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Special Issue

The Condition of Uniqueness in Manufacturing Process Representation by Performance/Quality Indicators

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One of the most critical aspects in operations management is making firm goals representable. This is usually done by translating the organization results and objectives in to 'performance measures'. The scientific literature shows many applications in different fields such as quality, production, logistics, marketing, etc. Nevertheless, a general theory formalizing the basic and application concepts is still lacking. This paper represents a first attempt to provide a mathematical structure to the concept of an indicator. A set of basic definitions is introduced with the aim of giving a rigorous explanation of the concept of a 'performance indicator'. Particular attention is dedicated to the condition of 'uniqueness'. When dealing, for example, with performance evaluations of a given manufacturing plant, a practical way is to define some indicators which make tangible the different aspects of the system at hand. In this case, indicators such as throughput, defectiveness, output variability, efficiency, etc. are commonly employed. However, going on into the problem, many questions arise: 'How many indicators shall we use?', 'Is there an optimal set?', 'Is this set unique?', 'If not, what is the best one (if it exists)?', 'Can all these indicators be aggregated in a unique one?', 'Are indicators the same as measurements?', and so on. The aim of this paper is to give an answer to all these questions. A general theory which specifically faces this topic is presented. All the introduced concepts are explained and discussed by the use of practical examples. Copyright © 2006 John Wiley & Sons, Ltd.

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1. INTRODUCTION

Since the early 1980s, the concept of 'metric' in operations management has been widely investigated[1–3]. In the current scientific literature, terms such as 'metric', 'performance measure' and 'performance indicator' are usually used as synonyms.

Metrics are utilized for a variety of purposes. Several authors have suggested many performance measures for the analysis of manufacturing systems. The most used are throughput, product defectiveness, product quality, material flow smoothness, due date attainment, output variability and flexibility[4]. Special emphasis is also given to the so-called 'design metrics', i.e. those factors that are inherent in product design and affect one or more product lifecycle stages[5]. As is properly indicated by Hauser and Katz[6], metrics such as market share, sales increase, margins and customer satisfaction surveys help firms to individuate their market position and to plan for the future.

Historically, logistics and manufacturing functions are two of the first factory functions to be concerned with the use of performance indicators. An interesting survey about 'logistic metrics' is presented by Caplice and Sheffi[7]. Basing their idea on the conviction that a performance measurement system that is strategically well designed can be defective on the individual metric level, they state that there is no need for the development of new performance metrics (in logistics there is a great abundance of adequate metrics), but there is a lack of methods to evaluate them. Hence, they suggest a set of evaluation criteria for individual logistics performance metrics as well as a taxonomy of the existing ones.

In management by the governments of financial projects, the concept of 'performance measurement' is far from new, as states Perrin[8] in his review of performance measurement theory and practice. Performance measures have been widely promoted by governments for more than 20 years for the purpose of increasing management's focus on achieving results[9]. This is further demonstrated by the publication in 2001 of '*The Performance-Based Management Handbook*' by Oak Ridge Institute for Science and Education (ORISE), U.S. Department of Energy[10].

The concept of a performance measure/indicator is also not new in Quality Management[11]. Recent years have been characterized by a widespread interest in this area. This phenomenon is mostly related to the new edition of ISO 9000 standards, which emphasize the concepts of 'Quality Measurement' and 'Customer Satisfaction Measurement'[12–14].

Metrics are also used in many other sectors very different from the business domain. Indicators are employed for determining the final score of athletes (or teams) in sport competitions; for example, think of a decathlon score, artistic gymnastics or Formula 1 car racing[15,16].

Currently, most knowledge about metrics is derived from the managerial literature[17–21]. Recent studies have focused on the development, implementation, management, use and effects of metrics in the operations management area or in the supply chain[22–24].

Many authors have tried to address their studies towards the definition of basic rules to assist practitioners in metrics definition[6–8,10,25]. Hauser and Katz[6], for example, in their article summarize seven 'pitfalls' in the use of metrics which can cause them to be counter-productive and fail, as well as outlining a seven-step system to design effective 'lean' metrics.

