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Measurements, evaluations and preferences: A scheme of classification according to the representational theory

P. Cecconi, F. Franceschini *, M. Galetto

Politecnico di Torino, Dipartimento di Sistemi di Produzione ed Economia dell'Azienda, Corso Duca degli Abruzzi 24, 10129 Torino, Italy

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Abstract

The paper traces out a scheme for classification of the three ways by which observable or non-directly observable properties of an object can be judged/described. The definition of *measurement* as an *empirical* and *objective* operation is the starting point. We compare the concept of *representational measurement* with those of *evaluation* and *preference*. *Evaluation* maintains the empiricity but not the objectivity of *measurement*: there is no unanimously acknowledged reference for the description of latent constructs. *Preference* is neither empirical nor objective: every subject has his/her own *relation* to express the judgment and this relation is not exogenously known. Representational theory is effective in giving a precise definition for empiricity but it is not exhaustive in defining objectivity in the case of preferences and evaluations. Finally, we trace possible research paths to be undertaken for further analysis of the proposed problem.

Keywords: Representational theory of measurement; Measurement; Evaluation; Preference empiricity; Objectivity; Quality

1. Introduction

The properties of an object, either directly or non-directly observable, may essentially be judged and described by three basic operations: measurements, evaluations and preferences. The aim of the present work is to propose a scheme for classification of these three operations.

The question has a particular scientific importance and some possible repercussions involve many disciplines like metrology, decision-making, and quality measurement.

Considering this latter aspect, when we ask a sample of people to express opinions about the quality of a good, we carry out an operation which is in suspense among measurement, preference and evaluation. It is necessary to know what the conceptual paradigms at heart of these three operations are.

The representational theory of measurement based on the properties of binary relations is the used instrument of investigation.

Given a set of objects or alternatives A, a binary relation R on A is a subset of the Cartesian product $A \times A$. Binary relations arise very frequently from

* Corresponding author. Tel.: +39 011 5647225; fax: +39 011 5647299.
E-mail address: fiorenzo.franceschini@polito.it (F. Franceschini).

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everyday language: for example, if A is the set of all people in a certain country, then the set:

$$F = \{(a,b) : a \in A,\ b \in A \text{ and } a \text{ is brother of } b)\}$$

defines a binary relation on A, which we may call "brother of".

The properties (reflexivity, symmetry, transitivity, etc...) of a generic relation R must be defined on a specific reference set. For example, the relation "brother of" is symmetric on the set of all males in a certain country, but it is not symmetric on the set of all people in that country. Formally, rather than a simple relation, we speak of a *relational system* (A, R), that is a relation applied to a set of objects. This concept, introduced by Tarski [26], has been the natural vehicle for the subsequent development of the representational theory of measurement [23,14].

When a subject judges or describes objects (physical or abstract), he/she considers one or more comparison relations. These relations may be tangible or intangible, uniformly or not uniformly interpretable. For example, the relations "more beautiful than", "more elegant than", "worthier than" "preferred to" are relations intangible and arbitrarily interpretable by different subjects, whereas the relations "heavier than", "longer than", "warmer than" are not. These latter are observable relations, which do not enable a free interpretation by the subjects. There is a direct reference to the scales of the International System of measurement.

In this work, the analysis of the differences among the three operations takes place considering only a specific property of the objects. "*The subjects of measurement are properties. Of course, properties exist only in connection with empirical objects. Usually, one object shows various properties. In measuring one property, we neglect all the other properties the object in question may have*" [19].

Moreover, the considerations here referred to are exclusively limited to rankings. We only consider the cases in which the individuals are able to establish a priority ranking, that is a hierarchy among objects.

2. Two criteria of discrimination between the three operations: Empiricity and objectivity

"*Measurement is the assignment of numbers to properties of objects or events in the real world by means of an objective empirical operation, in such a way as to describe them. The modern form of measurement theory is representational: numbers assigned to objects/events must represent the perceived relations between the properties of those objects/events*" [8].

The definition of a measurement refers to two fundamental concepts: empiricity and objectivity. This allow us to discriminate between the three operations: measurements, evaluations and preferences.

