

---

## **A taxonomy of model-based redundancy methods for CMM online performance verification**

---

Fiorenzo Franceschini\* and Maurizio Galetto

Department of Manufacturing Systems and Economics  
Polytechnic Institute of Turin  
Corso Duca degli Abruzzi, 24 – 10129 Torino, Italy  
E-mail: fiorenzo.franceschini@polito.it  
E-mail: maurizio.galetto@polito.it  
\*Corresponding author

**Abstract:** This paper presents a taxonomy of methods for online performance verification of three-axes Coordinates Measuring Machines (CMMs). The attention is focused on analytical redundancy methods, which are based on the use of mathematical models to describe physical relationships between measured quantities. A general framework for the application of the analytical redundancy methods is presented. A series of examples, together with an analysis of advantages and disadvantages of the applied methods, is described. In the end, one of these methods is experimentally tested and the obtained results are discussed.

**Keywords:** Coordinates Measuring Machines (CMM); performance verification; diagnostics; measurement reliability.

**Reference** to this paper should be made as follows: Franceschini, F. and Galetto, M. (2007) 'A taxonomy of model-based redundancy methods for CMM online performance verification', *Int. J. Technology Management*, Vol. 37, Nos. 1/2, pp.104–124.

**Biographical notes:** Fiorenzo Franceschini is Professor of Quality Management at the Department of Manufacturing Systems and Economics of Polytechnic Institute of Turin, Italy. He is author or co-author of three books and many published papers in scientific journals, and international conference proceedings. His current research interests are in the areas of quality engineering, QFD and quality management. He is a member of the editorial board of *Quality Engineering*, and *International Journal of Quality and Reliability Management*. He is a Senior Member of ASQ.

Maurizio Galetto is Assistant Professor at the Department of Manufacturing Systems and Economics at Polytechnic Institute of Turin. He holds a PhD in Metrology from Polytechnic Institute of Turin, Italy. His current research interests are in the areas of quality management and statistical process control.

## 1 Introduction

The growing demand for quality control in production processes, caused by more and more accurate mechanical machining, brought about a continuous and increasing diffusion of Coordinate Measuring Machines (CMMs) in manufacturing environments. CMMs represent a basic ring in the production chain of advanced manufacturing systems. From a functional point of view, they are often integrated in modern production cells for firm automation.

CMMs are able to measure, in a short time and in a structured way, a large number of geometrical characteristics of complex mechanical components (Bosch, 1995; Cauchick-Miguel *et al.*, 1996). This results in lower inspection costs and increased productivity.

The main features of a CMM are:

- high metrological performances (low uncertainty, high accuracy, traceability to international standards, high repeatability and reproducibility in standard conditions)
- high-speed of measurement
- user-friendliness
- measurement automation (basic requirement for quality control in large series production)
- integration with specific CAD, CAM and GD&T software.

Several different causes may affect CMM metrological performances over time, including variations of environmental factors, deterioration of some CMM subsystems and other uncontrollable effects (Bosch, 1995; Cauchick-Miguel *et al.*, 1996; Franceschini *et al.*, 2002; ISO 10360–2, 2001). The option for an online evaluation of this deterioration is particularly interesting for CMM users. It would allow a timely correction of anomalies, limiting scraps and low-quality production. Furthermore, keeping CMM metrological characteristics under online control, for ISO 9000-certified companies, is a mandatory requirement (ISO 10012–2, 1997).

A rising number of methods and standards denotes the interest towards these issues (de Aquino Silva and Burdekin, 2002; Savio *et al.*, 2002). Cauchick-Miguel *et al.* (1996) report an interesting survey about CMM offline verification techniques. They can be classified into three groups: kinematic reference standard techniques, parametric calibration techniques (or synthesis methods), and transfer standard techniques. These practices alternatively show different advantages and disadvantages in terms of time consumption, ease of use and management, diagnosis of errors, time stability and application costs (de Aquino Silva and Burdekin, 2002; Cauchick-Miguel *et al.*, 1996; ISO 10360–2, 2001; Savio *et al.*, 2002).

Furthermore, it is worth noting that a CMM can typically be subject to three types of verifications (ISO 10360–2, 2001):

- 1 the initial verification or acceptance test (the acceptance test is normally long, complex and expensive)
- 2 periodic verifications (verifications must be brief, simple to perform and low-cost)
- 3 the irregular/occasional controls.

The typical elements of such verifications are the use of more or less complex and costly artefacts, the use of experienced and qualified personnel, and the need to operate offline when the machine does not work. Furthermore, all these verifications allow the detection of a possible damage-state only at the time when they are carried out. They do not allow establishing the moment when such a damage occurred, nor the causes.

Therefore, it is evident that there is a need for a method that, placed side by side with the above verification strategies, should be able to automatically display the occurrence of deterioration in the machine performances, or in the environment where the CMM works.

