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Customer requirement prioritization on QFD: a new proposal based on the generalized Yager's algorithm

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Abstract The focus of this paper is on the prioritization of customer requirements (CRs) in the quality function deployment (QFD) process. There are numerous techniques for this task, which are based on different types of judgements (e.g., in the form of preference orderings, relative importance ratings, paired comparisons), generally collected by questionnaires/interviews. Unfortunately, many of these techniques violate some properties of the scales on which judgements are defined and/or are not flexible and user-friendly for respondents. This paper introduces a prioritization technique based on a recent algorithm—denominated as generalized Yager's algorithm—aimed at fusing the preference orderings by multiple respondents into a single ordering. This technique can be applied even when (1) some CRs are tied or omitted in the individual preference orderings, and (2) respondents have a hierarchical importance ranking. Also, the procedure is automatable and relatively robust and provides a solution which is consistent with the preference orderings. The description is supported by a realistic application example, concerning the prioritization of QFD's CRs in the design of an aircraft seat.

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1 Introduction and literature review

Quality function deployment (QFD) is a consolidated and powerful tool to improve the design/development process of products and services, in terms of customer satisfaction. The implementation of QFD may generate significant improvements, such as fewer and earlier design changes, improved cross-functional communications, improved product/service quality and reduced development time and cost (Hauser and Clausing 1988; Griffin and Hauser 1993; Tran and Sherif 1995; Franceschini 2002).

The great diffusion of QFD is demonstrated by the literally thousands of scientific publications, illustrating a variety of industrial applications, methodological improvements, new variants and possible integrations with other tools.

Typically, QFD utilizes four sets of matrices—the so-called Houses of Quality (HoQs). The four HoQs, respectively, translate (1) customer requirements (CRs) into engineering characteristics and, in turn, into (2) parts' characteristics, (3) process plans and (4) production requirements (Franceschini 2002). For detailed information, we refer the reader to the vast literature and extensive reviews (Chan and Wua 2002; Sharma et al. 2008).

The customer input, also defined as Voice of the Customer (VoC), is the key starting point for QFD process; if it does not accurately reflect what the customer expects from the product/service of interest, the process may lead to inaccurate forecasts (Sireli et al. 2007). Therefore, the first HoQ, also defined as Product Planning HoQ, is of strategic importance (Gonzalez et al. 2003). The Product Planning HoQ construction process can be summarized into ten phases, as shown in Fig. 1.

Among these phases—described in detail in the literature, e.g., see (Franceschini 2002; Franceschini and Rossetto 2002; Jiao and Chen 2006; Chen and Chen 2014)—particularly significant are those related to the VoC collection and analysis. For this task, it is necessary to select a representative sample of (potential) customers, with reasonable knowledge of the product/service to be designed. The VoC is generally collected through interviews, questionnaires, focus groups or other approaches.

This paper focuses on the prioritization of CRs (i.e., Phase 2, "Relative Importance Ratings"), which presumes that the main CRs related to the product/service of interest have already been identified. The expression "Relative Importance Ratings" indicates that this prioritization is aimed at discriminating a CR based on its importance over the others. Instead, Phase 4, "Final Importance Ratings," denotes a prioritization that combines the relative importance of a CR with its impact on the quality performance of the products/services of the company and its competitors.

There are many approaches for prioritizing the QFD's CRs, including point direct scoring method (Hauser and Clausing 1988; Griffin and Hauser 1993), analytic hierarchy process (AHP) (Li et al. 2009; Chuang 2001), analytic network process (ANP) (Karsak et al. 2002; Lee et al. 2008), outranking methods (Figueira et al. 2005), fuzzy variants (Kwong and Bai 2002; Buyukozkan et al. 2004, 2007; Chen and Huang 2011; Kwong et al. 2011; Yan et al. 2013; Song et al. 2014; Wang 2012) and techniques derived from the Kano model (Matzler and Hinterhuber 1998; Sireli et al. 2007; Chaudha et al. 2011). Without going into these techniques in detail, we remark that they may require different kinds of response data, judgements and/or elaborations from respondents. Even though all these techniques are supposed to reflect the VoC, they may sometimes lead to misleading results, especially when they violate some properties of the scales on which the evaluations concerning CRs are defined. Here are some examples:

- Techniques based on the AHP and ANP method require judgments in the form of paired-comparison data, defined on a *ratio* scale; e.g., considering two CRs (*a* and *b*) "*a* is three times as important as *b*" (Chuang 2001; Franceschini 2002; Kwong and Bai 2002; Lee et al. 2008; Li et al. 2009). These judgements are inevitably arbitrary, subjective and not very intuitive for respondents.
- Techniques that integrate the Kano model in the QFD process need the definition of arbitrary weights related to the (qualitative) Kano categories [i.e., *basic* or *must-be* (B), *one dimensional* (O), *attractive* (A), *indifferent* (I), *reverse* (R) and *questionable* (Q)] (Tan and Shen 2000). This operation implies an improper "promotion" of the *nominal* scale, on which categories are defined, to an *ordinal*, *interval* or even *ratio* scale.

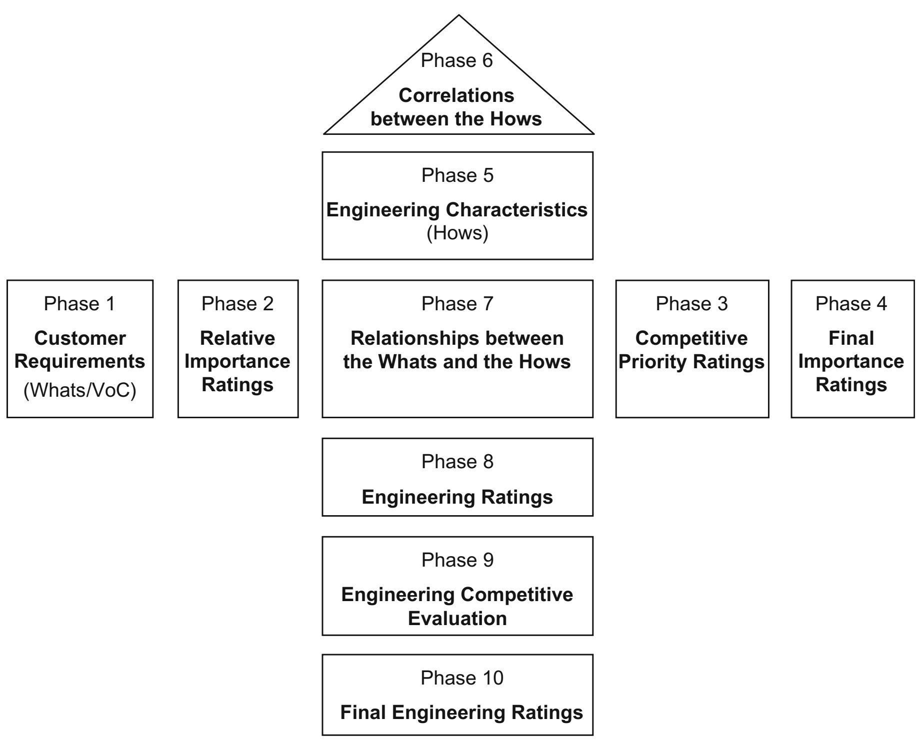

Fig. 1 Main phases of the construction of the Product Planning HoQ

- Other sophisticated techniques, such as that proposed by Nahm et al. (2013), model the uncertainty in CRs, taking into account the uncertainty of customer's judgment. Unfortunately, they require relatively complex elaborations from respondents and include questionable operations in the response-data processing, such as turning rank positions from different preference orderings into numbers and then summing them up together.
- In the classical questionnaires for prioritizing QFD's CRs, respondent judgements are defined on a five-level rating response scale (1 = not at all important, 2 = low importance, 3 = medium importance, 4 = high importance and 5 = very high importance). Since this response scale is an *ordinal* one, it only allows comparisons like "*a* is more important than *b*." Unfortunately, a typical abuse is "promoting" this scale to an *interval* or even *ratio* one. This abuse is implicitly committed when aggregating different preference judgements through their arithmetic average; in fact, this central tendency indicator should be applied only to variables defined on scales, such as *interval* or *ratio* ones, which take into account the "distance" between the objects represented, not to variables defined on *nominal* or *ordinal* scales (Roberts 1979; Burke et al. 2002). Another issue is that these scales are used subjectively as there is no absolute reference shared by all respondents. Consequently, judgments by a severe respondent will tend to be concentrated in the lower levels of the scale, while those by an indulgent respondent in the higher ones. This is another reason why it can be questionable to aggregate judgments by different respondents through indicators of central tendency, such as the mode, median or average value.

A relatively little-discussed problem is that of the ease of comprehension and simplicity of data collection process from the respondent perspective. Methods that require too much elaborate information may become very delicate to manage and lead to inconsistencies in judgment. For example, a typical issue of processes based on paired-comparison data is the violation of the *transitive* property (i.e., if $a > b$ and $b > c$, not necessarily $a > c$—where a, b and c are three CRs and the symbol "$>$" denotes the "strict preference" relationship).

According to the authors, a possible data collection process that represents a good compromise between simplicity and reliability of the input data is that in which each respondent formulates a preference ordering of the CRs. For being flexible and user-friendly for respondents, it seems reasonable that these orderings satisfy the four conditions in Table 1.

