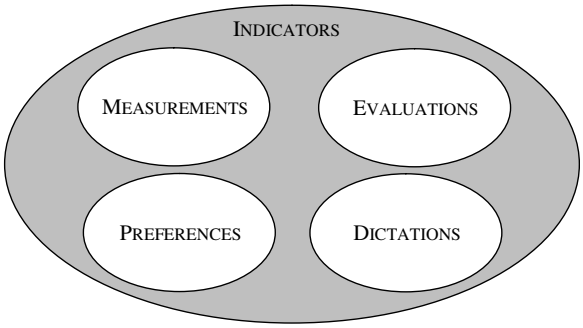

Fig. 7. Measurements, evaluations, preferences and dictations can be considered as a subset of indicators.

A dictation is objective because it produces always the same result, independently from the subject who performs the mapping. It is not empirical because there is not a set of empiric rules to produce that result (see Fig. 6). This is the case, for example, of fixed competitions, in which the winner is a priori defined and the selection rules are arbitrarily proposed in order to obtain the imposed result.

Dictations can be interpreted as a particular case of indicators. They also represent a map from an empirical system onto a representational one (see Fig. 7).

8. Conclusions

In general, objects and events are described and classified on the basis of specific indicators. Measurements, evaluations, preferences and dictations are the four basic operations which can be used for this purpose. The difference between them lies on the way of treating relations among objects/events.

Measurements set the empirical relations that must be represented without ambiguity by means of a measurement system which realizes the ''internal consistency check'' and makes feasible the fundamental requirement of objectivity.

On the other hand, it is the aim of an evaluation process to provide all possible ''tools'' in order to reduce the problems of semantic interpretation of the items and of the scale categories. In this way, individual judgments are treated as representational evaluations.

In the case of preference, relations are thought by subjects and transferred directly to the result of the representation under an endogenous decision-making activity, different from subject to subject. The inability to exogenously know the way in which things have been interpreted by subjects leads to the total absence of empiricity and objectivity of the operation.

The notion of homomorphism is effective to mark the difference between preference and measurement, but it does not define a clear border between the operation of measurement and evaluation. The source of the difference between the two operations is that evaluations give space to the possibility of choosing among different dimensions of representation from the outset.

Representational theory allows giving a precise and effective definition of empiricity in the three operations, but not that of objectivity. Some authors have emphasized the inability to incorporate the fundamental requirement of objectivity of a measurement in the formalization of the approach as the limit of the representational point of view. This way, someone claims the necessity of an operational-representational approach for the sake of an exhaustive definition of measurement, with the introduction of a measurement system (Mari, 2003).

Referring to the representational theory, measurements, evaluations and preferences can be interpreted as separate subsets of indicators, which represent the widest family of tools for describing objects/events.

According to the proposed scheme of classification, a further operation, the dictation, can be defined. Dictation is objective but not empirical: it produces a predefined (by a dictator) result, independently by the subject who performs the mapping.

Particular attention should be posed to the problem of aggregation of individual evaluations or preferences into ''social'' or group results. Problems of aggregation do not appear in the field of measurements because of their objective nature. On the contrary, an abundant literature about the aggregation of preferences is available. In this framework, renowned for its relevance, remains the Arrow Impossibility Theorem (1963) which represents the milestone of Social Choice Theory. By means of the theory of relations, Arrow proved the non-existence of aggregation mechanisms of individual preferences into a social preference such that four fundamental axioms are satisfied: Unrestrained Domain of Preferences, Independence of irrelevant alternatives, Pareto weakness, and Non-dictatorship (Arrow, 1963).

With the aim of tracing a possible path for a future research, the traditional problem of aggregation of individual preferences can be reformulated in the problem of aggregation of individual evaluations in order to see if the imposition of exogenous bonds relaxes Arrow's conditions. In substance, it is natural wondering if Arrow's Theorem is still valid when the input is changed from preferences into "canalized" preferences as evaluations.

Arrow's theorem can be adopted as a second possible instrument of investigation to further highlight differences between preferences and evaluations.

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