The present paper aims to give an analytical description of CMM online performance verification techniques, which may not make use of external artefacts and require only a few additional extra measurements.

In existing literature, some seminal papers have proposed online techniques for the verification of CMM performance. However, until today, standard methodologies for an online verification of CMM performances and, in general, for 'intelligent' measurement systems, are still lacking (Franceschini *et al.*, 2002).

An initial result of online approaches is the possibility to indicate the need for a more accurate test or even for a complete calibration (a very expensive procedure) only when this is really necessary. A second aspect concerns the 'guarantee' that the dimensions of the measured part are really those declared by the instrument.

There are two main novelties of the present research: a taxonomic approach for the analysis of the CMM online verification methods; and the introduction of a series of online statistical control techniques to monitor CMM metrological performance deterioration.

The paper is organised into three sections:

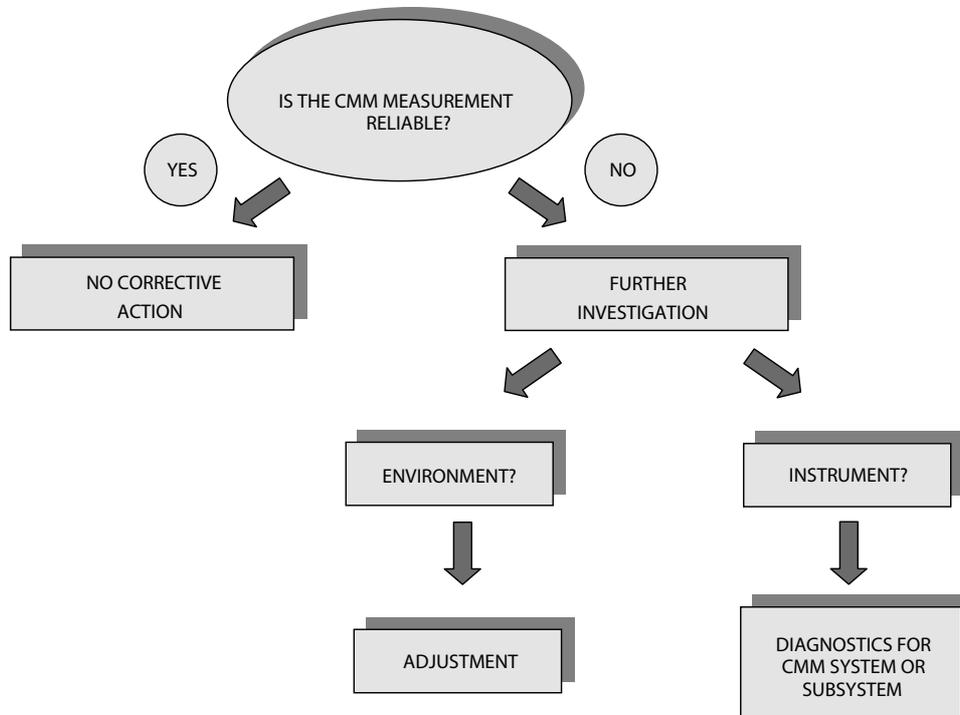
- 1 A definition of the 'self-diagnostic' concept for the online performance verification of automated measurement systems.
- 2 The introduction of a set of criteria for a taxonomy of the main methodologies adopted for an online CMM performance verification.
- 3 An experimental investigation of a specific online method based on the concept of 'analytical redundancy'.

## **2 Online metrological performance verification of a CMM**

The concept of 'online metrological performance verification' is strictly related to the notion of 'online self-diagnostics' in traditional automated systems (Gertler, 1998). This approach is complementary to that of uncertainty evaluation (ISO/TS 15530-6, 2000; Savio *et al.*, 2002). The online verification is a guarantee for the preservation of the measurement system characteristics (including accuracy reproducibility, and uncertainty).

Techniques for 'online metrological performance verification' allow individuating anomalous CMM behaviours, and automatically reconfiguring the control system to assure the preservation of metrological characteristics.

The effect of a CMM degradation is the production of non-reliable measurements. The indication of possible faults can be followed by more accurate tests or by a complete calibration (see Figure 1).

**Figure 1** General schematisation of the approach for CMM online performance verification

The problem of ‘self-diagnostics’ in automated systems is not a recent matter, and traditionally many strategies have been proposed (Clarke, 1995; Henry and Clarke, 1992; Isermann, 1984). In many critical sectors, such as the aeronautical and nuclear ones, the most used techniques are based on the replication of the instrumentation and the control equipments. This approach is usually very expensive.