Table 1 Conditions concerning the formulation of the preference ordering, in the prioritization of the QFD's CRs

1. Each respondent can have his/her own individual preference ordering over the CRs
2. The preference orderings should include the possibility of ties (i.e., indifference relationship) between two or more CRs
3. The preference orderings should include the possibility of omitting one or more CRs
4. Respondents (can) have a hierarchical importance ordering

Points 3 and 4 deserve special attention. The former comes from the fact that forcing respondents to include all the CRs in their orderings—even when some CRs are not very clear or familiar to them—could lead to inaccurate or arbitrary judgments. Therefore, it seems appropriate to include the possibility of omitting one or more CRs. Regarding the latter point, it is assumed that respondents who are more likely to provide accurate preference orderings could be considered as more important than other ones. This importance could depend on:

- their degree of experience and familiarity with respect to the product/service of interest;
- their level of education;
- their level of attention during the questionnaire/interview; e.g., the attention paid by a respondent in an "street interview" is reasonably lower than that for an online questionnaire.

Hence, it is not so unrealistic to assume that respondents are not necessarily equi-important (from the point of view of the information used in the product/service development stage), but there exists a hierarchical importance ranking; the fact that the majority of the classical approaches to the prioritization of QFD's CRs fail to consider this aspect limits their range of application.

The objective of this paper is to introduce a technique for the prioritization of multi-respondent preference orderings of CRs, which is able to merge them into a single "fused" ordering. Regarding the formulation of the preference orderings, we impose that the technique satisfies the conditions in Table 1. In addition, the fusion mechanism should (1) avoid undue promotions of the (ordinal) relationships among CRs and (2) produce a fused ordering reflecting the majority of the preference orderings. The suggested technique is based on the so-called generalized Yager's algorithm, hereafter abbreviated as GYA (Franceschini et al. 2015). The GYA represents a generalization of an algorithm proposed by Yager (2001) and can be used in different application fields, such as multi-criteria decision processes (Figueira et al. 2005), management of data displayed on Internet sites (Yager 2001) and marketing analysis in the quality engineering field (Griffin and Hauser 1993). The main advantages of the GYA, with respect to the algorithm by Yager are that: (1) The fused ordering better reflects the multi-respondent preference orderings and (2) the GYA admits preference orderings with omitted or incomparable alternatives; for details, see (Franceschini et al. 2015).

The proposed technique can be used when the importance hierarchy of respondents is given by a simple ordering and not a classical set of weights. This method is particularly interesting when it is difficult to define weights or they are not available.

The remainder of this paper is organized into three sections. Section 2 illustrates the characteristics of the input/output data of the GYA. Section 3 presents a practical application of the GYA to the problem of the prioritization of QFD's CRs, in the design of an aircraft seat. The concluding section summarizes the original contributions of this paper, its implications, limitations and possible suggestions for future research. To facilitate the understanding of the paper, the appendix includes a simplified description of the GYA's working principle.

2 Characteristics of the GYA's input/output data

The definition of the problem deserves special attention. Let us assume that there are M respondents (D_1, D_2, …, D_M), each of which defines an ordering of some CRs (a, b, c, etc.). The goal is to fuse the individual respondents' preference orderings into a single "fused" ordering. Not least is the fact that there can be a hierarchical importance ranking of respondents.

Consistent with the conditions in Table 1, the GYA admits preference orderings with ties between two or more CRs and omissions of one or more CRs. In addition to these two conditions, it also admits preference orderings with *incomparabilities*, i.e., there is neither preference nor indifference relationship, for some pairs of CRs (Nederpelt and Kamareddine 2004). These features make the GYA flexible and potentially adaptable to a variety of practical contexts.

In the special context of the prioritization of QFD's CRs, admitting incomparabilities could complicate the task of respondents and/or confuse them. Preference orderings where two generic CRs are comparable—i.e., non-strict *linear* orderings (Nederpelt and Kamareddine 2004)—are probably more practical for respondents. For the purpose of example, Fig. 2 illustrates five non-strict linear orderings—which are diagrammed as lines or chains of elements—by five fictitious respondents. Although there are six total CRs, some of them may be omitted in a certain vector; the number of elements can therefore change from an ith vector to one other.

We remark that these orderings should not be defined on unique scales. Each respondent can have its own individual ordering over the CRs, completely independent of that of another respondent. Linear orderings are also practical because they can be obtained from judgements expressed through a classical five-level rating response scale.

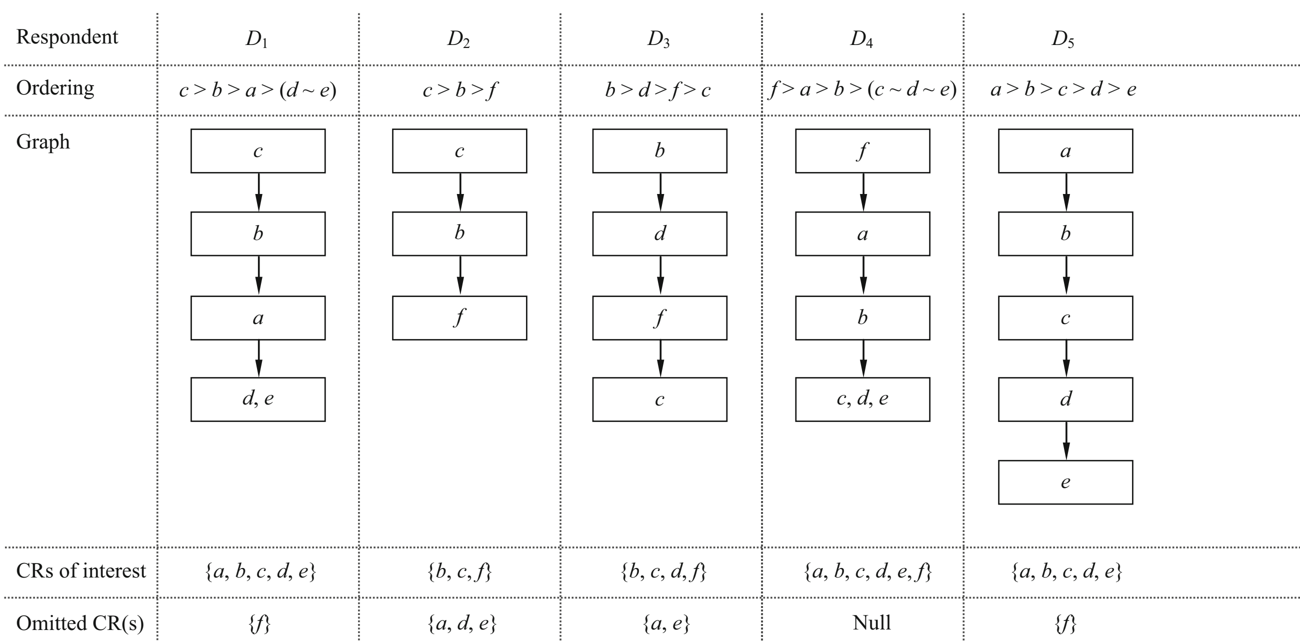

Fig. 2 Graphical representation of the (non-strict linear) preference orderings by five fictitious respondents (D_1–D_5). Symbols "$\sim$" and ">," respectively, mean "indifferent to" and "preferred to." The CRs in the prioritization problem are a, b, c, d, e and f. The respondents' importance ordering is $D_5 > (D_3 \sim D_4) > (D_1 \sim D_2)$

Figure 3 shows an example concerning the judgments by a fictitious respondent. This "transformation" can be practical when the number of CRs is very high, and hence the direct formulation of a preference ordering may be not very intuitive for respondents.

Another peculiar feature of the CR's prioritization problem of interest is that the importance hierarchy of respondents is given by a (non-strict linear) ordering and not defined on an *interval* or *ratio* scale. For example, given two generic respondents, we can identify the most important in relative terms but not their "distance" in terms of importance (Roberts 1979). Thus, contrary to many other problems—e.g., the recent fuzzy-based techniques by Buyukozkan et al. (2007), Kwong et al., (2011), Wang (2012), Iqbal et al. (2014), Song et al. (2014) or Zhong et al. (2014)—a weight depicting the absolute importance of each respondent cannot be defined. For example, considering the five respondents in Fig. 2, their importance ordering is assumed to be: $D_5 > (D_3 \sim D_4) > (D_1 \sim D_2)$.

The decision-making framework of interest can be denominated as "ordinal semi-democratic"; the adjective "semi-democratic" indicates that respondents do not necessarily have the same importance, while "ordinal" indicates that their hierarchy is defined by a crude ordering.

The output of the GYA is a consensus fused ordering of the CRs. Likewise the respondents' preference orderings, this ordering is non-strict linear since it admits the preference and indifference relationships but not the incomparability one between two or more CRs. The adjective "consensus" means that the fused ordering should reflect the majority of the individual preference orderings. The GYA solution is midway between the two extremes of *full dictatorship*—in which the fused ordering coincides with the preference ordering by the most important respondent (dictator)—and *full democracy*—where all respondents' orderings are considered as equi-important. This solution tends to give priority to the preference orderings related to the most important agents. "Description of the GYA" section in "Appendix" contains a simplified description of the GYA's characteristic phases.