Some authors assert that every metric, whether it is used explicitly to influence behaviour, to evaluate future strategies or simply to take stocks, will affect actions and decisions[6]. This is empirically demonstrated by a series of 'on field' studies[5,26]. The concept is quite intuitive. If in a firm some particular aspects are observed, let us say, for example, absenteeism, telephone costs and employee productivity, then managers (and the whole organization) will pay more attention to these aspects, rather than to others. This mechanism follows a rapid escalation, which in a short period drives the firm to 'become what it measures'[6]. Metrics gain control of the enterprise, with the risk that, if they lead to counter-productive decisions and actions, the result can be deleterious.

The current pressing interest around performance measurement, and the 'confusion' still existing are highlighted by the article of Melnyk *et al.*[25]. With the aim of giving some initial theoretical grounding for the metrics research topic, these authors provide a general definition of metric: 'A metric is a variable measure, stated in either quantitative or qualitative terms and defined with respect to a reference point. Ideally, metrics are

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consistent with how the operation delivers value to its customer as stated in meaningful terms'. The authors also give three basic functions which metrics must provide: Control, Communication and Improvement. Metrics can be classified on the basis of their 'focus' (e.g., quality, manufacturing, operational and financial focus) and their 'tense' (i.e. how the metrics are intended to be used: e.g., for outcome analysis, prediction, comparison among competitors). Furthermore, the authors give three levels of metrics: the 'individual metrics', the 'metrics set' and the 'overall performance measurement system'. Starting from the highest level, the 'performance measurement system' aims to identify a synthetic structure of the overall metrics that can be utilized in an organization.

Many authors have proposed different approaches for developing such an integrative system. The mostly cited are the 'balance scorecard'[27–29] and the 'strategic profit impact model'[30]. An 'individual metric' is defined as the basic block of a three-level structure which has as vertex the 'performance measurement system'. A collection of 'individual metrics' constitutes a 'metrics set'.

If there is no doubt about the importance of metrics, a general theory which faces the metrics problem from a rigorous mathematical point of view is still lacking. Some studies have tried to construct a framework within a theoretical context. The most cited are the 'agency theory' approach[31], which is based on the idea that the metric motivates and directs the actions in the dyadic relationship between 'principal' and 'agent', and the 'dependency theory'[32].

The theme of communications among the 'actors' of an organization is considered by Galbraith[33] in an 'information processing perspective'. A richer 'metric set' creates the basis for richer communication, but this can generate limits to the ability to process larger set of metrics; hence, increasing numbers of metrics could lead to greater conflict in the implied properties. This is also related to another important aspect evidenced by Melnyk *et al.*[25], which is that the trade-offs between metrics set richness and complexity. From a practical point of view, it reflects questions regarding the optimal size of a metrics set.

The aim of the present paper is to suggest some basic ideas for a general theory of metrics. In Section 2 we provide a definition of the concept of *indicator*. In Section 3 the condition of *uniqueness* is introduced as well as other basic properties. Practical effects of these properties are shown on a series of application examples.

2. THE CONCEPT OF 'PERFORMANCE INDICATOR'

When dealing with operations management, one of the most critical aspects is to make an organization's purposes and goals representable. This can be done by translating the organization results and objectives into 'performance measures' or, more properly, 'performance indicators'.

Consider, for example, a manufacturing plant for the production of a given component, say a specific model of automotive exhaust systems[34]. The firm's management could be interested in observing the plant performances in order to verify the production state, formulate predictions for the future or make comparisons with other similar plants. A practical way to do this is to define some 'indicators' which make tangible the basic aspect of the production system at hand.

In this case indicators such as throughput, process defectiveness, output variability, efficiency, etc. can be employed. However, as we enter into the problem, many questions arise: 'How many indicators shall we use?', 'Is there an optimal set?', 'Is this set unique?', 'If not, what is the best one (if it exists)?', 'Is it possible to aggregate all these indicators in an unique one?', 'Is there a difference between indicators and measurements?' and so on.

An answer to these questions can be obtained only if a general theory, based on the definition of *indicator* and its properties, is properly delineated.

2.1. *Definition of indicator*

The definition of *indicator* is strictly related to the notion of *representation-target*. A representation-target is the operation aimed to make a *context*, or part of it, 'tangible' in order to perform evaluations, make comparisons, formulate predictions, take decisions, etc. Examples of contexts are a manufacturing process (if we are dealing with production management), a distribution/supply chain (if dealing with logistics), a market (if dealing with

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business management) or a result of a competition (if dealing with sports). Given a context P, one or more different representation-targets Q_P can be defined.

