Quality Measurements. Visual controls by means of linguistic scales.

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ABSTRACT

Qualitative or linguistic scales are often used to collect evaluator judgements while performing visual controls on manufactured products. The analysis of these data is often performed through techniques that add mathematical properties to the evaluation scale. Numerical conversion of scale levels, for example, implies the introduction of the concept of distance between them.
In this paper we introduce the so-called 'ordinal distribution' to describe the population of judgements expressed on a linguistic scale, without making use of the concept of distance. A new dispersion measure, the 'ordinal range', as an extension of the cardinal range to ordinal scales, is then proposed. A practical application in the field of quality is developed throughout the article.

1 INTRODUCTION

Many manufacturing lines require the assessment of the attributes of products or services by means of a qualitative or linguistic scale, whose levels are judgements such as 'good', 'bad' or 'medium'. An example is given in (1) for a production line for fine liqueurs, where an operator in charge of visual control of the corking and closing process might have the following assessment possibilities:
- 'reject' if the cork does not work;
- 'poor quality' if the cork works but has some defects;
- 'medium quality' if the cork only has relevant aesthetic flaws;
- 'good quality' if the cork only has small aesthetic flaws;
- 'excellent quality' if the cork is perfect.

The results of an inspection of a hypothetical sample of thirty corks are reported in Table 1.
A simplistic analysis of these data could be performed through the codification of the levels of the linguistic scale, that is the assignment of a numerical value to each level of the linguistic

scale. After this operation, traditional statistics could be used for data analysis. However, the results heavily depend on the codification that has been chosen. For example, a codification of the type {1, 2, 3, 4, 5} would give an arithmetic mean of 3.7, while a codification of the type {1, 3, 9, 27, 81} would lead to an arithmetic mean of 32.9. The first codification suggests us that the arithmetic mean of the sample is between ‘medium quality’ and ‘good quality’, while the second codification gives an arithmetic mean between ‘good quality’ and ‘excellent quality’. The difference between the results is due to the fact that the choice of a particular codification implies fixing the distances between the levels of the scale. Since each type of codification is always arbitrary, it is necessary to develop a methodology of data analysis that does not exploit a ‘device’ of codification.

Table 1 Results of the analysis of a sample of thirty corks.

‘reject’	‘poor quality’	‘medium quality’	‘good quality’	‘excellent quality’
1	2	10	9	8

2 THE CONCEPT OF ORDINAL DISTRIBUTION

The possible values of an ordinal random variable S are judgements belonging to a set $\{S_1, S_2, \ldots, S_t\}$, where t is the number of levels of the linguistic scale. The distribution of S is given by:

$$f_S(S) = \begin{cases} \mathbb{P}\left[\mathrm{S} = S_j\right] & \text{if } S = S_j \text{, with } j = 1, 2, \ldots, t \\ 0 & \text{if } S \neq S_j \end{cases}$$

Its main feature is the lack of the concept of distance between the levels of the judgement scale. This is the reason why we introduce the statement ‘ordinal distribution’ to denote the equivalent of the frequency distribution in an ordinal environment. Fig. 1 reports a representation of the empirical ordinal distribution of sample reported in Table 1. The relative frequency of each judgement $S_i, \forall i = 1, 2, \ldots, t$, is given by $f_i = n_i / n$, where n_i is the number of judgements of the sample at level S_i and n is the sample size. The positions of the vertical bars are not fixed at precise points on the horizontal axis because the distances among them are not defined. To underline this concept, a ‘skate’ symbol is used.

3 THE CONCEPT OF ORDINAL RANGE

All location and dispersion measures that can be used in an ordinal environment mustn’t require the definition of the concept of distance between the levels of the scale.
Location measures that do not require the definition of the concept of distance and therefore can be properly used in an ordinal environment are, for example, the median, the mode and the OWA emulator of arithmetic mean. A survey of the median and the mode properties can be found in (3), while a review of the OWA emulator can be found in (1, 2, 4).
None of the traditional dispersion measures can be used in an ordinal environment. They all require to make a difference between the levels of the scale, that in turn requires the

introduction of the concept of distance. As an alternative, we think that a dispersion measure should also include some information on the location of sample at hand.