The *empiricity* arises when a judgment of a ranking "*is the result of observations and not, for example, of a thought experiment. Further, the concept of the property measured must be based on empirically determinable relations and not, say, on convention*" [9].

There is empiricity when the type of relation is observable, that is the property of the object proves to be, in a precise moment, in a well-defined state characterized without ambiguity. Empiricity means that there is "*an objective rule for classifying some aspect of observable objects as manifestations of the property*" [9].

The *objectivity* concerns the kind of results that the judgment produces, "*within the limits of error independent of the observer*" [7]. "*Experiments may be repeated by different observers and each will get the same result*" [25]. Full objectivity means independence of the subjects. The result of the operation gives only information about the measured property.

The *measurement* requires both empiricity and objectivity. It is an operation of objective description: the results of n different measurements, in the same operating conditions, are univocal and independent by subjects. We suppose there is no "error" and uncertainty in an ideal measurement process (the environmental and other influential variables are considered non-existent). It is also an empirical operation: "*Measurement has something to do with assigning numbers that correspond to or represent or "preserve" certain observed relations*" [20].

The *preference* is neither empirical nor objective. Preferences are, by definition, subjective and conflicting. We are not able to know exogenously, in detail, the relation that each subject applies when assigning a ranking. An outside observer will have considerable difficulties in interpreting the results generated by this kind of operation. In other words, different subjects interpret the relation in different ways and can establish disagreeing orderings. In this

Table 1
Scheme of classification of the three operations

	Objective	Empirical
Measurement	Yes	Yes
Evaluation	No	Yes
Preference	No	No

case, the uncertainty concerns deeply the kind of relation applied by each subject.

The *evaluation* is somewhere between measurement and preference. It is not objective because evaluations are individual perceptions, performed without the use of an univocal instrument of measurement. Nevertheless, it is an operation that *wants* to be empirical: the meaning of intangible relations is circumscribed by means of an exogenous process of semantic definition from the outset. Subjects are called on to conform to this process. Operatively, there is uncertainty in the interpretation that subjects give to the provided dimension of observation.

The three operations can be classified as illustrated in Table 1:

As explanation, it is convenient to make explicit the meaning of the following terms [2]:

- *Exogenous*: The expression of a description/ordering is subordinated to a coactive, explicit and declared constraint. An outside observer imposes rules (concerning the dimension of observation, interpretation of the scales, etc...) to which subjects conform from the outset.
- *Endogenous*: The expression of a description/ordering happens according to a latent, implicit, non-declared point of view. Each subject decides to adopt the rules he considers more convenient, without declaring them.

3. The representational definition of measurement

The assumption that the relations are observable is at heart of the representational point of view.

In a measurement, subjects are called on to "judge" an observable relation, on which there are no doubts about meaning and interpretation. Relations like "longer than", "heavier than", "warmer than", etc. are some possible examples.

The representational theory of measurement related to a quality or a property of an object has four fundamental parts [9]:

(1) *An empirical relational system*. Consider some quality (for example the length of an object) and let q_i represent an individual manifestation of the quality Q, so that we can define a set of all possible manifestations as $Q = \{q_1 \ldots\}$. Let there be a family of empirical relations R_i on Q, $R = \{R_1, \ldots, R_n\}$. Then the quality can be represented by an empirical relational system $\mathcal{L} = \langle Q; R \rangle$.

(2) *A numerical relational system.* Let N represent a class of numbers $N = \{n_1 \ldots\}$. Let there be a family of relations $P = \{P_1, \ldots, P_n\}$ defined on N. Then $\mathcal{N} = \langle N; P \rangle$ represents a numerical relational system.