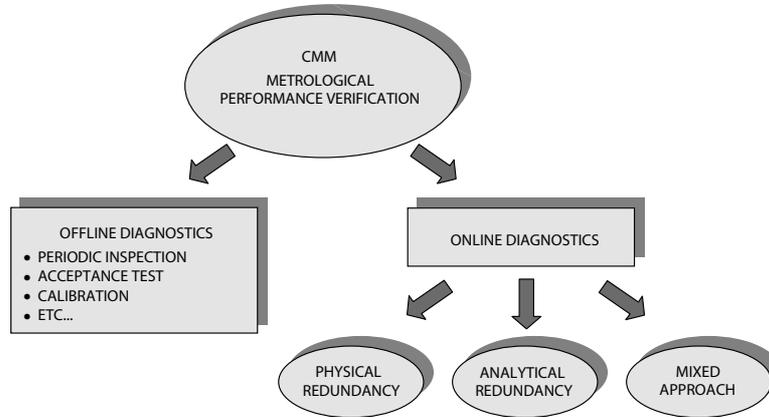
In general, we distinguish two main types of redundancy (see Figure 2):

- 1 The ‘physical redundancy’ (Gertler, 1998), which consists of replicating the instrumentation and the control equipment (for example, by integrating the touch probe with an optical device or by creating a redundancy of the probe itself).
- 2 The ‘analytical’ or ‘model-based redundancy’ (Frank, 1990; Isermann, 1984; 1993), which substitutes the replication of a physical instrumentation by the use of appropriate mathematical models. These latter may derive from physical laws applied to experimental data or from self-learning method (for example, neural networks).

Sometimes the two approaches are employed together. In this case, the analytical redundancy method is enhanced with the use of an external ‘witness-part’ (Franceschini *et al.*, 2002).

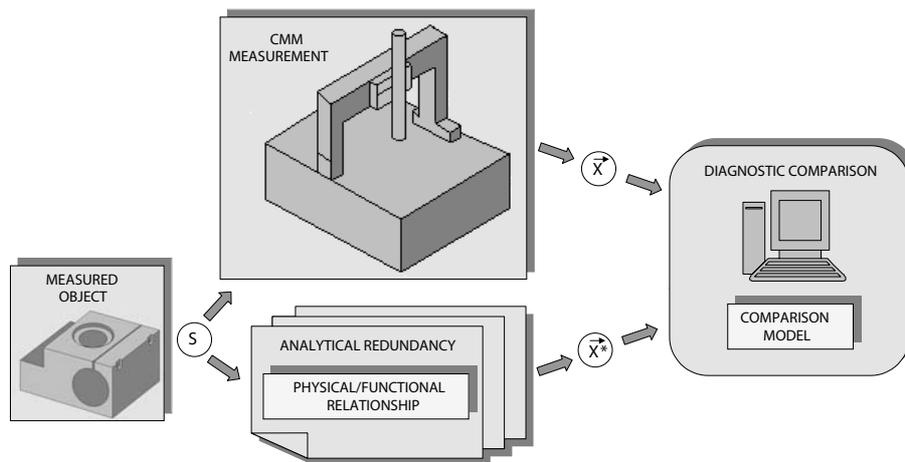
Offline CMM verification approaches also make use of both analytical and physical redundancy. This is the case of acceptance test, when a comparison is performed by an external reference gauge or by using specific analytical models for compensation of the influence parameters (Cauchick-Miguel *et al.*, 1996).

**Figure 2** Summary of different approaches for CMM metrological performance verification



Referring to online methods, a simplified scheme of the analytical redundancy approach for CMM online self-diagnostics is shown in Figure 3.

**Figure 3** Simplified scheme of the analytical redundancy approach for CMM self-diagnostics



Three different entities can be defined:

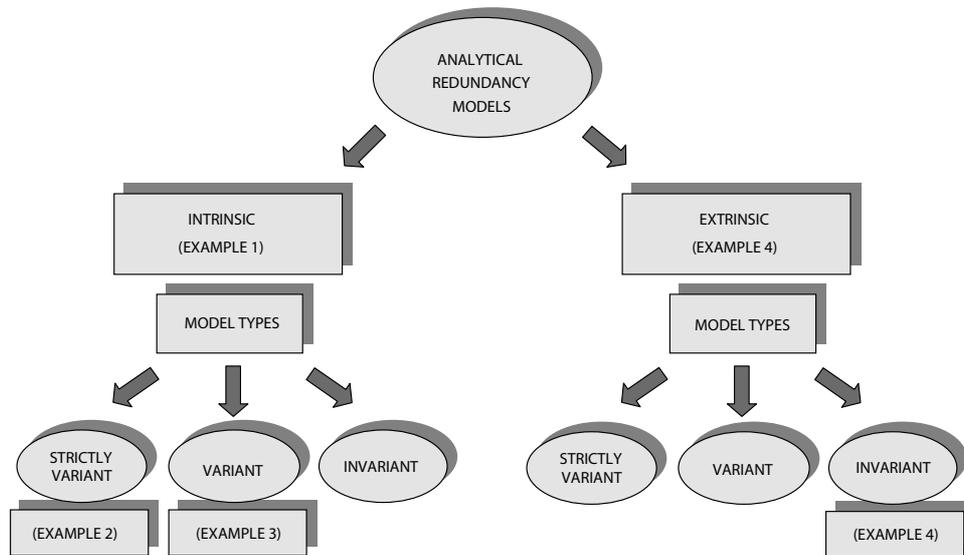
- 1 the physical stimulus  $S$  (CMM probe contact)
- 2 the measured variable  $\bar{X}$  (i.e., the coordinates of the contact point)
- 3 the ‘complementary’ of the measured variable  $\bar{X}^*$ , elaborated by an analytical redundancy model.