A *set of indicators* S_Q is a tool which operationalizes the concept of representation-target, referring to a given context:

$$\mathrm{S}_Q = \{I_i\}_Q, \quad i = 1, 2, \ldots, n, \ n \in \mathbb{N} \tag{2.1}$$

For example, if the context is the 'logistic process' of a company and the representation-target is 'the classification of suppliers', the 'delivery time' and the 'lead time' can be two of the possible related indicators.

In general, it can be shown that, given a representation-target, a set of associated indicators is not algorithmically generable[35].

2.2. *The representational approach for indicators*

To better understand the definition of indicator, the concept of *measurement* must be recalled. According to the Representation Theory of Measurement, a measurement is a 'map' from an *empirical relational system* (the 'real world') into a *representational relational system* (usually, a *numerical system*)[36,37].

Given a set of all possible manifestations of a specific property of a well-defined representation context,

$$\mathrm{A} = \{a_1, \ldots, a_i, \ldots\} \tag{2.2}$$

and a family of empirical relations among the elements of A,

$$\mathrm{R} = \{R_1, \ldots, R_m\} \tag{2.3}$$

then the following empirical relational system can be defined:

$$\mathfrak{A} = \langle \mathrm{A}, \mathrm{R} \rangle \tag{2.4}$$

Analogously, if Z is a set of symbols

$$\mathrm{Z} = \{z_1, \ldots, z_i, \ldots\} \tag{2.5}$$

and P is a family of relations on Z

$$\mathrm{P} = \{P_1, \ldots, P_m\} \tag{2.6}$$

then

$$\mathfrak{Z} = \langle \mathrm{Z}, \mathrm{P} \rangle \tag{2.7}$$

is a symbol relational system.

In general, according to the so-called 'Symbolic Representation Theory', a measurement is an objective empirical function which maps homomorphically the empirical relational system $\mathfrak{A} = \langle \mathrm{A}, \mathrm{R} \rangle$ into the symbol relational system $\mathfrak{Z} = \langle \mathrm{Z}, \mathrm{P} \rangle$ (see Figure 1)[37].

Two mappings are defined:

$$\mathrm{M} : \mathrm{A} \to \mathrm{Z} \quad \text{(homomorphism)} \tag{2.8}$$

and

$$\mathrm{F} : \mathrm{R} \to \mathrm{P} \quad \text{(isomorphism)} \tag{2.9}$$

so that $M(a) = z$ is the image in Z of a generic element a, and $F(R) = P$ is the image in P of a generic relation R.

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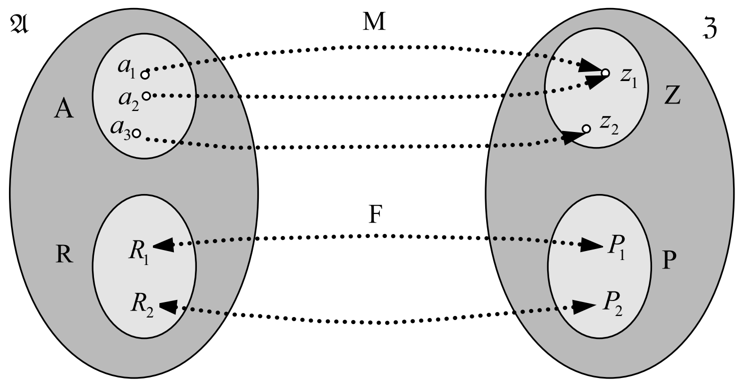

Figure 1. Schematic representation of the concept of measurement

M is a homomorphism. The mapping is not one-to-one. Separate but indistinguishable manifestations are mapped into the same symbol.

The 'representation code' for $\mathfrak{U}$ is defined as follows:

$$C = \langle \mathfrak{U}, \mathfrak{Z}, \mathrm{M}, \mathrm{F} \rangle \tag{2.10}$$

The inverse of C is called the 'interpretation code'. z is the symbol of a.

In most applications the mapping is performed into a numerical relational system, defined as

$$\mathfrak{N} = \langle \mathrm{N}, \mathrm{P} \rangle \tag{2.11}$$

where N is a class of numbers

$$\mathrm{N} = \{n_1, \ldots, n_i, \ldots\}, \quad n_i \in \mathbb{R} \tag{2.12}$$

and P is a subset of relations on $\mathbb{R}$.