(3) *A representation condition.* Measurement is defined as an objective empirical operation such that $\mathcal{L} = \langle Q; R \rangle$ is mapped homomorphically into (onto) $\mathcal{N} = \langle N; P \rangle$ by M and F. Specifically, M is the function mapping Q to N, so that $n_h = M(q_h)$ $(M: Q \to N)$. F is the function mapping one-to-one the relations of R on P $(F: R \to P)$.
The above homomorphism is the representation condition.
Firstly, it implies that if q_h is related to q_m by an empirical relation R_k, that is $R_k(q_n, q_m)$, P_k is the numerical relation corresponding to R_k, $n_h = M(q_h)$ is the image of q_h in N under M then $R_k(q_h, q_m)$ implies and is implied by $P_k(n_h, n_m)$.
Measurement is a homomorphism—not an isomorphism—because M is not a one-to-one function. It maps separate but indistinguishable property manifestations into the same number.
$S = \langle \mathcal{L}, \mathcal{N}, M, F \rangle$ constitutes a scale of measurement for $\mathcal{L}$.
The definition of a representational measurement is illustrated in Fig. 1.

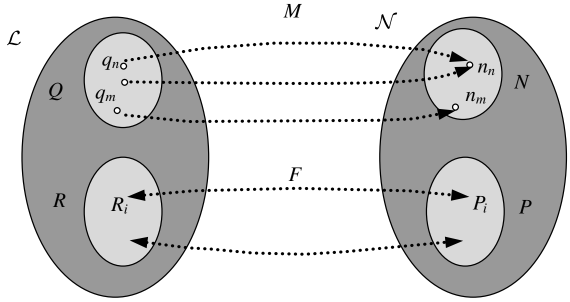

Fig. 1. Homomorphism of $\mathcal{L}$ onto $\mathcal{N}$. Two elements in Q may be mapped into the same number in N.

"*In measurement we start with an observed or empirical relational system and we seek a mapping to a numerical relational system which "preserves" all the relations and operations observed in the empirical one*" [20].

"*Whatever inferences can be made in the numerical relational system apply to the empirical one*" [5].

(4) *Uniqueness condition*. The representation condition may be valid for more than one mapping function M. There are admissible transformations from one scale to another scale without invalidating the representation condition. The uniqueness condition defines the class of transformations for which the representation condition is valid. For ordinal scales, all monotone increasing functions are admissible transformations [9].

3.1. An example of ordinal measurement

A measurement can be viewed as a representation of some properties of the real world by symbols; nominal and ordinal scales are examples of representation by non-numeric symbols. The ability to order a set of objects/events is related to the notion of the "amount" of a property manifested by each one. In ordinal measurement, the relation "$\succ$" ("it has got the property more than") is transitive and reflexive. Therefore,

$$q \succ r \Longleftrightarrow M(q) > M(r)$$

Consider the hardness measurement of minerals (the relation R "harder than"). It is a typical example of ordinal measurement.

The Mohs scale orders minerals from diamond to talc, on the basis of which scratches which. The ability to scratch (i.e. to etch, to cut in surface) is the empirical relation and the ordering is the formal relation.

The scale is built as follows: ten standard minerals are arranged in an ordered sequence so that precedent ones in the sequence can be scratched by succeeding ones and cannot scratch them. The standards are assigned numbers 1–10. The sequence is talc 1, gypsum 2, calcite 3, fluorite 4, apatite 5, orthoclase 6, quartz 7, topaz 8, corundum 9, diamond 10. A mineral sample of unknown hardness which cannot be scratched by quartz and cannot scratch it, is assigned measure 7 [7].

3.2. Some clarifications about the concept of measurement

The *homomorphism* is a faithful representation of empirical relations by symbolic relations. The condition of homomorphism, namely of an assignment of symbols (in general numbers) to objects\events according to the degree of presence of a certain property, is considered by the followers of the representational theory as a necessary and sufficient condition to define a measurement.

Some authors [5] assert that not all possible rules of assignment yield right measurements. The assignment of numbers is a representational measurement only if three requirements are satisfied. "*The assignment of numbers must:*"

(a) *be orderly;*
(b) *represent meaningful attributes;*
(c) *yield meaningful predictions.*

According to the authors, the presence of these three conditions defines the "*automatic consistency check*", which realizes the difference between a representational measurement and non-representational one: "*when mineral* (*a*) *scratches mineral* (*b*), *then* (*a*) *is represented above* (*b*) *in the order*" [5].