The comparison between the expected value  $\bar{X}^*$  and the experimental one  $\bar{X}$  provides an estimation of CMM performances. Many different typologies of redundancy models can be utilised. All of these may produce more or less advantageous results according to the characteristics of the process at hand (Reznik and Solopchenko, 1985).

### 3 Analytical redundancy methods

In this section, a set of criteria for classification of CMM analytical redundancy methods is proposed. The physical significance of the analytical model and the diagnostic ability to individuate possible ‘incongruence’ in the machine performance are the two basic aspects considered in the analysis. Figure 4 shows the schematic structure of the proposed classification.

**Figure 4** Scheme of the structure suggested for the taxonomy of CMM analytical redundancy models



#### 3.1 Relationship classification

A mathematical redundancy model is based on physical relationships among measured quantities (Reznik and Solopchenko, 1985). We define *intrinsic* as those relationships yielded by specific characteristics of the measurement process at hand.

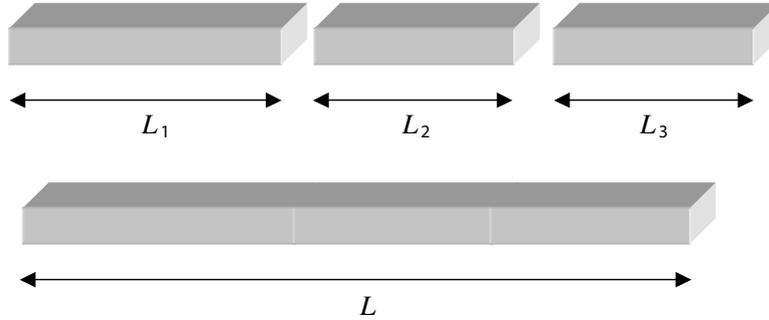
**Example 1**

Consider the measurement of  $n$  linear components obtained by the segmentation of an original metallic bar whose length  $L$  is known (see Figure 5). The relationship among the  $n$  elements is the following:

$$L_1 + L_2 + \dots + L_n = L$$

where  $L_1, L_2, \dots, L_n$  are the measurements of each single element.

The presence of faults in the measurement process may cause some incongruence in the above relationship. The model can be utilised for a diagnostic application.

**Figure 5** Simplified scheme of the measurement of three linear components obtained by the segmentation of a metallic bar

On the other hand, a relationship is *extrinsic* (or *fundamental*) if it can be directly obtained from the basic laws of physics or mathematics. For example, this is the case of the sum of the internal angles of a polygon. The sum of internal angles must be equal to  $\pi \cdot (m - 2)$  rad ( $m$  is the number of polygon angles). If this sum does not satisfy this condition (within the uncertainty limits), a perturbation in the measurement process has occurred.

### 3.2 Model classification

In general, given a set of  $n$  points  $M \equiv \{P_1, P_2, \dots, P_n\}$ , measured by a three-axes CMM,  $P_1 \equiv (X_1, Y_1, Z_1)$ ,  $P_2 \equiv (X_2, Y_2, Z_2), \dots, P_n \equiv (X_n, Y_n, Z_n)$ , three types of mathematical models can be individuated:

- 1 *strictly-variant* model ( $f_{SV}(M)$ ), defined as:

$$f_{SV}(P_1, P_2, \dots, P_n) = K_{SV}$$

where  $K_{SV}$  is a parameter depending on the set of points  $M \equiv \{P_1, P_2, \dots, P_n\}$ .

$f_{SV}(P'_1, P'_2, \dots, P'_n) \neq f_{SV}(P_1, P_2, \dots, P_n)$  if there is at least one different point between the two sets.

#### Example 2

Consider a measurement cycle of a product. The CMM performance verification can be performed by monitoring the behaviour of repeated measurements of a set of specific points over time (Franceschini *et al.*, 2002).