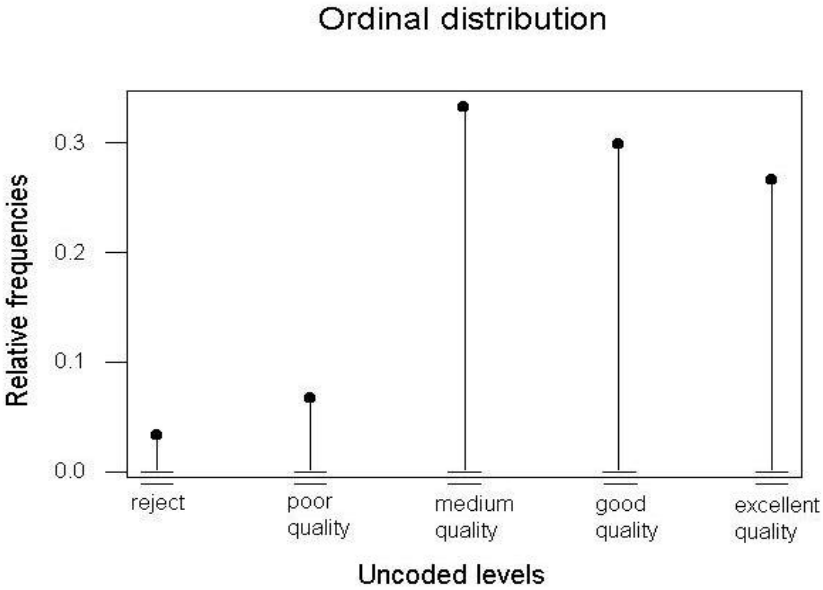

Fig. 1 A representation of the empirical ordinal distribution of sample reported in Table 1. The concept of distance between the vertical bars is not defined.

We propose a new dispersion measure, the so-called 'ordinal range', whose value is associated with a unique couple of judgements representing the minimum and the maximum value of the sample. The ordinal range is measured on a scale with $t(t+1)/2$ levels, where t is the number of levels of the judgement scale. In the example of the corking process, $t = 5$: therefore, the ordinal range scale will have 15 levels. These levels can be ordered in increasing 'dangerousness' of dispersion of the sample. The dispersion of the sample is less 'dangerous' if the number of levels of the judgement scale between the minimum and the maximum sample value is minor. In case this number of levels were the same, dispersion would be more 'dangerous' for a sample centred at a more 'dangerous' level of the judgement scale (that is, a level nearer to 'reject' in the example of the corking process). Table 2 reports all possible values of the ordinal range and the corresponding couple of minimum and maximum sample value for the example of the corking process.
Table 3 shows two different samples with the same value of range of ranks (the total number of scale levels contained between the maximum and the minimum value of a sample of evaluations), which would have the same dispersion with a traditional approach (assuming that scale levels are equally spaced) (1). The respective ordinal ranges, however, are not the same. Dispersion of sample *b* is more 'dangerous' than dispersion of sample *a*. Sample *b* is centred at a lower (more 'dangerous') value of the judgement scale.

4 CONCLUSIONS

In this paper we point out the problems that arise when dealing with ordinal scales. We extend the concept of frequency distribution to an ordinal environment by introducing the so-called 'ordinal distribution'. We then provide a new dispersion measure, the 'ordinal range', that can be used in an ordinal environment.

Future research will aim at finding an asymptotic ($n \to \infty$) ordinal distribution for each location and dispersion measure, provided that it actually exists. Current studies are leading to the development of statistical tools for on-line monitoring of quality characteristics expressed on an ordinal scale.

Table 2 Values of the ordinal range and the corresponding couple of minimum and maximum sample value for the example of the corking process.

Minimum sample value	Maximum sample value	Values of the ordinal range
Excellent quality	Excellent quality	R_1
Good quality	Good quality	R_2
Medium quality	Medium quality	R_3
Poor quality	Poor quality	R_4
Reject	Reject	R_5
Good quality	Excellent quality	R_6
Medium quality	Good quality	R_7
Poor quality	Medium quality	R_8
Reject	Poor quality	R_9
Medium quality	Excellent quality	R_{10}
Poor quality	Good quality	R_{11}
Reject	Medium quality	R_{12}
Poor quality	Excellent quality	R_{13}
Reject	Good quality	R_{14}
Reject	Excellent quality	R_{15}

Table 3 Two different samples with the same sample size and the same value of range of ranks, but different values of ordinal range.

	'reject'	'poor quality'	'medium quality'	'good quality'	'excellent quality'	Ordinal range
Sample *a*	0	3	10	9	8	R_{13}
Sample *b*	3	10	9	8	0	R_{14}

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