3 Application example

This section exemplifies the application of the GYA to the problem of the CRs' prioritization, when designing a new civilian aircraft seat. Sections 3.1 and 3.2 refer to the data collection and analysis, respectively; Sect. 3.3 shows a possible sensitivity analysis and an application of the GYA in the case of full democracy (i.e., equi-important respondents).

3.1 Data collection

Through a market survey, a sample of twenty respondents—i.e., regular air passengers—were selected to identify the CRs by questionnaires/interviews and/or focus groups, in which they were encouraged to describe their needs. The list of requirements gathered in such an exercise were refined and structured by a cross-functional QFD team of experts (i.e., including the marketing, design, quality, technology, production, logistics and supplier function), before its entry into the HoQ. To support this operation, the affinity diagrams and tree diagrams were used (Franceschini and Rossetto 2002; Shahin and Zairi 2009). The resulting structure was then documented in the CR portion of the HoQ matrix: 12 major CRs (reported in Table 2) were identified to represent the major concerns of passengers.

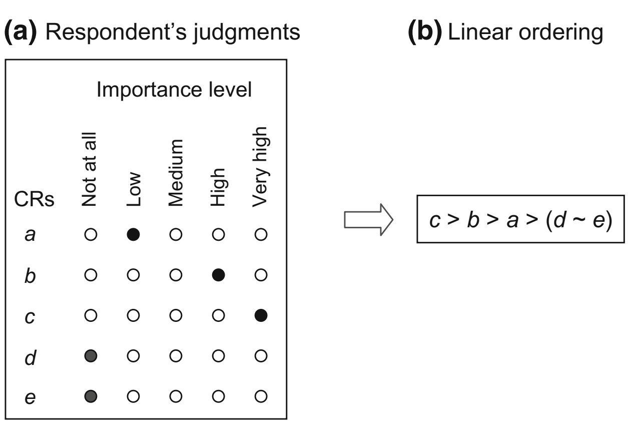

Fig. 3 Example of linear ordering derived from one respondent's judgments, expressed through a five-level rating scale. Symbols "$\sim$" and "$>$," respectively, mean "indifferent to" and "preferred to"

Table 2 List of the major CRs related to an aircraft seat, from the perspective of passengers

Abbr.	Description
a	Comfortable (does not give you back ache)
b	Enough leg room
c	Comfortable when you recline
d	Does not hit person behind when you recline
e	Comfortable seat belt
f	Seat belt feels safe
g	Arm rests not too narrow
h	Arm rest folds right away
i	Does not make you sweat
j	Does not soak up a spilt drink
k	Hole in tray for coffee cup
l	Magazines can be easily removed from rack

Respondents are divided into four classes of importance (i.e., A, B, C and D, in decreasing order), based on two analysis dimensions: (1) the "average frequency of flights" (e.g., one flight every x months) and (2) the "level of education of the respondent" (e.g., secondary school, high school, bachelor, master, doctorate). The QFD team of experts selected these two dimensions, as they may significantly influence the accuracy of the response while being relatively easy to evaluate. The two dimensions can be described through the two-dimensional map in Fig. 4. The most important respondents, in class A, are those with relatively high values in both the analysis dimensions. According to a lexicographic ordering, which favors the former dimension with respect to the latter, the second and third most important classes are, respectively, B and C. The least important respondents, in class D, are those with relatively low values in both the analysis dimensions. Of course, this classification could be based on additional and/or substitute analysis dimensions (e.g., "level of attention of the respondents," "average duration of flights,") or different aggregation criteria.

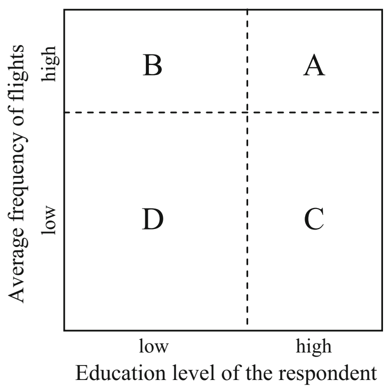

Fig. 4 Qualitative map to discriminate the importance classes (A, B, C and D) of respondents, according to two dimensions of analysis (i.e., "average frequency of flights" and "education level of the respondent")

Next, respondents are required to formulate individual preference orderings of the CRs. The importance class of each respondent and the relevant preference ordering are reported in Table 3. It can be noticed that many of the preference orderings are incomplete because they do not contain the totality of the CRs; the number of elements ($n_i \leq 12$) can therefore change from an ith preference ordering to one other. Coincidentally, the twenty respondents are divided uniformly in the four importance classes (i.e., five respondents per class).

Next, preference orderings are turned into the so-called preference vectors. In this construction, we place the CRs as they appear in one ordering, with the most preferred one(s) in the top positions. Defining t as the number of tied

Table 3 Preference orderings related to the twenty respondents surveyed (D_1–D_{20})

Respondent	Importance class	Preference ordering	Number of CRs (n_i)	Omitted CR (s)
D_1	B	$b > a > (c \sim f) > h > e > (g \sim l) > k > (d \sim i \sim j)$	12	Null
D_2	A	$b > e > (c \sim f \sim g) > (a \sim i) > k$	8	$\{d, h, j, l\}$
D_3	D	$b > (a \sim e \sim g) > f > i > d > c > (j \sim k)$	10	$\{h, l\}$
D_4	B	$b > e > a > g > (c \sim d \sim f) > (h \sim i \sim l) > (j \sim k)$	12	Null
D_5	C	$a > i > f > b > e > (c \sim j) > (h \sim l) > d > k$	11	$\{g\}$
D_6	C	$b > a > (e \sim f \sim g) > i > c > (h \sim k) > l$	10	$\{d, j\}$
D_7	B	$b > g > (a \sim f \sim i) > e > k > (c \sim d) > j$	10	$\{h, l\}$
D_8	A	$b > (a \sim g) > e > (c \sim f) > d > i > k$	9	$\{h, j, l\}$
D_9	B	$e > f > (a \sim g) > (b \sim c \sim d) > h > k > l$	10	$\{i, j\}$
D_{10}	B	$e > b > h > a > f > g > i > c > k > (d \sim j)$	11	$\{l\}$
D_{11}	C	$g > (a \sim b) > c > (e \sim f) > l > d > j > k$	10	$\{h, i\}$
D_{12}	A	$b > (a \sim e \sim f) > i > d > (c \sim g) > (h \sim j \sim k) > l$	12	Null
D_{13}	D	$b > e > (a \sim f) > i > (c \sim g) > j > (h \sim k) > l$	11	$\{d\}$
D_{14}	D	$b > e > g > f > (a \sim c) > (j \sim k) > (d \sim l)$	10	$\{h, i\}$
D_{15}	C	$b > a > c > e > f > g > (d \sim j) > l > k$	10	$\{h, i\}$
D_{16}	D	$(a \sim b) > (c \sim e \sim f) > i > h > l > j > k$	10	$\{d, g\}$
D_{17}	D	$b > a > h > f > (d \sim e) > (c \sim g) > j > k$	10	$\{i, l\}$
D_{18}	A	$b > a > g > (c \sim f) > (d \sim e \sim h) > j > k > l$	11	$\{i\}$
D_{19}	A	$b > e > (f \sim g) > a > (c \sim i) > h > d > k > l$	11	$\{j\}$
D_{20}	C	$e > b > l > i > c > a > (d \sim f) > (h \sim k) > j$	11	$\{g\}$

(i.e., indifferent) CRs at any point, if $t > 1$, we place the tied CRs in the same element and then place the null set ("Null") in the next $t - 1$ lower positions. By adopting this convention, the number of elements of a vector will coincide with the number of CRs.

Table 4 exemplifies the construction of the preference vectors related to the twenty preference orderings in Table 3. For simplicity, they will be denominated as the corresponding respondents (i.e., D_i). Each vector element is associated with a relative-position indicator: $F_{i,j} = j/n_i$, i.e., the cumulative relative frequency referring to a certain vector element (j being the position of an element, starting from the bottom, and n_i being the total number of elements).

3.2 Data analysis

3.2.1 Construction and reorganization of preference vectors

The goal of this phase is to aggregate and reorganize the preference vectors. The vector reorganization stage is based on two steps: (1) D_i vectors are sorted decreasingly with respect to their importance, and (2) those with indifferent importance are aggregated. The aggregation is performed by merging the elements of the vectors with indifferent importance and sorting them in descending order with respect to their $F_{i,j}$ values. When the $F_{i,j}$ values of two (or more) elements coincide, the CRs that they contain are considered as indifferent. For more information on this operation, see "Construction and reorganization of preference vectors" section in "Appendix".

Since respondents are divided into four importance classes, the resulting "reorganized vectors" (D_i^*) will be four, each of which is obtained by aggregating five equi-important preference vectors. The resulting D_i^* vectors are reported in Table 5; each vector element is associated with a corresponding $F_{i,j}^*$ indicator, which contains the information on the relative position of the elements in the original orderings.