If consistency check fails, we cannot speak about representational measurements. Consider, for example, a subject expressing his/her own opinion about a given product. The judgment is stated on a *rating* scale of the type indicated in Fig. 2. In this

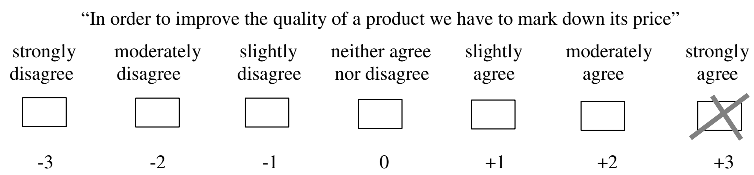

Fig. 2. An example of a measurement with no consistency check.

particular case, suppose that the subject selects the label "+3".

This statement "*does not represent a specific behaviour*" of the subject. In fact, we may wish to make many inferences on the basis of this behaviour. For example that he/she will turn his/her own attention to those firms adopting this policy, that he/she is deceiving himself/herself about something that will not happen, or that he/she believes in paying suppliers by manufacturing money. "*But we cannot make a firm prediction about some other response to this or another rating scale. There is no consistency check, hence no representational measurement*" [5].

Vice versa, measurements of hardness, mass, length, etc. present a consistency check. This check is performed by an appropriate measurement system realizing the homomorphism from the empirical relational system to the numerical one.

The measurement system guarantees the two fundamental components of a measurement: "*assignment and empirical determination*" [16]. It circumscribes without ambiguity the empirical relations (harder than, heavier than, longer than, etc...), and performs the assignment of numbers to the objects according to the rule of the corresponding homomorphism.

The presence of a conventional, non-ambiguous, empirical reference is the reason at the base of the objectivity of measurement results.

4. Evaluations

In general, the qualities of an object/event can be classified as physical or non-physical, observable or non-directly observable. The utility, the quality of a performance, the customer satisfaction, the attitude, etc. are examples of non-physical and non-directly observable magnitudes. Attempts to measure these latter cause many problems.

The first problem is "*the difficulty of establishing an adequate objective concept, or theoretical construct, of these qualities based on empirical operations*" [7].

The description of these attributes or latent constructs is not objective. It may origin a free interpretation of the meaning by the subjects.

In these situations, we cannot speak about real measurement, but just about evaluations or attributions of values to individual judgments.

Descriptions about non-tangible qualities are possible only by means of subjective judgments expressed on adequate scales (Fig. 3).

Fig. 3 illustrates the concept of evaluation of a non-physical magnitude. The subject ideally compares the object (first scale pan) with the reference terms on the scale (second scale pan). The evaluation consists in identifying the judgment on the scale which balances the two scale pans.

The first fundamental component of a measurement (empiricity) runs short. Consider, for example,

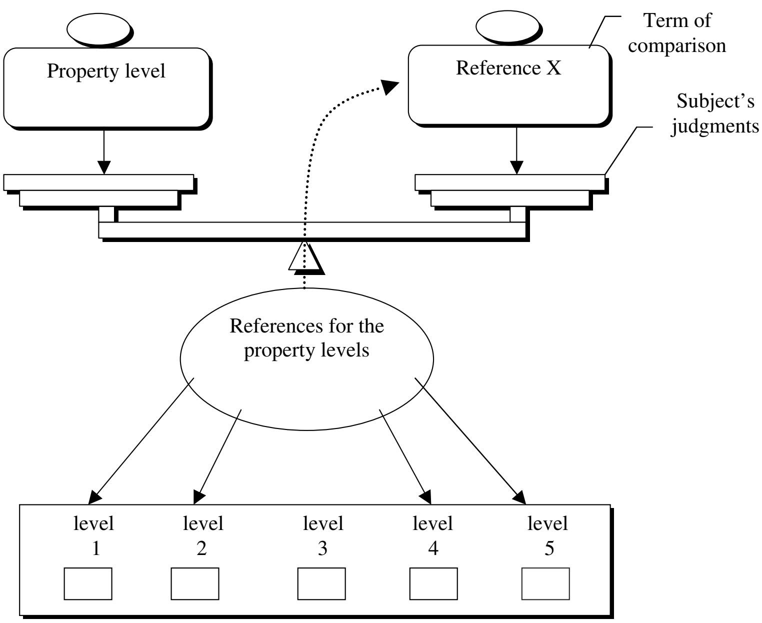

Fig. 3. Basilar scheme of evaluation of a non-physical magnitude [12].