Referring to the Representation Theory of Measurement, an indicator I_Q can be considered as a 'map' from an empirical system (the 'real world') into a representational system (usually, a numerical system). However, the mapping between the empirical and symbol relations (2.9), unlike measurement, is not required:

$$I_Q : a \in \mathrm{A} \to I_Q(u) \in \mathrm{E}_Q \tag{2.13}$$

where E_Q is the set of representation elements in the representational system $\mathfrak{I}_Q$, A is a set of manifestations of the empirical system $\mathfrak{U}$, a is a manifestation of A and $I_Q(a)$ is the representation of a into the representational system $\mathfrak{I}_Q$.

Recalling that an empirical system is said to be relational if there exists a set of empirical relations among empirical manifestations (2.4), the identification of relations is conditioned both by the context and the way we are able to interpret it. The context is filtered by how we perceive and model it. For indicators, the mapping of the empirical system into a representational one may introduce new relations or modify the existing ones.

In accordance with this approach, three elements have to be considered: the model (i.e. the conceptualization of the real world), the representation-target and the rules to determine the related set of indicators together with their associated relations. The representation does hold if these three elements are delineated (see Figure 2).

For example, if we want to identify the winner of a competitive tender:

- the model is given by 'how we evaluate the competitors' credentials';
- the representation-target is 'finding a winner';
- the indicators and the associated relations originate from the rules we establish for obtaining a final score.

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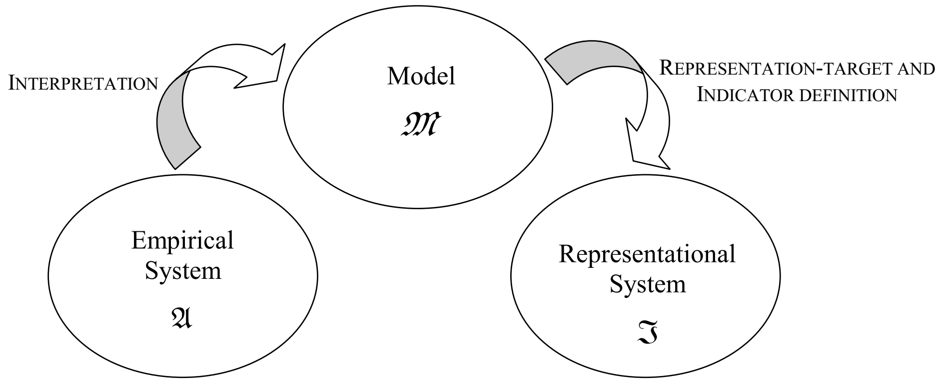

Figure 2. A scheme of the representational approach of an empirical system through the concepts of model, representation-target and indicators

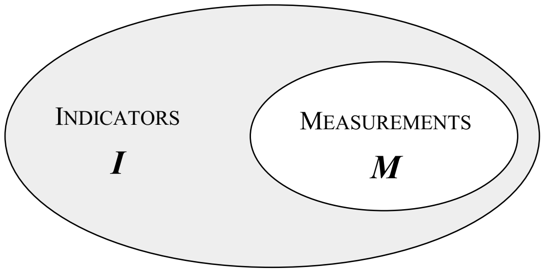

Figure 3. Measurements interpreted as a subset of indicators

On the basis of the representational approach, measurements may be interpreted as a subset of indicators. The basic difference between measurements and indicators is the way the relations of the empirical systems are mapped. Indicators do not require an isomorphism between empirical and representational relations (2.9). This means that, while a measurement is certainly an indicator, the *vice versa* is not true (Figure 3).

Let us consider, for example, the problem of the choice of a car. The customer preference is an indicator, which maps the empirical system (different car models) into a representational system (ranking of the most desired cars). It is not a measurement. No order relation is defined among empirical manifestations.

In general, if the representational system is a numerical system, an indicator is defined as a real value function on the set of empirical system manifestations:

$$I_Q : a \in \mathrm{A} \to I_Q(a) \in \mathrm{N} \qquad (2.14)$$

where N is a class of numbers, defined as in (2.12).