3.2.2 Definition of the reading sequence

The reading sequence of the vector elements is determined by applying the algorithm illustrated in "Definition of the reading sequence" section in "Appendix", according to a "top-down" approach, i.e., starting from reading the vector elements in the upper levels and gradually going down. The resulting sequence numbers (S) are shown in Table 5. The total number of steps is 102, which—of course—corresponds to the total number of elements of the reorganized vectors.

3.2.3 Construction of the fused ordering

As illustrated in "Construction of the fused ordering" section in "Appendix", a kth CR is included into the fused ordering when the gradual number of occurrences (O_k) in the reading sequence reaches a certain threshold ($T_{k,x}$), which is a conventional percentage (x) of the total number of occurrences (O_k^{TOT}) in the D_i^* vectors' elements. Table 6 shows the $T_{k,50\%}$ values related to the CRs (x conventionally set to 50 %[1]).

Table 7 provides a visual representation of the step-by-step cumulative count of the occurrences (O_k) of each CR, in the gradual construction of the fused ordering. Each row reports the chronological collection of the sequence numbers (S) related to elements that contain a certain CR. For example, the first two S values related to a are 3 and 4, since they correspond to the first two elements containing a, according to the reading sequence. This representation also includes the information on the $T_{k,x}$ values of each CR: for each kth CR, the italicized sequence numbers denote O_k values included into the corresponding $T_{k,x}$ value. We note that, in the case that the same vector element contains $t > 1$ CRs (even repeated), the corresponding S value will appear in t different cells; e.g., the vector element with $S = 2$ contains three occurrences of b and two of e; therefore, this value is associated with S total cells. On the other hand, the S values corresponding to a "Null" vector element are not reported in any cell.

The scheme in Table 7 allows to determine the fused ordering relatively quickly: Having defined the $T_{k,x}$ thresholds to be used, we (1) identify the cells in which the thresholds relating to each CR are reached and (2) sort the CRs in decreasing order with respect to the S values contained in these cells. The resulting fused ordering is: $b > a > e > g > f > c > i > d > h > k > j > l$.

3.3 Further remarks

3.3.1 Consistency of the fused ordering

In general, the GYA provides a fused ordering which is consistent with the input preference orderings. This can be demonstrated by a simple exercise in which the fused ordering and the preference orderings are compared, at the level of paired comparisons between CRs. For simplicity, we restrict the exercise to the first five CRs (i.e., a, b, e,

[1] In general, it is recommended to use neither too small nor too large values of x: for low values, the fused ordering may not well reflect the preference orderings, while for large ones, the respondents' importance hierarchy may be overlooked. This is the reason why x is conventionally set to 50 %; for details see (Franceschini et al. 2015).

Table 4 Preference vectors related to the twenty orderings in Table 3

D_1		D_2		D_3		D_4		D_5		D_6		D_7		D_8		D_9		D_{10}	
Class B		Class A		Class D		Class B		Class C		Class C		Class B		Class A		Class B		Class B	
$F_{1,j}$	Elem.	$F_{2,j}$	Elem.	$F_{3,j}$	Elem.	$F_{4,j}$	Elem.	$F_{5,j}$	Elem.	$F_{6,j}$	Elem.	$F_{7,j}$	Elem.	$F_{8,j}$	Elem.	$F_{9,j}$	Elem.	$F_{10,j}$	Elem.
1.00	{b}	1.00	{b}	1.00	{b}	1.00	{b}	1.00	{a}	1.00	{b}	1.00	{b}	1.00	{b}	1.00	{e}	1.00	{e}
0.92	{a}	0.88	{e}	0.90	{a, e, g}	0.92	{e}	0.91	{i}	0.90	{a}	0.90	{g}	0.89	{a, g}	0.90	{f}	0.91	{b}
0.83	{c, f}	0.75	{c, f, g}	0.80	Null	0.83	{a}	0.82	{f}	0.80	{e, f, g}	0.80	{a, f, i}	0.78	Null	0.80	{a, g}	0.82	{h}
0.75	Null	0.63	Null	0.70	Null	0.75	{g}	0.73	{b}	0.70	Null	0.70	Null	0.67	{e}	0.70	Null	0.73	{a}
0.67	{h}	0.50	Null	0.60	{f}	0.67	{c, d, f}	0.64	{e}	0.60	Null	0.60	Null	0.56	{c, f}	0.60	{b, c, d}	0.64	{f}
0.58	{e}	0.38	{a, i}	0.50	{i}	0.58	Null	0.55	{c, j}	0.50	{i}	0.50	{e}	0.44	Null	0.50	Null	0.55	{g}
0.50	{g, l}	0.25	Null	0.40	{d}	0.50	Null	0.45	Null	0.40	{c}	0.40	{k}	0.33	{d}	0.40	Null	0.45	{i}
0.42	Null	0.13	{k}	0.30	{c}	0.42	{h, i, l}	0.36	{h, l}	0.30	{h, k}	0.30	{c, d}	0.22	{i}	0.30	{h}	0.36	{c}
0.33	{k}			0.20	{j, k}	0.33	Null	0.27	Null	0.20	Null	0.20	Null	0.11	{k}	0.20	{k}	0.27	{k}
0.25	{d, i, j}			0.10	Null	0.25	Null	0.18	{d}	0.10	{l}	0.10	{j}			0.10	{l}	0.18	{d, j}
0.17	Null					0.17	{j, k}	0.09	{k}									0.09	Null
0.08	Null					0.08	Null												

D_{11}		D_{12}		D_{13}		D_{14}		D_{15}		D_{16}		D_{17}		D_{18}		D_{19}		D_{20}	
Class C		Class A		Class D		Class D		Class C		Class D		Class D		Class A		Class A		Class C	
$F_{11,j}$	Elem.	$F_{12,j}$	Elem.	$F_{13,j}$	Elem.	$F_{14,j}$	Elem.	$F_{15,j}$	Elem.	$F_{16,j}$	Elem.	$F_{17,j}$	Elem.	$F_{18,j}$	Elem.	$F_{19,j}$	Elem.	$F_{20,j}$	Elem.
1.00	{g}	1.00	{b}	1.00	{b}	1.00	{b}	1.00	{b}	1.00	{a, b}	1.00	{b}	1.00	{b}	1.00	{b}	1.00	{e}
0.90	{a, b}	0.92	{a, e, f}	0.91	{e}	0.90	{e}	0.90	{a}	0.90	Null	0.90	{a}	0.91	{a}	0.91	{e}	0.91	{b}
0.80	Null	0.83	Null	0.82	{a, f}	0.80	{g}	0.80	{c}	0.80	{c, e, f}	0.80	{h}	0.82	{g}	0.82	{f, g}	0.82	{l}
0.70	{c}	0.75	Null	0.73	Null	0.70	{f}	0.70	{e}	0.70	Null	0.70	{f}	0.73	{c, f}	0.73	Null	0.73	{i}
0.60	{e, f}	0.67	{i}	0.64	{i}	0.60	{a, c}	0.60	{f}	0.60	Null	0.60	{d, e}	0.64	Null	0.64	{a}	0.64	{c}
0.50	Null	0.58	{d}	0.55	{c, g}	0.50	Null	0.50	{g}	0.50	{i}	0.50	Null	0.55	{d, e, h}	0.55	{c, i}	0.55	{a}
0.40	{l}	0.50	{c, g}	0.45	Null	0.40	{j, k}	0.40	{d, j}	0.40	{h}	0.40	{c, g}	0.45	Null	0.45	Null	0.45	{d, f}
0.30	{d}	0.42	Null	0.36	{j}	0.30	Null	0.30	Null	0.30	{l}	0.30	Null	0.36	Null	0.36	{h}	0.36	Null
0.20	{j}	0.33	{h, j, k}	0.27	{h, k}	0.20	{d, l}	0.20	{l}	0.20	{j}	0.20	{j}	0.27	{j}	0.27	{d}	0.27	{h, k}
0.10	{k}	0.25	Null	0.18	Null	0.10	Null	0.10	{k}	0.10	{k}	0.10	{k}	0.18	{k}	0.18	{k}	0.18	Null
		0.17	Null	0.09	{l}									0.09	{l}	0.09	{l}	0.09	{j}
		0.08	{l}																

Twelve total CRs are considered: *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*, *j*, *k* and *l*

The respondents' importance ordering is $(D_2 \sim D_8 \sim D_{12} \sim D_{18} \sim D_{19}) > (D_1 \sim D_4 \sim D_7 \sim D_9 \sim D_{10}) > (D_5 \sim D_6 \sim D_{11} \sim D_{15} \sim D_{20}) > (D_3 \sim D_{13} \sim D_{14} \sim D_{16} \sim D_{17})$

$F_{i,j} = j/n_i$ is the cumulative relative frequency referring to a certain vector element (j being the position of an element, starting from the bottom, and n_i being the total number of elements) *f* and *g*) from the fused ordering determined in Sect. 3.2.3; in fact, these CRs are the most significant for the Product Planning HoQ. The fused ordering and the preference orderings are turned into paired-comparison relations; for example, from the fused ordering (i.e., $b > a > e > g > f$), we obtain $a < b$, $a > e$, $a > f$, $a > g$, $b > e$, $b > f$, $b > g$, $e > f$, $e > g$ and $f < g$, while from the preference ordering relating to D_1 (i.e., $b > a > f > e > g$), we obtain $a < b$, $a > e$, $a > f$, $a > g$, $b > e$, $b > f$, $b > g$, $e < f$, $e > g$ and $f > g$.