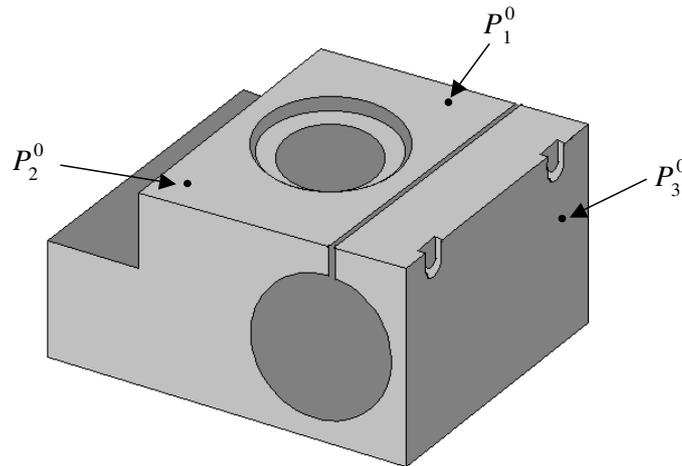
Let us define as  $\{P_1^k(X_1, Y_1, Z_1), P_2^k(X_2, Y_2, Z_2), \dots, P_t^k(X_t, Y_t, Z_t)\}$  the  $k$ -th replication of the coordinates of a specific nominal point set  $\{P_1^0(X_1, Y_1, Z_1), P_2^0(X_2, Y_2, Z_2), \dots, P_t^0(X_t, Y_t, Z_t)\}$ , carried out under a changed path of the touch-probe and different workpieces with same nominal shape (see Figure 6). The measurement reproducibility of a CMM is a multivariate random variable defined as follows:

$$\mathbf{D}^{(k)} = \{d(P_1^0, P_1^k), d(P_2^0, P_2^k), \dots, d(P_t^0, P_t^k)\} \quad \forall k = 1 \dots t$$

where  $d(P^0, P^k)$  is the Euclidean distance operator between the reference point and its  $k$ -th reproduction in the working volume,  $l$  the number of measured points, and  $t$  the total number of replications.

If during CMM operation, one or more influencing factors affect the measurement process, the measurement process would not be able to yield 'credible' information (Oksman, 1993). CMM yields unreliable measurements when reproducibility measurements manifest a variation out of its natural tolerance. The continuous observation of  $D^{(k)}$  can allow monitoring of the performances of the whole CMM/environment/part/subsystems with respect to some reference conditions. Useful tools able to monitor, at the same time, the central tendency and the dispersion of each component of the random variables  $D^{(k)}$  are the process  $\bar{X}-R$  control charts (Montgomery, 2001; Franceschini *et al.*, 2002).

**Figure 6** Example of CMM performance verification performed by monitoring the behaviour of repeated measurement points over time.  $\{P_1^0(X_1, Y_1, Z_1), P_2^0(X_2, Y_2, Z_2), P_3^0(X_3, Y_3, Z_3)\}$  is the nominal point set of the sample surface



2 variant model ( $f_V(M)$ ), defined as:

$$f_V(P_1, P_2, \dots, P_n) = K_V$$

where  $K_V$  is a parameter not univocally depending on the set of points

$$M \equiv \{P_1, P_2, \dots, P_n\}.$$

**Example 3**

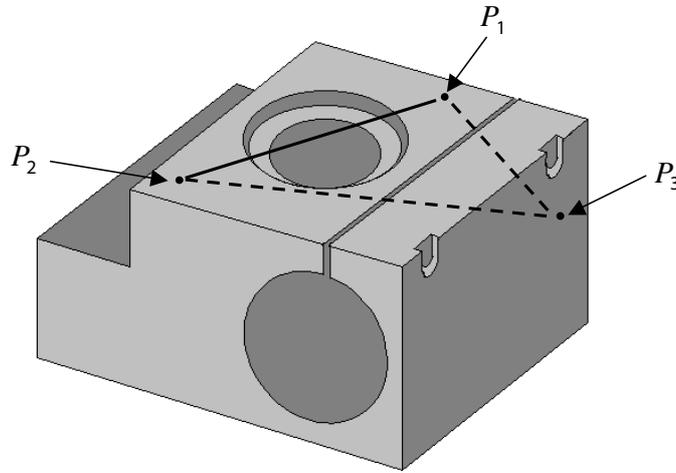
Let us consider the difference between the areas of triangles provided by the measure replications of three different points of an object (see Figure 7):

$$\Delta A^{(k)} = A^k(P_1^k, P_2^k, P_3^k) - A^0(P_1^0, P_2^0, P_3^0) \quad \forall k = 1 \dots t$$

where the notation  $A^k(P_1^k, P_2^k, P_3^k)$  indicates the area of a triangle defined by points  $P_1^k, P_2^k, P_3^k$ ,  $A^0(P_1^0, P_2^0, P_3^0)$  the reference area defined by three points  $P_1^0, P_2^0, P_3^0$  and  $t$  the number of replications.

The logic is the same as the previous case. A variation on the  $k$ -th replication gives a variation on  $\Delta A^{(k)}$ . The model is variant: a contemporary rigid roto-translation of the three points cannot be detected ( $\Delta A^{(k)} = 0$ ).