the aesthetical beauty of an object: in this case "*there is not an objective rule for classifying some aspect of observable objects as manifestations of the beauty. Similarly, there are no objective empirical relations such as indistinguishability or precedence, in respect of beauty. The basis for the measurement of beauty is thus absent from the outset*" [7].

The aim of an evaluation is that of "building" and "imposing" to evaluators, in some way, this empiricity component.

This way, Pawson [18] asserts that: "*First, evaluation deals with the real, that is we evaluate things and empirical relations about things. Secondly, evaluation should follow a realist methodology. Thirdly, evaluation, perhaps above all, needs to be realistic*".

4.1. The evaluation of non-tangible qualities: The exogenous imposition of rules for the subjects

Let A be a set of objects and R the relation "heavier than". That is, for any pair of object x, y in A we define:

xRy x is heavier than y

"*Note that R can be defined either by a balance or by psychophysical experiments using an observer to compare the weights. The two procedures yield similar empirical relational systems with the same object set. The interpretation of the relation "heavier than", however, is physical in the former system and psychological in the latter*" [4].

In the second case, we should speak of evaluations.

The evaluation is typically a normative process. It is based on exogenous rules driving subjects in the attribution of values to intangible and interpretable qualities. Essentially, there are two kinds of rules:

- clear and precise definition of non-physical attributes (abstract constructs);
- definition of operating evaluation scales.

The first fundamental step of an evaluation process consists in the definition of a *reference axis*. The evaluation process wants to make the initial latent construct observable and less interpretable, giving to it "*empirical substance*", by specific empirical rules. The second phase consists in providing suitable evaluation scales.

An evaluation requires that there is "uniformity" for subjects in accepting the rules provided. This uniformity does not exist in preference judgments, where everyone is free to choose the interpretation with which the judgment takes place.

4.2. Evaluation: A subjective homomorphism

We said that evaluation is a partially empirical and subjective operation. We are going to justify this position by means of the representational theory.

Psychological tests or Questionnaires for the evaluation of product or service quality are examples of evaluation processes.

Paraphrasing measurement definition, the evaluation becomes the assignment of numbers, or labels to properties or events of the real world by means of an empirical subjective operation, in such a way as to describe them.

In the evaluation context, the mapped relation R is not empirical. The relation becomes empirical by means of a set of semantic (what the construct means) and operating rules (evaluation scale). We exogenously impose the dimension or relation that subjects have to observe.

What is perceived as "having the property more than", in the empirical world, has an immediate translation in the ranking performed by the subject. In ordinal evaluation, there is a homomorphism from the empirical world onto the symbolic (numeric) one, that is to say, there is a faithful representation of empirical objects and their relations in the numerical world. The representational form is, therefore, maintained. From this point of view, the homomorphism is not able to formalize the difference between the operation of evaluation and measurement [17].

The question is that we deal with a subjective operation. The ranking performed by a subject will not coincide with that of another subject. It is not univocal. Different subjects can observe different degrees of the property in the same object.

This dependence may cause some important formal consequences.

The concepts of indifference threshold δ and the distinction between the indifference relation I (reflexive and symmetric) and the equivalence relation E (reflexive, symmetric and transitive) are strictly connected with each subjective operation. This latter distinction supports the difference between evaluation and measurement.

There is an indifference relation between two stimuli-objects (for example, two sounds of different intensity) every time each subject is not able to discriminate them.

The equivalence relation E is stronger than the indifference relation I. It does not confine itself simply to asserting the non-difference between two elements (for example the two auditory stimuli mentioned before), but it establishes their equality [20].