Table 8 shows that, for each paired-comparison (in the first column), the relation obtained from the fused ordering (highlighted in bold) is always the most frequent among those obtained from the preference orderings (frequencies in brackets).

This exercise confirms the consistency of the GYA solution, even in situations—such as that exemplified—in which some CRs are characterized by relatively large fluctuations in the preference orderings (Franceschini et al. 2015).

Table 5 Reorganized vectors (D_i^*) related to the preference vectors (D_i) in Table 4

D_1^* (from $D_2 \sim D_8 \sim D_{12} \sim D_{18} \sim D_{19}$)			D_2^* (from $D_1 \sim D_4 \sim D_7 \sim D_9 \sim D_{10}$)			D_3^* (from $D_5 \sim D_6 \sim D_{11} \sim D_{15} \sim D_{20}$)			D_4^* (from $D_3 \sim D_{13} \sim D_{14} \sim D_{16} \sim D_{17}$)		
A class			B class			C class			D class		
$F_{1,j}^*$	*S*	Elements	$F_{2,j}^*$	*S*	Elements	$F_{3,j}^*$	*S*	Elements	$F_{4,j}^*$	*S*	Elements
1.00	1	{5*b*}[1]	1.00	2	{3*b*, 2*e*}[1]	1.00	3	{*a*, 2*b*, *e*, *g*}[1]	1.00	4	{*a*, 5*b*}[1]
0.92	5	{*a*, *e*, *f*}	0.92	6	{*a*, *e*}	0.91	9	{*b*, *i*}	0.91	10	{*e*}
0.91	7	{*a*, *e*}	0.91	8	{*b*}	0.90	13	{3*a*, *b*}[1]	0.90	14	{2*a*, 2*e*, *g*}[1]
0.89	11	{*a*, *g*}	0.90	12	{*f*, *g*}	0.82	20	{*f*, *l*}	0.82	21	{*a*, *f*}
0.88	15	{*e*}	0.83	17	{*a*, *c*, *f*}	0.80	24	{*c*, *e*, *f*, *g*}	0.80	25	{*c*, *e*, *f*, *g*, *h*}
0.83	16	Null	0.82	19	{*h*}	0.73	30	{*b*, *i*}	0.73	31	Null
0.82	18	{*f*, 2*g*}[1]	0.80	23	{2*a*, *f*, *g*, *i*}[1]	0.70	35	{*c*, *e*}	0.70	36	{2*f*}[1]
0.78	22	Null	0.75	27	{*g*}	0.64	39	{*c*, *e*}	0.64	40	{*i*}
0.75	26	{*c*, *f*, *g*}	0.73	29	{*a*}	0.60	45	{*e*, 2*f*}[1]	0.60	46	{*a*, *c*, *d*, *e*, *f*}
0.73	28	{*c*, *f*}	0.70	33	Null	0.55	50	{*a*, *c*, *j*}	0.55	51	{*c*, *g*}
0.67	32	{*e*, *i*}	0.67	34	{*c*, *d*, *f*, *h*}	0.50	54	{*g*, *i*}	0.50	55	{2*i*}[1]
0.64	37	{*a*}	0.64	38	{*f*}	0.45	58	{*d*, *f*}	0.45	59	Null
0.63	41	Null	0.60	43	{*b*, *c*, *d*}	0.40	65	{*c*, *d*, *j*, *l*}	0.40	66	{*c*, *d*, *g*, *h*, *j*, *k*}
0.58	42	{*d*}	0.58	44	{*e*}	0.36	69	{*h*, *l*}	0.36	70	{*j*}
0.56	47	{*c*, *f*}	0.55	49	{*g*}	0.30	76	{*d*, *h*, *k*}	0.30	78	{*c*, *l*}
0.55	48	{*c*, *d*, *e*, *h*, *i*}	0.50	53	{*e*, *g*, *l*}	0.27	77	{*h*, *k*}	0.27	79	{*h*, *k*}
0.50	52	{*c*, *g*}	0.45	57	{*i*}	0.20	86	{*j*, *l*}	0.20	88	{*d*, 3*j*, *k*, *l*}[1]
0.45	56	Null	0.42	62	{*h*, *i*, *l*}	0.18	87	{*d*}	0.18	89	Null
0.44	60	Null	0.40	64	{*k*}	0.10	97	{2*k*, *l*}[1]	0.10	99	{2*k*}[1]
0.42	61	Null	0.36	68	{*c*}	0.09	98	{*j*, *k*}	0.09	100	{*l*}
0.38	63	{*a*, *i*}	0.33	72	{*k*}						
0.36	67	{*h*}	0.30	74	{*c*, *d*, *h*}						
0.33	71	{*d*, *h*, *j*, *k*}	0.27	75	{*k*}						
0.27	73	{*d*, *j*}	0.25	81	{*d*, *i*, *j*}						
0.25	80	Null	0.20	84	{*k*}						
0.22	82	{*i*}	0.18	85	{*d*, *j*}						
0.18	83	{2*k*}[1]	0.17	91	{*j*, *k*}						
0.17	90	Null	0.10	95	{*j*, *l*}						
0.13	92	{*k*}	0.09	96	Null						
0.11	93	{*k*}	0.08	102	Null						
0.09	94	{2*l*}[1]									
0.08	101	{*l*}									

For each element, the relevant $F_{i,j}^*$ value and the sequence number (*S*) are reported

[1] Coefficients "2," "3," …, "*t*" indicate that the CR has two, three, …, *t* occurrences in that vector element

3.3.2 Sensitivity analysis

The robustness of the fused ordering can be evaluated through a *sensitivity analysis* with respect to small variations in the $T_{k,x}$ values—i.e., setting x to 40, 50 and 60 %, and analyzing the stability of the GYA solution. Table 9a reports the three different groups of $T_{k,x}$ values, while Table 9b the resulting fused orderings.

Of course, as x increases, vectors are read in more "depth," i.e., examining a larger amount of lower-level elements. This mechanism justifies possible differences in the resulting fused orderings.

It can be noticed that despite the use of different $T_{k,x}$ values, fused orderings are very close to each other, especially regarding the upper positions. For example, when switching from $T_{k,50\%}$ to $T_{k,60\%}$, there is just a single rank reversal between j and l. This result confirms the robustness of the GYA (Franceschini et al. 2015). We also notice that fluctuations in the three fused orderings look relatively small if compared with those in the respondents' preference orderings (in Table 3).

3.3.3 Fully democratic case

The GYA can be applied effectively even in the case in which all respondents have the same importance. For the purpose of example, let us consider the same preference orderings in Table 3, under the assumption that respondents are all equi-important. The individual orderings would be merged into a single "reorganized" vector (in Table 10), and the reading sequence of the vector elements would be trivial: i.e., from the top to the bottom. When using $x = 50$ %, the resulting fused ordering would be: $b > a > e > g > f > c > i > d > h > (j \sim k \sim l)$, which is not very dissimilar from that obtained in Sect. 3.2.3.

Table 6 $T_{k,x}$ values for the selection of the CRs, determined setting x to 50 %

CRs	O_k^{TOT}	$T_{k,50\ \%}$
a	20	10.0
b	20	10.0
c	20	10.0
d	16	8.0
e	20	10.0
f	20	10.0
g	17	8.5
h	13	6.5
i	14	7.0
j	15	7.5
k	20	10.0
l	14	7.0

4 Conclusions

This work suggested the implementation of the GYA for the prioritization of QFD's CRs. This algorithm was chosen since it is able to aggregate multi-respondent preference orderings into a single one and is relatively simple and flexible for respondents. In fact, (1) each respondent formulates a non-strict linear preference ordering of the CRs; (2) each ordering may include tied or omitted CRs, and (3) there can be an importance ranking between respondents. Other important strengths of the GYA are that it is automatable and relatively robust.

We remark that the GYA could also be applied in the case preference orderings include incomparabilities between CRs or when judgements are expressed through a classical five-level rating response scale.