**Figure 7** Example of CMM performance verification performed by monitoring the difference between the areas of triangles defined by three repeated points



3 *invariant* model ( $f_I(M)$ ), defined as:

$$f_I(P_1, P_2, \dots, P_n) = K_I$$

where  $K_I$  is a constant that does not depend on the considered set of points  $M \equiv \{P_1, P_2, \dots, P_n\}$ .

**Example 4**

Referring again to Example 2, one could be tempted to use as a reference model the sum of internal angles of a triangle:

$$\Theta = \hat{P}_1^k + \hat{P}_2^k + \hat{P}_3^k \quad \forall k = 1 \dots t$$

where the notation  $\hat{P}_i^k$  denotes the angle having as a vertex  $P_i^k$ , and  $t$  the number of replications.

This is an extrinsic relationship. The nominal result of the sum of internal angles is  $\pi$  rad independently by the  $k$ -th replication. It is an invariant model. It gives no indication about the CMM performance behaviour. It is evident that invariant models, being insensitive to model variable variations, are useless for online CMM metrological performance verification.

Strictly variant models are the most appropriate for online performance verification. They provide necessary and sufficient conditions to detect the appearance of some ‘incongruence’ in the measurement process.

In the design phase of an online CMM performance verification method, it is important to define the more appropriate model to utilise according to the typology of measures and the measuring cycle at hand. It must be highlighted that, in principle, it is not possible to define a universal model with general validity, but specific models should be individuated case by case, according to the measurement cycle at hand.

Some models are preferable to others because they are easier to apply or are able to amplify the occurrence of perturbation effects. This is very helpful in cases of small variations of monitored variables.

### 3.3 Test detection and discrimination

For a given measurement system, it is necessary to define the ability of ‘detection’ of any specific test. That means the capacity of detecting the various sources of perturbation.

To do this, the so called ‘residual matrix’  $\mathbf{S}$  (in literature also indicated as  $\mathbf{R}$ ) is utilised (Gertler, 1998). It helps to formally define the presence of a correlation between the used mathematical relationship and the possible sources of perturbation in the measurement process.

In the case of three-axes CMMs, the residual matrix is composed of  $n$  rows, which represent the  $n$  possible measurement faults, and one column for each test response. A generic element  $r_{ij}$  is one, if the test is sensible to the measurement  $i$ -fault, otherwise it is zero.

For example, referring to Example 2, the corresponding residual matrix (regarding fault contributions due to rotations and translations) is given in Table 1.

**Table 1** ‘Residual matrix’ for the online performance evaluation of a three axes CMM using the analytic approach described in Example 2

<i>Measurement fault</i>	<i>Reproducibility test</i>
Translation along $X$ -axis	1
Translation along $Y$ -axis	1
Translation along $Z$ -axis	1
Rigid rotations	1

Another basic property of a diagnosis model is its ability to isolate each cause of incorrect behaviour (‘discrimination’ property).

The case described in Example 2 (see Table 1) represents a model that is not able to discriminate the source of perturbation. When the test shows the appearance of a fault, the model is not able to distinguish if the cause is due to a translation along one of the three axes or to a rotation.

#### **Example 5**

The same approach as that for Example 2 can be applied to the difference between the repeated values and the nominal one of each single coordinate. In this case, the model individuates and isolates each perturbation effect acting on the three coordinates  $X$ ,  $Y$ , or  $Z$ .

Further techniques for improving the discrimination ability of a model can be based on mixed approaches, *i.e.*, on the use of both analytical redundancy and some external references (for example, a built-in reference gauge or a previously characterised workpiece). For these approaches the online performances verification is more expensive in terms of equipment, number of measured points and process time. On the other hand, the use of an external reference assures a better robustness in terms of accuracy and traceability of the reference.

### Example 6

Let us again consider the approach described in Example 2. With the aim of discriminating each single variability contribution, a series of reproducibility tests on an external witness-part (reference gauge) is also executed during the measurement cycle (Franceschini *et al.*, 2002). As a result, we can obtain a separate estimation of two main contributions to the random variables  $D^{(k)}$ :

- The component strictly related to the machine reproducibility, that is, the composition of the contributions because of the CMM subsystems, to the touched point position in the working volume of the machine, to the environmental conditions, *etc.*
- The component related to the measured object, that is, the composition of the variability because of its geometry, to the reference system alignment, *etc.*

This approach does not allow discrimination if the causes of non-reproducibility are due to translations along some of the three axes ( $X, Y, Z$ ) or to some rigid rotation, *etc.*

An overall comparison of all the examined examples is summarised in Table 2.