The different significance of the two relations is fundamental in psychological evaluations. Imagine three weigths a, b and c; if, weighing them by hands, aIb (a is indifferent to b) and bIc, it cannot be said that also aIc. A little difference between a and b and between b and c can become noticeable when the conjoint effects of the single differences are considered. The absence of transitive property in the indifference relation shows this possibility. This property, on the other hand, belongs to the equivalence relation E: if aEb (a is equivalent to b) and bEc, then aEc.

The problem of discrimination between the equivalence and indifference relation is connected to the concept of "*Just Noticeable Differences*", originally introduced by Fechner [6], and to the definition of *semiorders* [15]. *Just Noticeable Differences* arise when different stimuli are judged indifferent from a subject who is not able to discriminate among them. These differences become noticeable using measurement instruments more accurate than a simple subjective evaluation. The homomorphism performed by the single subject will not coincide with the homomorphism performed by means of an adequate measurement system.

4.3. Problems and questions still open

Some fundamental problems arise from ordinal evaluations.

4.3.1. Dimension of representation "compatible" with the way of thinking of subjects

The priority aim of an evaluation consists in yielding empirical predictions, for example, about the purchasing intentions of customers, the undertaking of an investment, etc. Therefore, it is necessary that the provided dimension of representation is compatible with the way of thinking of the subjects, otherwise the evaluations will be neither significant nor useful.

Setting up a good evaluation scale means having to know the dimension of representation which is most important for the subjects. "*Clearly, not all rating scales are compatible with intuitive thought, nor does compatibility imply that rating scales are isomorphic with such thought*" [5].

4.3.2. The design of a scale reflecting the real capacity of discrimination of subjects

This problem mostly arises from rating scales with enumerated categories, where the subject expresses himself on a verbal category and not on a linear continuum. In fact, problems of interpretation of the categories meaning can subsist. A verbal label, "very satisfying", may not have the same meaning for both a very demanding subject and one who is easy to please. This poses a real problem to codify information, being usually unknown the interpretation of the scale adopted by each subject [12].

The response category can be interpreted by subjects as too wide or too narrow. When the category is too wide, the subject is forced to classify as equal objects that he perceives as slightly different. In this case, there are no representational measurements, because a faithful representation of observed relations in numerical relations does not occur: "*it is essential that the relations among the objects of the world be properly reflected by the relations among the numbers assigned to them*" [4].

The opposite problem can also appear. The response categories could be too detailed and the subject is confused in giving an answer. He may find himself in more than one response category.

The adequate definition of the width and of the meaning of each scale category brings evaluations to the stage of homomorphisms from an empirical relational system to a numerical one, that is to say, representational evaluations. This result remains an actionable target only after "*numerous interactions with evaluator subjects*" [7].

4.3.3. The aggregation of evaluations expressed by many subjects

This problem is connected with any subjective operation. The aggregation of individual rankings in a social global ranking is object of study of many disciplines: social and behavioural sciences, operational research and economics [11,13,21].

5. Preference

Preference is the act by which, in presence of two or more possible objects, one of them can be chosen over the other, because it is considered more pleasant, more convenient, more conform to own tastes, interests, ideals, etc.

"*Preference is necessarily relative to a subject. A preference is always somebody's preference. A preference, moreover, is relative not only to a subject but also to a certain moment or occasion or situation in the life of a subjects. Not only may have different people with different preferences, but one and the same man may revise his preferences in the course of his life...the concept of preference is related to the notion of betterness*" [27].
"*Preferences, to a greater or lesser extent, govern decisions... into our axiomatic system an individual's preference relation on a set of alternatives enters as a primitive or a basic notion. This means that we shall not attempt to define preferences in terms of other concepts...preferences between decision alternatives might be characterized in terms of several factors relating to the alternatives*" [10].

When a subject says that he/she prefers alternative a to b, he/she makes a relation between a and b which seems perfectly mouldable with the mathematic notion of a binary relation.

Suppose A is a collection of alternatives among which you are choosing, and suppose

$$P = \{(a, b) \in A \times A: \text{ you(strictly)prefer } a \text{ to } b\}$$

Then P is called (strict) preference relation.