2.3. *Basic and derived indicators*

An indicator is basic if it is obtained as a direct observation of an empirical system. Examples of basic indicators are the number of defectives in a production line, the number of manufactured parts and the lapse time between events. An indicator is derived if it is obtained by the synthesis of two or more indicators. Examples of derived indicators are the ratio of defectives for a given time unit in a production line or the production rate in a manufacturing plant.

3. *THE CONDITION OF 'UNIQUENESS'*

In general, given a representation-target, the related indicator (or set of indicators) is not univocally defined. This can be shown for both basic and derived indicators by a series of simple examples.

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Table I. Experimental data of four equivalent production lines for exhaust systems in a manufacturing plant

	Motorizations			
Indicators	α	β	γ	δ
Daily production (number per day)	360	362	359	358
Daily defectiveness (number per day)	35	32	36	40
Unavailability equipment ratio (%)	4.00	5.50	4.50	5.00

3.1. 'Non-uniqueness' for derived indicators

Let us consider an automotive exhaust-systems production plant composed of four equivalent production lines (motorizations): α, β, γ and δ[38].

In this case, the context is the 'manufacturing plant' and the representation-target is 'the identification of the best production line'.

Production line performances are defined by the following three *indicators*:

- daily production (the average number of items produced in a day);
- daily defectiveness (the average number of rejected items in a day);
- unavailability equipment ratio (the average percentage of breakdown hours in a day).

Given these indicators, at least two different derived indicators which operationalize the given representation-target can be found. Let us consider the experimental data reported in Table I.

For each indicator we may establish the following rankings:

- daily production: $\beta \succ \alpha \succ \gamma \succ \delta$;
- daily defectiveness: $\beta \succ \alpha \succ \gamma \succ \delta$;
- unavailability equipment ratio: $\alpha \succ \gamma \succ \delta \succ \beta$.

The way to aggregate these three indicators is conditioned by a series of constraints, with first of all the scale properties and their meaning[36–38].

The assignment of weights, demerits and so on to reflect the degree of importance of each indicator is adopted in many circumstances[39]. This is a subjective approach. It suffers from the absence of consistent criteria to determine (*a priori*) the weighting values. Changing the numerical encoding may determine a change in the obtained results. In this way the person who analyses the problem does influence directly the aggregation results. Any conclusions drawn from the analysis on 'equivalent' numerical data could be partially or wholly distorted.

The choice of special codification techniques based on the use of substitution rates or cost utility functions is, in principle, also not correct. The arbitrary application of subjective codification rules can produce radical alterations of final results[36,40].

In this paper we consider the following two *derived indicators*.

1. *Borda's indicator* (I_B). Referring to the order of each indicator (see Table I), each motorization has a rank: 1 for the first position in the ranking; 2 for the second, . . . ; and n for the last. The Borda score for each motorization is the sum of every motorization's rank. The winner is the motorization with the lowest Borda score[41]:

$$I_B(x) = \sum_{i=1}^{m} I_i(x) \tag{3.1}$$

where $I_i(x)$ is the ranking obtained by a motorization x with regard to ith indicator and M is the total number of indicators (in this case, $m = 3$). The winner (the best motorization x^*) is given by

$$I_B(x^*) = \min_{x \in A}\{I_B(x)\} \tag{3.2}$$

where A is the set of compared motorization. In this example, $A \equiv \{\alpha, \beta, \gamma, \delta\}$.

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Table II. Pair comparisons of the data in Table I. Global ranking according to Condorcet's method

	α	β	γ	δ	I_C	Ranking
α	—	1	3	3	1	2nd
β	2	—	2	2	2	1st
γ	0	1	—	3	0	3rd
δ	0	1	0	—	0	3rd

2. *Condorcet's indicator* (I_C). For each pair of motorizations, it is determined how many times a motorization is ranked higher than the other. Motorization x is preferred to motorization y if the number of indicators in which x exceeds y is larger than the number of indicators in which y exceeds x. A motorization that is preferred to all other motorizations is called the (Condorcet) winner. A (Condorcet) winner is an alternative that, opposed to each of the other $(n-1)$ alternatives, wins by a majority. It can be demonstrated that there is never more than one (Condorcet) winner[42]:

$$I_C(x) = \min_{y \in A-\{x\}} \{i : xPy\} \tag{3.3}$$

where i is the number of indicators in which x exceeds y, and P is the preference operator. The winner (the best motorization x^*) is given by

$$I_C(x^*) = \max_{x \in A} \{i_C(x)\} \tag{3.4}$$

Applying Borda's method to the data in Table I, we obtain the following results:

$$I_B(\alpha) = 2 + 2 + 1 = 5$$
$$I_B(\beta) = 1 + 1 + 4 = 6$$
$$I_B(\gamma) = 3 + 3 + 2 = 8$$
$$I_B(\delta) = 4 + 4 + 3 = 11$$