Table 7 Visual representation of the step-by-step cumulative count of the occurrences of each CR

CRs	O_k																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
a	*3*	*4*	*5*	*6*	*7*	*11*	*13*	*13*	*13*	*14*	14	17	21	23	23	29	37	46	50	63
b	*1*	*1*	*1*	*1*	*1*	*2*	*2*	*2*	*3*	*3*	4	4	4	4	4	8	9	13	30	43
c	*17*	*24*	*25*	*26*	*28*	*34*	*35*	*39*	*43*	*46*	47	48	50	51	52	65	66	68	74	78
d	*34*	*42*	*43*	*46*	*48*	*58*	*65*	*66*	71	73	74	76	81	85	87	88	–	–	–	–
e	*2*	*2*	*3*	*5*	*6*	*7*	*10*	*14*	*14*	*15*	24	25	32	35	39	44	45	46	48	53
f	*5*	*12*	*17*	*18*	*20*	*21*	*23*	*24*	*25*	*26*	28	34	36	36	38	45	45	46	47	58
g	*3*	*11*	*12*	*14*	*18*	*18*	*23*	*24*	25	26	27	49	51	52	53	54	66	–	–	–
h	*19*	*25*	*34*	*48*	*62*	*66*	67	69	71	74	76	77	79	–	–	–	–	–	–	–
i	*9*	*23*	*30*	*32*	*40*	*48*	*54*	55	55	57	62	63	81	82	–	–	–	–	–	–
j	*50*	*65*	*66*	*70*	*71*	*73*	*81*	85	86	88	88	88	91	95	98	–	–	–	–	–
k	*64*	*66*	*71*	*72*	*75*	*76*	*77*	*79*	*83*	*83*	84	88	91	92	93	97	97	98	99	99
l	*20*	*53*	*62*	*65*	*69*	*78*	*86*	88	94	94	95	97	100	101	–	–	–	–	–	–

For each kth CR, the italicized sequence numbers denote O_k values included into the corresponding $T_{k,x}$ value (x was conventionally set to 50 %)

Table 8 Comparison between the fused ordering and the preference orderings by respondents, at the level of paired comparisons

Paired comparison	Preference orderings (total no., respondents and importance class)							
	>		<		∼		N/A	
a, b	(2)	A: Null	**(16)**	**A: D_8, D_2, D_{12}, D_{18}, D_{19}**	(2)	A: Null	(0)	A: Null
		B: D_9		**B: D_1, D_4, D_7, D_{10}**		B: Null		B: Null
		C: D_5		**C: D_6, D_{15}, D_{20}**		C: D_{11}		C: Null
		D: Null		**D: D_3, D_{13}, D_{14}, D_{17}**		D: D_{16}		D: Null
a, e	**(10)**	**A: D_8, D_{18}**	(8)	A: D_2, D_{19}	(2)	A: D_{12}	(0)	A: Null
		B: D_1, D_7		B: D_4, D_9, D_{10}		B: Null		B: Null
		C: D_5, D_6, D_{11}, D_{15}		C: D_{20}		C: Null		C: Null
		D: D_{16}, D_{17}		D: D_{13}, D_{14}		D: D_3		D: Null
a, f	**(13)**	**A: D_8, D_{18}**	(4)	A: D_2, D_{19}	(3)	A: D_{12}	(0)	A: Null
		B: D_1, D_4, D_{10}		B: D_9		B: D_7		B: Null
		C: D_5, D_6, D_{11}, D_{15}, D_{20}		C: Null		C: Null		C: Null
		D: D_3, D_{16}, D_{17}		D: D_{14}		D: D_{13}		D: Null
a, g	**(9)**	**A: D_{12}, D_{18}**	(5)	A: D_2, D_{19}	(3)	A: D_8	(3)	A: Null
		B: D_1, D_4, D_{10}		B: D_7		B: D_9		B: Null
		C: D_6, D_{15}		C: D_{11}		C: Null		C: D_5, D_{20}
		D: D_{13}, D_{17}		D: D_{14}		D: D_3		D: D_{16}
b, e	**(17)**	**A: D_2, D_8, D_{12}, D_{18}, D_{19}**	(3)	A: Null	(0)	A: Null	(0)	A: Null
		B: D_1, D_4, D_7		B: D_9, D_{10}		B: Null		B: Null
		C: D_5, D_6, D_{11}, D_{15}		C: D_{20}		C: Null		C: Null
		D: D_3, D_{13}, D_{14}, D_{16}, D_{17}		D: Null		D: Null		D: Null
b, f	**(19)**	**A: D_2, D_8, D_{12}, D_{18}, D_{19}**	(1)	A: Null	(0)	A: Null	(0)	A: Null
		B: D_1, D_4, D_7, D_{10}		B: D_9		B: Null		B: Null
		C: D_5, D_6, D_{11}, D_{15}, D_{20}		C: Null		C: Null		C: Null
		D: D_3, D_{13}, D_{14}, D_{16}, D_{17}		D: Null		D: Null		D: Null
b, g	**(15)**	**A: D_2, D_8, D_{12}, D_{18}, D_{19}**	(2)	A: Null	(0)	A: Null	(3)	A: Null
		B: D_1, D_4, D_7, D_{10}		B: D_9		B: Null		B: Null
		C: D_6, D_{15}		C: D_{11}		C: Null		C: D_5, D_{20}
		D: D_3, D_{13}, D_{14}, D_{17}		D: Null		D: Null		D: D_{16}
e, f	**(11)**	**A: D_2, D_8, D_{19}**	(5)	A: D_{18}	(4)	A: D_{12}	(0)	A: Null
		B: D_4, D_9, D_{10}		B: D_1, D_7		B: Null		B: Null
		C: D_{15}, D_{20}		C: D_5		C: D_6, D_{11}		C: Null
		D: D_3, D_{13}, D_{14}		D: D_{17}		D: D_{16}		D: Null
e, g	**(11)**	**A: D_2, D_{12}, D_{19}**	(4)	A: D_8, D_{18}	(2)	A: Null	(3)	A: Null
		B: D_1, D_4, D_9, D_{10}		B: D_7		B: Null		B: Null
		C: D_{15}		C: D_{11}		C: D_6		C: D_5, D_{20}
		D: D_{13}, D_{14}, D_{17}		D: Null		D: D_3		D: D_{16}
f, g	(7)	A: D_{12}	**(7)**	**A: D_8, D_{18}**	(3)	A: D_2, D_{19}	(3)	A: Null
		B: D_1, D_9, D_{10}		**B: D_4, D_7**		B: Null		B: Null
		C: D_{15}		**C: D_{11}**		C: D_6		C: D_5, D_{20}
		D: D_{13}, D_{17}		**D: D_3, D_{14}**		D: Null		D: D_{16}

Symbols ">," "<," and " ∼ " denote, respectively, the strict preference, reverse strict preference, and indifference; "N/A" means that the paired-comparison relationship is not defined since (at least) one of the two CRs of interest is omitted. For each relation, we report the preference orderings from which it can be obtained and the relevant importance classes (i.e., A, B, C and D). In brackets we report the number of preference orderings associated with the relationship of interest. The paired-comparison relationships obtained from the fused ordering are highlighted in bold

Table 9 Data concerning sensitivity analysis

CRs	O_k^{TOT}	$T_{k,40\ \%}$	$T_{k,50\ \%}$	$T_{k,60\ \%}$
a	20	8.0	10.0	12.0
b	20	8.0	10.0	12.0
c	20	8.0	10.0	12.0
d	16	6.4	8.0	9.6
e	20	8.0	10.0	12.0
f	20	8.0	10.0	12.0
g	17	6.8	8.5	10.2
h	13	5.2	6.5	7.8
i	14	5.6	7.0	8.4
j	15	6.0	7.5	9.0
k	20	8.0	10.0	12.0
l	14	5.6	7.0	8.4

Threshold	Fused ordering
Relevant fused orderings	
$T_{k,40\ \%}$	$b > a > e > g > f > c > i > d > h > j > k > l$
$T_{k,50\ \%}$	$b > a > e > g > f > c > i > d > h > k > j > l$
$T_{k,60\ \%}$	$b > a > e > g > f > c > i > h > d > j > (k \sim l)$

Table 10 Single reorganized vector, obtained by aggregating the totality of the preference vectors in Table 4, assuming that they are equi-important

$F^*_{1,j}$	*S*	Elements	$F^*_{1,j}$	*S*	Elements
1.00	1	{2*a*, 15*b*, 3*e*, *g*}[1]	0.50	21	{*c*, *e*, 3*g*, 3*i*, *l*}[1]
0.92	2	{2*a*, 2*e*, *f*}[1]	0.45	22	{*d*, *f*, *i*}
0.91	3	{*a*, 2*b*, 2*e*, *i*}[1]	0.44	23	Null
0.90	4	{5*a*, *b*, 2*e*, *f*, 2*g*}[1]	0.42	24	{*h*, *i*, *l*}
0.89	5	{*a*, *g*}	0.40	25	{2*c*, 2*d*, *g*, *h*, 2*j*, 2*k*, *l*}[1]
0.88	6	{*e*}	0.38	26	{*a*, *i*}
0.83	7	{*a*, *c*, *f*}	0.36	27	{*c*, 2*h*, *j*, *l*}[1]
0.82	8	{*a*, 3*f*, 2*g*, *h*, *l*}[1]	0.33	28	{*d*, *h*, *j*, 2*k*}[1]
0.80	9	{2*a*, 2*c*, 2*e*, 3*f*, 3*g*, *h*, *i*}[1]	0.30	29	{2*c*, 2*d*, 2*h*, *k*, *l*}[1]
0.78	10	Null	0.27	30	{*d*, 2*h*, *j*, 3*k*}[1]
0.75	11	{*c*, *f*, 2*g*}[1]	0.25	31	{*d*, *i*, *j*}
0.73	12	{*a*, *b*, *c*, *f*, *i*}	0.22	32	{*i*}
0.70	13	{*c*, *e*, 2*f*}[1]	0.20	33	{*d*, 4*j*, 2*k*, 2*l*}[1]
0.67	14	{*c*, *d*, *e*, *f*, *h*, *i*}	0.18	34	{2*d*, *j*, 2*k*}[1]
0.64	15	{*a*, *c*, *e*, *f*, *i*}	0.17	35	{*j*, *k*}
0.63	16	Null	0.13	36	{*k*}
0.60	17	{*a*, *b*, 2*c*, 2*d*, 2*e*, 3*f*}[1]	0.11	37	{*k*}
0.58	18	{*d*, *e*}	0.10	38	{*j*, 4*k*, 2*l*}[1]
0.56	19	{*c*, *f*}	0.09	39	{*j*, *k*, 3*l*}[1]
0.55	20	{*a*, 3*c*, *d*, *e*, 2*g*, *h*, *j*, *i*}[1]	0.08	40	{*l*}

[1] To facilitate the visualization, the vector is split into two columns

The limitations of the proposed technique are those of the GYA; e.g., potentially questionable conventions for (1) merging equi-important vectors and (2) reading vector elements.