If A is a set of alternatives, aPb holds if and only if you prefer (strictly) a to b, it is possible to assign a real number $u(a)$ to each $a \in A$, such that for all a, $b \in A$

$$aPb \Longleftrightarrow u(a) \succ u(b)$$

The function u is often called ordinal utility function.

This assignment allows the relation "preferred to" of the single subject to be observed.

Nevertheless, as Roberts asserts, "*often, "preferred to" does not define a relation*" (20, pp. 272–273). With these remarks, Roberts wants to underline the absolute peculiarity of this relation that enjoys neither the property of consistency nor that of transitivity.

Consider the following example: a subject is called on to vote among three candidates A, B and C. If the subject prefers alternative A to alternative B and B to C, he/she will not necessarily prefer alternative A to C, as the transitivity property requires. It can occur that C is preferred to A. In the background concerning the problems connected with the aggregation of preferences, the situation just described is noted as Condorcet Paradox [3].

The subject assigning preferences could have an implicit model of preferences such that it can not be mapped on any numerical structure (it can contain intransitivity chains).

The lack of transitivity often arises for intangible relations like "preferred to", "more beautiful than", "more elegant than", etc. ...relations arbitrarily interpretable, because non-directly observable.

5.1. The impossibility of the representational form for preferences

What differences emerge comparing the definition of preference with that of measurement?

In general, we are faced with a preference assignment every time the subject is called on to perform a ranking among things without a "measurement system" or a set of predefined rules, such as in the evaluation process. The preference ranking among alternatives is the result of an endogenous activity of a subject who chooses, in an arbitrary way, how to represent the relation.

Preference becomes "*an arbitrary measurement conceived as a decision-making activity*" [22].

Examples of arbitrarily interpretable relations are: "worthier than", "better than", "more beautiful than", etc.

The assignment of preferences is neither empirical, nor objective. It does not deal with an empirical operation because the subject chooses arbitrarily the relation considered remarkable for the ranking. The above choice is completely endogenous, different from subject to subject. We are not able to understand exogenously what is the dimension of representation selected and followed by the subject and the interpretation he/she has given to it.

There is not a transfer of observable relations into numerical relations. We cannot speak of a homomorphism as defined by the representational point of view. There is no mapping onto a numerical relational system "*preserving both relations (and operations) observed in the empirical relational system*" [20].

Paraphrasing a Stevens' expression, preference can be defined as "*a measurement according to any rule*" [24]. Subjects may have chosen to observe one of the infinite possible relations on the objects in order to perform a preference ranking.

It is obvious that this operation is completely subjective: "*the measurement value is not so much a property of the thing measured as something which expresses an appreciation of the measurer towards the thing itself. What counts, does not count because it counts in itself, but because it is judged to count by someone*" [16].

When subjects report their own opinions about not adequately detailed constructs, an attribution of values to individual preference judgments takes place.

Constructs like the utility of a service, the aesthetic of a product, the guidablity of a vehicle, must be specified and detailed to transform preferences into evaluations. Otherwise, each subject will interpret the construct as he/she considers more convenient. The reason for this is a "*semantic ambiguity*" in the constructs [5]. As an example, consider the responses to the following two attitude questions:

- "*The adoption of a Certified Quality System is a necessary burden*".
- "*Advantages of a Certified Quality System are larger than disadvantages*".

A positive answer to both items seems to indicate a favourable attitude of the management toward quality certification, but how should that response be interpreted and represented? Who answers "yes" has a mildly positive attitude. Those with either a strongly positive or strongly negative attitude would answer "no". In contrast, an affirmative response to the latter item should be interpreted as meaning that the responders' favorability surpasses neutrality, but how far we do not know. "*The point is that they involve choice based on semantic knowledge, our semantic knowledge. There is nothing in the observation of affirmative answers themselves that dictates how they are to be interpreted and represented*" [5].

The presence of this ambiguity is the main difference between preference and evaluation. In an evaluation process we will try to circumscribe this semantic ambiguity, fully defining in this case the concepts of "necessary burden", "advantage" and "disadvantage".