**Table 2** Overall comparison of the methods introduced by Examples 1 to 6

<i>Example</i>	<i>Extra measures</i>	<i>External reference</i>	<i>Detection</i>	<i>Discrimination</i>	<i>Model typology</i>	<i>General application</i>
1	No	No	Yes	No	VAR.	No
2	Yes	No	Yes	No	STR.-VAR.	Yes
3	Yes	No	No	No	VAR.	Yes
4	Yes	No	No	No	INVAR.	Yes
5	Yes	No	Yes	Yes	STR.-VAR.	Yes
6	Yes	Yes	Yes	No	STR.-VAR.	Yes

Notes: Comparison criteria are the following: need of extra measures, need of an external reference, detection ability (regarding fault contributions due to rotations and translations), discrimination property, model typology (invariant, variant and strictly variant), and possibility of a general application of the method in every kind of CMM measurement.

The reported cases, which represent only a sample of the innumerable different applications of the online approaches, prove that in some situations the performance verification can be carried out by using only the measurement data, with no additional measures or external artefacts (see Example 1). In other situations, extra measurements (see Examples 2 to 6) or even external references (see Example 6) may be needed.

#### 4 A procedure for CMM online performance verification

The taxonomic analysis of self-diagnostics methods for online CMM verification shows that the measurement design phase is basilar. The organisation of a procedure for online evaluation of CMM performances should be based on the following steps:

- 1 Identification of the most appropriate analytical redundancy model. This action requires a deep knowledge of the measurement process and of the involved physical phenomena.
- 2 Use of the residual matrices in order to define the types of information utilised and transmitted by the model. This will show what perturbations can be evidenced and discriminated.
- 3 Setting up of the monitoring parameters (number and typology of points to be probed, numbers of replications, frequency of monitoring, *etc.*).
- 4 Analysis of results (statistical tests, frequency diagrams, comparison matrices, control charts, *etc.*).

#### 5 Experimental investigation

In order to give a practical example of an online CMM performance verification, an application case based on Example 6 will be described.

Let us define  $s$  as the total number of measurement points collected on the surface of a workpiece for its geometrical characterisation, and  $n$  as a random subset of  $s$  over which to carry out the reproducibility tests (see Example 6). These latter tests are the group of measurement points used to set up a set of  $\bar{X} - R$  control charts (one for each point). In order to individuate position effects, these points are uniformly distributed over the measurement surface.

The frequency of verifications and the number  $n$  are established on the bases of the part's complexity and measurement costs. A simplified model for the computation of the test frequency can be found in Franceschini and Settineri (2000).

##### 5.1 Definition of variables

Let us define the reproducibility for measurements along  $X$ -axis as follows:

$$\mathbf{D}_X^{(k)} = \{X_1^k - X_1^0, X_2^k - X_2^0, \dots, X_j^k - X_j^0, \dots, X_l^k - X_l^0\} \quad \forall k = 1 \dots t$$

where  $X_j^k$  is the  $X$  coordinate value of the  $k$ -th replication of the  $j$ -th point, and  $X_j^0$  is its nominal value.  $\mathbf{D}_X^{(k)}$  is a multivariate random variable on  $\mathbb{R}^l$ .

Analogous variables ( $\mathbf{D}_Y^{(k)}$  and  $\mathbf{D}_Z^{(k)}$ ) may be defined for  $Y$  and  $Z$  axes.

In the absence of perturbation sources for a conventional measurement cycle,  $\mathbf{D}_X^{(k)}$ ,  $\mathbf{D}_Y^{(k)}$  and  $\mathbf{D}_Z^{(k)}$  are normally multi-varied distributed with  $\sigma_{R_X^{(k)}}^2 = \sigma_X^2$ ,  $\mu_{R_X^{(k)}} = [0]$ ,  $\sigma_{R_Y^{(k)}}^2 = \sigma_Y^2$ ,  $\mu_{R_Y^{(k)}} = [0]$ ,  $\sigma_{R_Z^{(k)}}^2 = \sigma_Z^2$ , and  $\mu_{R_Z^{(k)}} = [0]$  where  $[0]$  is an all zeros matrix. Values of vectors  $\sigma_X$ ,  $\sigma_Y$  and  $\sigma_Z$  depend on the structural configuration of the machine, the part geometry, and the environmental operating conditions.

Five factors influence  $D_X^{(k)}$ ,  $D_Y^{(k)}$  and  $D_Z^{(k)}$  variables:

- 1 the object geometry ( $G_X^{(k)}$ ,  $G_Y^{(k)}$  and  $G_Z^{(k)}$ )
- 2 the CMM subsystems ( $M_X^{(k)}$ ,  $M_Y^{(k)}$  and  $M_Z^{(k)}$ )
- 3 the reference system alignment on the measured object ( $A_X^{(k)}$ ,  $A_Y^{(k)}$  and  $A_Z^{(k)}$ )
- 4 the environmental conditions affecting the measured object ( $C_X^{(k)}$ ,  $C_Y^{(k)}$  and  $C_Z^{(k)}$ )
- 5 the measurement point position in the working volume of the machine and the different types of contacts between probe and object surface ( $P_X^{(k)}$ ,  $P_Y^{(k)}$  and  $P_Z^{(k)}$ ).