This technique represents the first step of an ambitious research project of building a "fully ordinal" Product Planning HoQ, in which—using ordinal input data exclusively (e.g., concerning the CRs' preference orderings or the ordinal relationships between the Whats and the Hows)—the engineering characteristics of the product/service can be prioritized without "promotions" or violations of the scale properties of the data of interest (Franceschini et al. 2007).

Appendix

Description of the GYA

This section briefly describes the GYA. For a more detailed explanation, we refer the reader to (Franceschini et al. 2015).

In general, the GYA is aimed at fusing the preference orderings of *n* alternatives (*a*, *b*, *c*, etc.), by multiple decision-making agents[2] (D_1, D_2, etc.), into a consensus fused orderings. In the problem of the prioritization of QFD's CRs, decision-making agents would be the questionnaire/interview respondents and alternatives would be the CRs. For simplicity, in the remaining explanation, we will refer to this specific problem.

The GYA will be explained, assuming that the input preference orderings are non-strict *linear* (i.e., with no incomparabilities between CRs); however, this algorithm can be applied even in the more general case in which orderings are non-strict *partial*, i.e., when two or more CRs are incomparable (Franceschini et al. 2015).

The GYA description can be structured in three phases, described in the remaining subsections: (1) construction and reorganization of preference vectors, (2) definition of the reading sequence and (3) construction of the fused ordering.

Construction and reorganization of preference vectors

The goal of this phase is constructing preference vectors related to the input preference orderings. For the purpose of example, let us consider the same example presented in Fig. 2, about five fictitious respondents ($D_1 \sim D_5$) with five (non-strict linear) orderings of six CRs (*a*, *b*, *c*, *d*, *e* and *f*). It can be noticed that the only ordering including all the

[2] By a decision-making agent we can consider any of a wide variety of different types of entities. Examples could be human beings, individual criteria in a multi-criteria decision process or software based intelligent agents on the Internet.

Table 11 Construction of preference vectors related to the orderings in Fig. 2

Respondents	D_1			D_2			D_3			D_4			D_5		
Orderings	$c > b > a > (d \sim e)$			$c > b > f$			$b > d > f > c$			$f > a > b > (c \sim d \sim e)$			$a > b > c > d > e$		
Number of CRs (n_i)	5			3			4			6			5		
Omitted CR(s)	$\{f\}$			$\{a, d, e\}$			$\{a, e\}$			Null			$\{f\}$		
Preference vectors	j	$F_{1,j}$	Elem.	j	$F_{2,j}$	Elem.	j	$F_{3,j}$	Elem.	j	$F_{4,j}$	Elem.	j	$F_{5,j}$	Elem.
	5	$5/5 = 1.00$	$\{c\}$	3	$3/3 = 1.00$	$\{c\}$	4	$4/4 = 1.00$	$\{b\}$	6	$6/6 = 1.00$	$\{f\}$	5	$5/5 = 1.00$	$\{a\}$
	4	$4/5 = 0.80$	$\{b\}$	2	$2/3 = 0.67$	$\{b\}$	3	$3/4 = 0.75$	$\{d\}$	5	$5/6 = 0.83$	$\{a\}$	4	$4/5 = 0.80$	$\{b\}$
	3	$3/5 = 0.60$	$\{a\}$	1	$1/3 = 0.33$	$\{f\}$	2	$2/4 = 0.50$	$\{f\}$	4	$4/6 = 0.67$	$\{b\}$	3	$3/5 = 0.60$	$\{c\}$
	2	$2/5 = 0.40$	$\{d, e\}$				1	$1/4 = 0.25$	$\{c\}$	3	$3/6 = 0.50$	$\{c, d, e\}$	2	$2/5 = 0.40$	$\{d\}$
	1	$1/5 = 0.20$	Null							2	$2/6 = 0.33$	Null	1	$1/5 = 0.20$	$\{e\}$
										1	$1/6 = 0.17$	Null			

Six total CRs are considered: a, b, c, d, e and f

The respondents' importance ordering is $D_5 > (D_3 \sim D_4) > (D_1 \sim D_2)$

j denotes the position of an element, starting from the bottom

$F_{i,j}$ is the cumulative relative frequency referring to a certain vector element

Table 12 Construction of reorganized vectors related to the preference vectors in Table 11

D_1^* (from D_5)			D_2^* (from $D_3 \sim D_4$)			D_3^* (from $D_1 \sim D_2$)		
$F_{1,j}^*$	S	Elem.	$F_{2,j}^*$	S	Elem.	$F_{3,j}^*$	S	Elem.
1.00	1	$\{a\}$	1.00	2	$\{b, f\}$	1.00	3	$\{2c\}$[1]
0.80	4	$\{b\}$	0.83	5	$\{a\}$	0.80	9	$\{b\}$
0.60	6	$\{c\}$	0.75	7	$\{d\}$	0.67	10	$\{b\}$
0.40	11	$\{d\}$	0.67	8	$\{b\}$	0.60	13	$\{a\}$
0.20	14	$\{e\}$	0.50	12	$\{c, d, e, f\}$	0.40	16	$\{d, e\}$
			0.33	15	Null	0.33	17	$\{f\}$
			0.25	18	$\{c\}$	0.20	20	Null
			0.17	19	Null			

S is the resulting sequence numbers, obtained by applying the logic illustrated in Fig. 5a. The elements of D_3 and D_4 are merged into the vector D_2^*, while those of D_1 and D_2 into D_3^*

[1] Coefficient "2" indicates that the CR has two occurrences in the vector element

six CRs is that by D_4. In the other ones, one or more CRs are systematically omitted. It is assumed that the non-strict linear ordering depicting the importance hierarchy between respondents is: $D_5 > (D_3 \sim D_4) > (D_1 \sim D_2)$.

The preference orderings should be turned into preference vectors. In this construction, we place the CRs as they appear in one ordering, with the most preferred one(s) in the top positions. If at any point $t > 1$ CRs are tied (i.e., indifferent), we place them in the same element and then place the null set ("Null") in the next $t - 1$ lower positions. For example, when considering three CRs (a, b and c) with the ordering $(a \sim b) > c$, the resulting vector will conventionally be $[\{a \sim b\}, \text{Null}, \{c\}]^{\mathrm{T}}$. By adopting this convention, the number of elements of a vector will coincide with the number of CRs.

Table 11 exemplifies the construction of the preference vectors related to the five preference orderings in Fig. 2. For simplicity, they will be denominated as the corresponding respondents (i.e., D_i). Although there are six total CRs, some of them may be omitted in a certain vector; therefore, the number of elements (n_i) can change from an ith vector to one other. It can be seen that each vector element is associated with a relative-position indicator, given by the cumulative relative frequency $F_{i,j}$—i.e., the ratio between the position (j) of an element—starting from the bottom—and n_i.

Next, preference vectors are transformed into so-called reorganized vectors, conventionally denominated as D_i^*. The vector reorganization stage is based on two steps: (1) D_i vectors are sorted decreasingly with respect to their importance, and (2) those with indifferent importance are aggregated. For example, D_1 and D_2 vectors have indifferent importance, so they are aggregated; the same applies to D_3 and D_4.

The aggregation is performed by merging the elements of the vectors to be aggregated and sorting them in descending order with respect to their $F_{i,j}$ values. When the $F_{i,j}$ values of two (or more) elements coincide, the CRs that they contain are considered as indifferent. After this reorganization, the resulting vectors are conventionally denominated as D_i^*, while the $F_{i,j}$ values that they include are denominated as $F_{i,j}^*$ (see Table 12). It can be noted that D_2^* and D_3^*—given by the aggregation of two pairs of vectors with indifferent importance (respectively, D_3, D_4 and D_1, D_2)—may contain two occurrences of some CRs.

We remark that the vector aggregation is performed by using the information on the relative position of the elements (i.e., $F_{i,j}$). The underlying assumption is that the degree of preference of the CRs in different preference vectors depends on their relative position. For a certain aggregated vector, the relevant $F_{i,j}^*$ values which contain the information on the relative position of the elements in the original orderings.

Definition of the reading sequence

The object of this phase is determining a sequence for reading the elements of the reorganized vectors. We remark that (1) these elements can be read according to a *bottom-up* or *top-down* sequence and that (2) the importance of D_i^* vectors should be taken into account in this phase.

As regards the CRs' prioritization problem, the top-down approach is probably more appropriate since it is focused on the upper positions of the preference orderings. For this reason, the remaining description of the GYA will refer to this approach. The flowchart in Fig. 5a illustrates the algorithm for constructing the fused ordering.