We say that a preference has not the empiricity and objectivity requirements of a measurement. However, the representational theory considers that representational measurement of preference are possible. It is important to concisely define this position. When subjects assign a ranking to the elements of a certain set, they make their own preferences explicit and consequently their own relation "preferred to" on that set is made observable. According to the representational theory, this assignment is a real representational measurement. It is viewed as a homomorphic representation of the relation "preferred to" from the empirical world of the objects into the numerical world (even if the relation is not explicit in the empirical domain).

Roberts [20] presents the case of preferences among classical music composers. The author points out that it is possible to speak of representational measurements of individual preferences when these satisfy the axioms of Cantor's theorem. Given a set of objects and defined a relation of preference P on it, the relational system (A, P) has to satisfy the following conditions:

Asymmetry: if $aPb \Rightarrow \sim bPa$, $\forall a,b \in A$ (natural property of preferences).

Negative transitivity: if $\sim aRb$ & $\sim bRc \Rightarrow \sim aRc$, $\forall a,b,c \in A$.

Axioms of Cantor's theorem are necessary and sufficient conditions to have ordinal representational measurements.

However, enacting Roberts' position, "*some information can be obtained on the evaluating subject, about their way of seeing things, but surely not on the elements of the set, i.e. the empirical world*" [17]. Roberts does not dwell on the analysis of the meaning of the relation but he/she simply analyses and interprets the results of the representation. The author seems to neglect that the relation "preferred to" is not empirical because it is interpretable and therefore observable in an arbitrary way.

The problem is that we do not know what is the meaning of the relation "preferred to", in terms of empirical relations. Therefore, an empirical relational system for the preference cannot be identified.

6. Conclusions

Preference, evaluations and measurements are three basic operations by which it is possible to describe objects. The difference between them are the different way of treating relations among objects.

In the case of preference, relations are thought by subjects and transferred directly to the final ranking under an endogenous decision-making activity, different from subject to subject. The inability to exogenously know the way in which things have been interpreted by subjects leads to the concept of *non-representational measurements* (the representation is unknown).

Measurements set the empirical relations that must be represented without ambiguity by means of a measurement system which realizes the ''*internal consistency check*'' and makes feasible the fundamental requirement of objectivity.

On the other hand, it is the aim of an evaluation process to provide all possible ''tools'' in order to reduce the problems of semantic interpretation of the items and of the scale categories. In this way, individual judgments are treated as representational evaluations.

The notion of homomorphism, defining representational measurements, is effective to mark the difference between preference and measurement, but it does not define a clear border between the operation of measurement and evaluation. The source of the difference between the two operations is that evaluations give space to the possibility of choosing among different dimensions of representation from the outset.

Representational theory allows giving a precise and effective definition of empiricity in the three operations, but not that of objectivity. Some authors have emphasized the inability to incorporate the fundamental requirement of objectivity of a measurement in the formalization of the approach as the limit of the representational point of view. This way, someone claims the necessity of an operational-representational approach for the sake of an exhaustive definition of measurement, with the introduction of a measurement system [17].

Particular attention should be posed to the problem of aggregation of individual evaluations or preferences into ''social'' or group results. Problems of aggregation do not appear in the field of measurements because of their objective nature. On the contrary, an abundant literature about the aggregation of preferences is available. In this framework, renowned for its relevance, remains the Arrow Impossibility Theorem (1951) which represents the milestone of Social Choice Theory. By means of the theory of relations, Arrow proved the non-existence of aggregation mechanisms of individual preferences into a social preference such that four fundamental axioms are satisfied: Unrestrained Domain of Preferences, Independence of irrelevant alternatives, Pareto weakness, and Non-dictatorship [1].

With the aim of tracing a possible path for the future research, the traditional problem of aggregation of individual preferences can be reformulated in the problem of the aggregation of individual evaluations in order to see if the imposition of exogenous bonds relaxes Arrow's conditions. In substance, it is natural wondering if Arrow's Theorem is still valid when the input is changed from preferences into ''canalized'' preferences as evaluations.

Arrow's theorem can be viewed as a second possible instrument of investigation to highlight further the difference between preferences and evaluations.

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