According to (3.2), the final ranking is

$$\alpha \succ \beta \succ \gamma \succ \delta$$

The winner (i.e. the motorization with best performance) is motorization α.

Condorcet's method applied to the data in Table I gives the results (in Table II). According to (3.4), the best motorization is β ($\beta \succ \alpha \sim \gamma \succ \delta$).

The two approaches, although satisfying the same representation-target, provide different conclusions about the production plant performances.

A significant aspect regards the use of Borda's indicator. It is possible to demonstrate that it is sensitive to 'irrelevant alternatives'. According to this assertion, if x precedes y in a Borda order, there is no guarantee that x still precedes y if a third alternative z is added[43].

Consider again an exhaust-system production plant with three motorizations $\{\alpha, \beta, \gamma\}$. Suppose they are compared with regard to the daily production and the daily defectiveness (see Table III).

The two rankings are:

- daily production: $\alpha \succ \gamma \succ \beta$;
- daily defectiveness: $\beta \succ \alpha \succ \gamma$.

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Table III. Experimental data of three equivalent production lines for exhaust systems in a manufacturing plant

	Motorizations		
Indicators	α	β	γ
Daily production (number per day)	367	350	354
Daily defectiveness (number per day)	35	30	37

Table IV. Experimental data of three equivalent production lines for exhaust systems in a manufacturing plant

	Motorizations		
Indicators	α	β	γ
Daily production (number per day)	367	350	345
Daily defectiveness (number per day)	35	30	32

The resulting Borda scores are

$$I_{\mathrm{B}}(\alpha) = 1 + 2 = 3$$
$$I_{\mathrm{B}}(\beta) = 3 + 1 = 4$$
$$I_{\mathrm{B}}(\gamma) = 2 + 3 = 5$$

According to Borda's indicator (3.2), the best motorization is α.

Now suppose that γ varies its position in the orders (daily production, from 354 items to 345 items; daily defectiveness, from 37 items to 32 items), while the reciprocal position of α and β does not change (see Table IV).

The new rankings are:

- daily production: $\alpha \succ \beta \succ \gamma$;
- daily defectiveness: $\beta \succ \gamma \succ \alpha$.

The new resulting Borda scores are

$$I_{\mathrm{B}}(\alpha) = 1 + 3 = 4$$
$$I_{\mathrm{B}}(\beta) = 2 + 1 = 3$$
$$I_{\mathrm{B}}(\gamma) = 3 + 2 = 5$$

In this case, the best motorization is β.

On the other hand, it can be shown that the Condorcet's method does not guarantee the property of 'transitivity' between relations[43]. Consider, for example, the exhaust-system production plant with three motorizations $\{\alpha, \beta, \gamma\}$. Suppose they are compared with regard to the daily production, the daily defectiveness and the unavailability equipment ratio (see Table V).

The resulting rankings are:

- daily production: $\alpha \succ \beta \succ \gamma$;
- daily defectiveness: $\beta \succ \gamma \succ \alpha$;
- unavailability equipment ratio: $\gamma \succ \alpha \succ \beta$.

Condorcet's method gives the results in Table VI.

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Table V. Experimental data of three equivalent production lines for exhaust systems in a manufacturing plant

	Motorizations		
Indicators	α	β	γ
Daily production (number per day)	365	362	359
Daily defectiveness (number per day)	35	32	34
Unavailability equipment ratio (%)	5.50	6.00	4.50

Table VI. Pair comparisons of data in Table V. Global ranking according to Condorcet's method