Let us now focus on the criterion for switching from one element to one other. The first element to be read is that with highest position, in the most important vector (D_1^*). Having read a certain vector element, the next potentially readable D_i^* vectors are those for which the *not-yet-read* element with highest position has $F_{i,j}^*$ higher than or equal to that of the last element *read* in the preceding vector (i.e., D_{i-1}^*).

Reversing the perspective, a D_i^* vector is temporarily "locked" (i.e., it cannot be read) if the $F_{i,j}^*$ value of the not-yet-read element with highest position is overcome by that of the last element read in the preceding vector. The set A includes the subscripts of the potentially readable (or "unlocked") vectors. In formal terms:

$$A = \{i \in \{2, \ldots, m\}\} : F^*_{i,\max(\text{j:not-yet-read})} \geq F^*_{i-1,\min(\text{j:read})}. \quad (1)$$

Among the vectors indicated in A, the one to be read is that with subscript:

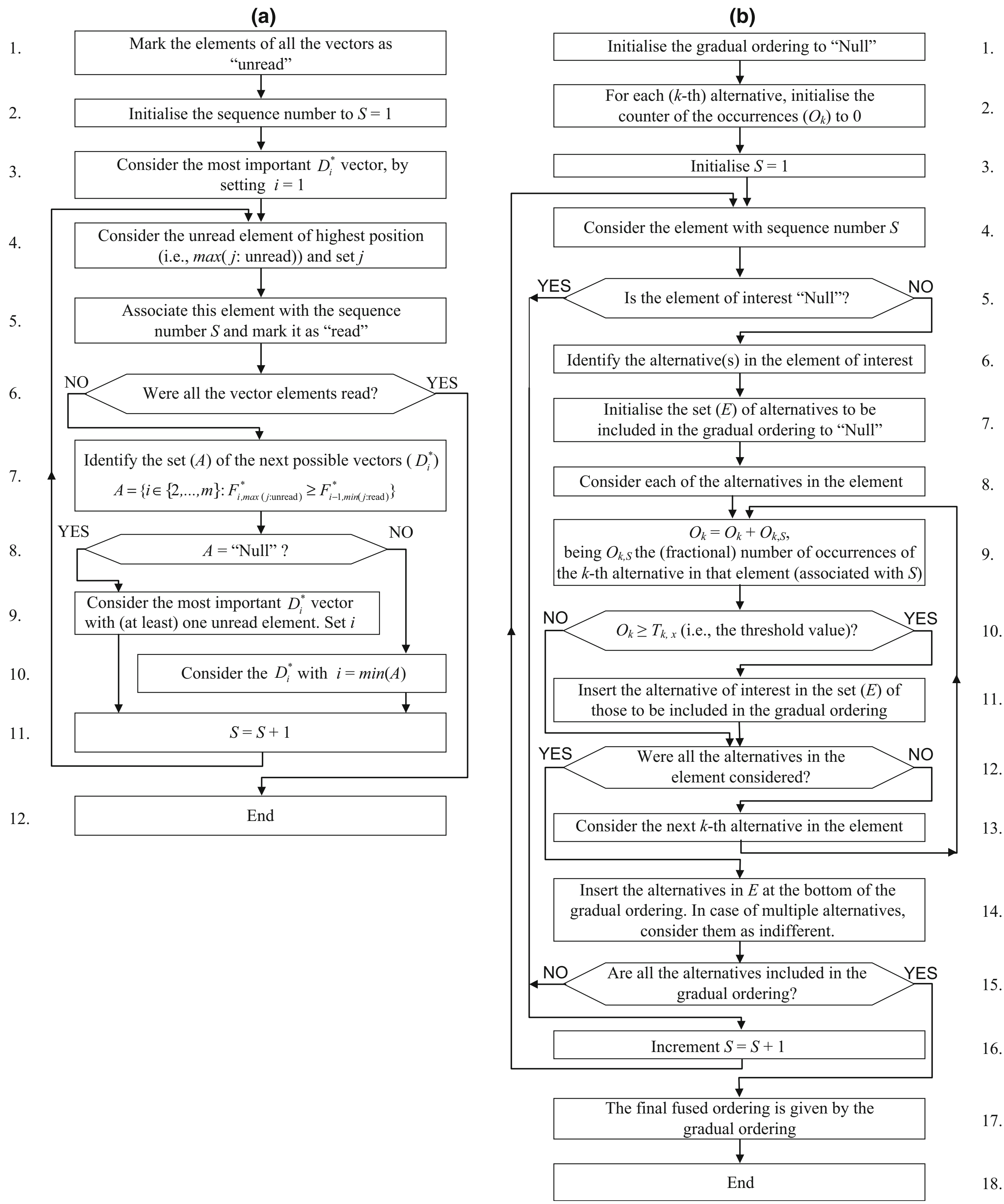

Fig. 5 Flowcharts illustrating the second and third phase of the *top-down* variant of the GYA: (a) definition of the sequence for reading the elements of the reorganized vectors and (b) procedure for constructing the fused ordering

Table 13 Thresholds for the selection of the CRs, in the case exemplified in Table 11; x is conventionally set to 50 %

CRs	a	b	c	d	e	f
Total number of occurrences O_k^{TOT}	3	5	5	4	3	3
$T_{k,50\%}$	1.5	2.5	2.5	2	1.5	1.5

Table 14 Step-by-step application of the GYA, in the case exemplified in Table 11

Step (S)	A	Vector selected	Selectable CR (s)	Occurrences (O_k)						Residual CRs	Gradual ordering
				a	b	c	d	e	f		
0	–	–	Null	–	–	–	–	–	–	$\{a, b, c, d, e, f\}$	Null
1	Null	1	$\{a\}$	1	0	0	0	0	0	$\{a, b, c, d, e, f\}$	Null
2	{2, 3}	2	$\{b, f\}$	1	1	0	0	0	1	$\{a, b, c, d, e, f\}$	Null
3	{3}	3	$\{2c\}$	1	1	2	0	0	1	$\{a, b, c, d, e, f\}$	Null
4	Null	1	$\{b\}$	1	2	2	0	0	1	$\{a, b, c, d, e, f\}$	Null
5	{2}	2	$\{a\}$	2	2	2	0	0	1	$\{b, c, d, e, f\}$	a
6	Null	1	$\{c\}$	2	2	3	0	0	1	$\{b, d, e, f\}$	$a > c$
7	{2}	2	$\{d\}$	2	2	3	1	0	1	$\{b, d, e, f\}$	$a > c$
8	{2, 3}	2	$\{b\}$	2	3	3	1	0	1	$\{d, e, f\}$	$a > c > b$
9	{3}	3	$\{b\}$	2	4	3	1	0	1	$\{d, e, f\}$	$a > c > b$
10	{3}	3	$\{b\}$	2	5	3	1	0	1	$\{d, e, f\}$	$a > c > b$
11	Null	1	$\{d\}$	2	5	3	2	0	1	$\{e, f\}$	$a > c > b > d$
12	{2}	2	$\{c, d, e, f\}$	2	5	4	3	1	2	$\{e\}$	$a > c > b > d > f$
13	{3}	3	$\{a\}$	3	5	4	3	1	2	$\{e\}$	$a > c > b > d > f$
14	Null	1	$\{e\}$	3	5	4	3	2	2	Null	$a > c > b > d > f > e$
End	–	–	Null	–	–	–	–	–	–	–	–

The first three columns are related to the reading sequence: The first contains the sequence number (S), the second indicates the potentially selectable vectors, and the third reports the subscript of the vector selected. The subsequent columns refer to the construction of the gradual ordering. We remark that a CR is added to the gradual ordering when the cumulative number of occurrences (O_k) reaches $T_{k,x}$ (see the numeric values in Table 13)

$$i = \min(A). \tag{2}$$

Equation 2 entails that, among the "unlocked" vectors, priority is given to the one of highest importance. Having determined the vector to be read, the next element is the one not-yet-read with highest position. If there is no unlocked vector (i.e., A = "Null"), the next element is that (not-yet-read) with highest position in the most important not-yet-completely-read vector.

For further information about this sequencing logic and the rationale behind it, we refer the reader to (Franceschini et al. 2015).

Construction of the fused ordering

The flowchart in Fig. 5b illustrates the procedure for determining the fused ordering. A kth CR is included into the fused ordering when the gradual number of occurrences (O_k) in the reading sequence reaches a certain threshold, i.e.,:

$$T_{k,x} = x \cdot O_k^{\mathrm{TOT}}, \tag{3}$$

being x a conventional percentage of the total number of occurrences (O_k^{TOT}) in the D_i^* vectors' elements. Table 13 shows the $T_{k,x}$ values related to the CRs; x was conventionally set to 50 %.

It is worth noting that for a CR to be in the upper positions of the fused ordering, a predetermined portion of the occurrences larger than or equal to (x) should be in the upper positions in any of the individual preference orderings.

Applying the algorithm to the reorganized vectors in Table 12 and using the thresholds in Table 13, the fused preference ordering is: $a > c > b > d > f > e$. Table 14 shows the step-by-step construction. The last columns contain the gradual ordering.

The robustness of the solution can be evaluated through a *sensitivity analysis* aimed at evaluating the stability of the fused ordering with respect to small variations in the $T_{k,x}$ values.

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