	α	β	γ	I_C	Ranking
α	—	2	1	1	1st
β	1	—	2	1	1st
γ	2	1	—	1	1st

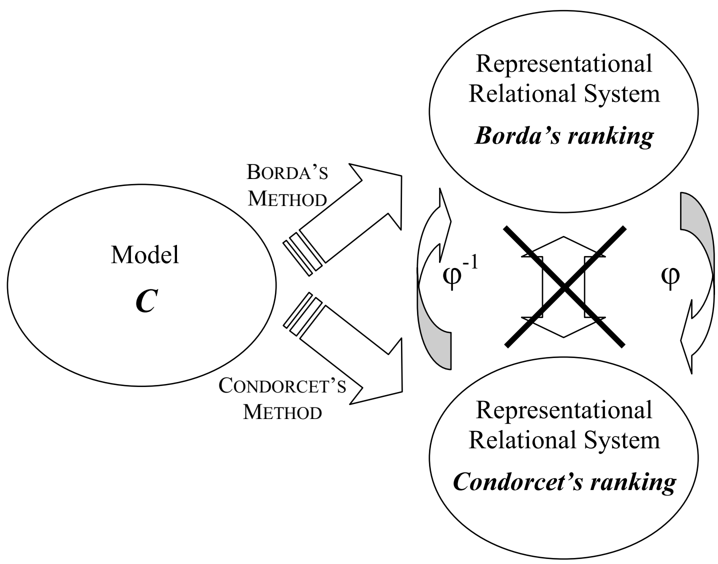

Figure 4. Schematic representation of the 'independence' between Borda's indicator and Condorcet's indicator

In this case, there is no winner. The transitivity property is not satisfied. According to direct comparisons, the result is

$$\alpha \succ \beta, \quad \beta \succ \gamma, \quad \gamma \succ \alpha$$

I_B and I_C are independent indicators. There is no mathematical transformation which maps Borda scores into Condorcet scores, or *vice versa*, maintaining the global order (see Figure 4)[43].

In conclusion, the results obtained by alternatively applying Borda's and Condorcet's indicators are different, although they have been provided for the same representation-target. We deduce that the representation condition is valid for more than one indicator. In general, uniqueness is not guaranteed for derived indicators.

3.2. *'Non-uniqueness' for basic indicators*

Let us consider again the four motorizations $\{\alpha, \beta, \gamma, \delta\}$ introduced in Table I. Suppose that the comparison is performed with regard to daily defectiveness. The representation-target is 'identifying the motorization with the lower number of rejected components'.

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At least two different indicators can be adopted.

1. *'Best in ten' indicator* (I_{SING}). For each motorization, I_{SING} is given by the best single performance registered in the last ten sample (ten days) inspections:

$$I_{SING}(x) = \min_{i=1,\ldots,10}\{D_i(x)\} \tag{3.5}$$

where $D_i(x)$ is the daily defectiveness shown by motorization x in the ith inspection. The better motorization (x^*) is the one with the minimum value,

$$I_{SING}(x^*) = \min_{x\in A}\{I_{SING}(x)\} \tag{3.6}$$

where $A \equiv \{\alpha, \beta, \gamma, \delta\}$.

2. *'Best average' indicator* (I_{AVE}). For each motorization, I_{AVE} is the average of the performance registered in the last ten sample (ten days) inspections:

$$I_{AVE}(x) = \frac{\sum_{i=1}^{10} D_i(x)}{10} \tag{3.7}$$

Again, the winning motorization (x^*) is the one with the minimum value,

$$I_{AVE}(x^*) = \min_{x\in A}\{I_{AVE}(x)\} \tag{3.8}$$

In this case too, it can be shown that the two indicators are independent of each other. This means that the representation condition is valid for more than one basic indicator. In general, the condition of uniqueness is not guaranteed for basic indicators.

3.3. Remarks about the condition of 'uniqueness'

The non-fulfilment of the uniqueness condition implies a series of consequences in the use of indicators. The most evident is that there is an arbitrary choice in setting up the mapping into the representational system. This entails that, given two or more indicators for a specified representation-target, there may be no transformation from one indicator into another. This causes the fact that, for example, analogous representation-targets might not be comparable if represented by different indicators.

On the other hand, it is interesting to recall that for measurements the requirement of a homomorphism for mapping empirical manifestations and an isomorphism for mapping relations defines a class of equivalent scales. Each equivalent scale can be mapped into another. All the possible transformations form the so-called 'class of admissible transformations'[37].

The imperfect objectivation of the model and/or the incomplete definition of the representation-target, as well as the non-fulfilment of the condition of uniqueness, give rise to an *uncertainty* concept. In particular, uncertainties of measurement are considered to be imperfections of the measurement process and/or the result of incorrect determinations of empirical observations or empirical laws[37]. A similar concept can be defined for indicators.

3.4. Condition of 'uniqueness' by specializing the representation-target

It is easy to show that a deeper specialization of the representation-target does not imply the automatic removal of the condition of non-uniqueness.

Let us consider again the four motorizations $\{\alpha, \beta, \gamma, \delta\}$ introduced in the example reported in Table I. We showed that 'the identification of the motorization with the lower number of rejected components' yields at least two different indicators.

Now, let us try to further specialize the representation-target definition in order to eliminate the non-uniqueness of related indicators. A more specialized definition may be 'identifying the motorization with the lower number of rejected components in the last three observations'.

Also in this case, at least two different indicators can be adopted.

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1. *'Best in three' indicator* (I'_{SING}). For each motorization, I'_{SING} is given by the best single performance registered in the last three days,

$$I'_{\mathrm{SING}}(x) = \min_{i=1,\ldots,3}\{D_i(x)\} \tag{3.9}$$

where $D_i(x)$ is the daily defectiveness showed by motorization x in the ith day. The winning motorization (x^*) is the one with the minimum value,

$$I'_{\mathrm{SING}}(x^*) = \min_{x\in A}\{I'_{\mathrm{SING}}(x)\} \tag{3.10}$$

where $A \equiv \{\alpha, \beta, \gamma, \delta\}$.

2. *'Best average' indicator* (I'_{AVE}). For each motorization, I'_{AVE} is given by the average of the performance registered in the last three days,

$$I'_{\mathrm{AVE}}(x) = \frac{\sum_{i=1}^{3} D_i(x)}{3} \tag{3.11}$$

The winning motorization (x^*) is the one with the minimum value,

$$I'_{\mathrm{AVE}}(x^*) = \min_{x\in A}\{I'_{\mathrm{AVE}}(x)\} \tag{3.12}$$

Again, we can try to further specialize the representation-target: 'identifying the motorization with the lower average number of rejected components in the last three observations'. This new definition does not imply the uniqueness of the indicator. We can still define at least two new different indicators. For example, we can adopt an indicator which excludes from the rejected components those which only have to be reworked, and another which includes them, and so on.

In general, we can affirm that a specialization of the representation-target implies a more accurate definition of the related indicator (or set of indicators), never reaching the condition of uniqueness. The result is that an univocal definition of an indicator (or a set of indicators) can never be obtained. The remaining differences, after the representation-target specialization, will contribute to uncertainty.

3.5. *The choice of the best set of indicators*

An immediate consequence of the non-uniqueness condition is that a representation-target can be described by a different set of indicators.

This leads to the need of establishing a series of rules (or an empirical procedure) to give the set of indicators which better embodies a given representation-target.

The choice of the best set of indicators involves the analysis of the impact that the indicators will produce on the observed system. Different sets of indicators may differently influence the overall behaviour of a system with uncontrollable consequences[6].

To select indicators, two different typologies of properties should be considered:

- *basic properties*, directly related to the mathematical definition of an indicator (uniqueness, exhaustiveness, monotony and non-redundancy)[35,36];
- *operational properties*, related to their practice application (validity, robustness, usefulness, integration, economy, compatibility, etc.)[7].

According to each application case, the final choice must be addressed towards the set which better meets the two families of properties and generates the most 'effective' impact.

4. CONCLUSIONS

Many rising theories try to provide an initial theoretical grounding to the linkage between performance measures and their impact on the observed system (agency theory, dependency theory, strategic fit theory, etc.). However,

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a general theory is still lacking. In this paper a first attempt to give a mathematical structure to the concept of indicator is presented. An indicator (or a set of indicators) is a tool which makes operational the contents of a representation-target. Particular attention has been focused on the condition of uniqueness. In general, indicators do not fulfil the condition of uniqueness. This entails that, for a given representation-target, two or more different indicators can be defined.

Specific care should be given to the choice of the so-called best set of indicators. This choice can be correctly done by observing two different families of properties: basic properties, and operational properties.

Future work will regard a deeper analysis of these properties as well as the impact that a set of indicators could induce on an observed system.

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