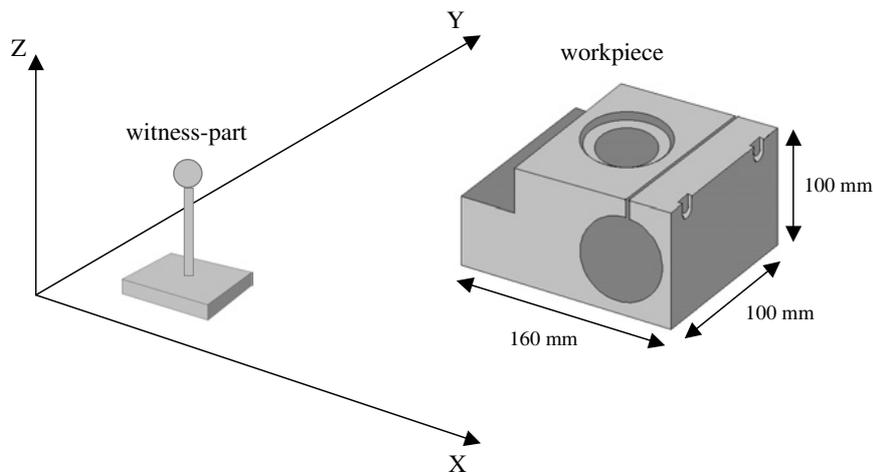
The same contribution holds for variable  $D^{(k)}$  (see Example 2).  $D^{(k)}$  distribution is not normal, it cannot assume negative values. Anyway it may be demonstrated that, as a first approximation, normality assumption (with  $\sigma_{R^{(k)}}^2 = \sigma$  and  $\mu_{R^{(k)}} = \mu$ ) produces results not far from the right distribution (Franceschini *et al.*, 2002).

Under the hypothesis of negligible position effects (that means working in a limited measurement space), some of these contributions can be separately estimated by using different data analysis strategies. This has been tested during a preliminary series of experiments, which show that the effects, owing to the location of the measurement points in a bounded volume (*i.e.*,  $200 \times 200 \times 200$  mm), are negligible if compared to other contributions to the overall reproducibility.

## 5.2 Experiment description

Tests have been carried out by the CMM in the laboratory of the Department of Manufacturing Systems and Economics at Polytechnic Institute of Turin (producer: DEA, model: IOTA 0101, standard motorised version). The machine has a moving bridge structure, with a measuring volume up to 555 mm (X-axis), 610 mm (Y-axis), 410 mm (Z-axis). Figure 8 shows the workpiece utilised for the investigation.

**Figure 8** Location of the measured workpiece and the external witness-part (15 mm diameter sphere)



As external witness-part, a 15 mm diameter sphere has been employed. The two objects have been positioned in two different zones of the measuring volume. The sphere was close to the machine's natural origin, and the measured object was located in the opposite corner (Figure 8).

In order to simulate a measurement process of a part in a production line, the following sequence of activities has been planned:

- Positioning of the workpiece within the measuring volume of the CMM (part positioning).
- Setting up of the coordinate system for the workpiece (reference system alignment).
- Measurement cycle of the workpiece (measurement cycle execution).
- Removing of the workpiece from the CMM (part removing).
- Introduction of a new workpiece in the measuring volume, and so on.

A measurement cycle of  $s = 45$  points for the geometrical characterisation of each workpiece was scheduled. For the reproducibility tests, five points have been considered on the workpiece ( $n = 5$ ) and five on the witness-part ( $m = 5$ ). Selected test-points on the workpiece are a random subset of the  $s$  measurement points. The test frequency has been fixed every 15 measurement points (*i.e.*, three replications have been performed on each test-point for every workpiece).

Without losing generality, we consider the following as nominal reference sets, for the workpiece and the witness-part, respectively:

$$\{P_1^0(X, Y, Z), \dots, P_5^0(X, Y, Z)\}_{\text{workpiece}} \quad \text{and} \quad \{P_1^0(X, Y, Z), \dots, P_5^0(X, Y, Z)\}_{\text{witness-part}}$$

The two sets of five points (nominal references) for the reproducibility tests have been measured once at the beginning of the control process.

The complete measuring cycle for each single workpiece was the following:

- Measurement of the first 15 points of the measurement cycle on the workpiece.
- First measurement of the five-point sets for the reproducibility test on the workpiece and on the witness-part.
- Measurement of the second 15 points of the measurement cycle on the workpiece.
- Second measurement of the five-point sets for the reproducibility test on the workpiece and on the witness-part.
- Measurement of the third 15 points of the measurement cycle on the workpiece.
- Third measurement of the five-point sets for the reproducibility test on the workpiece and on the witness-part.

In order to facilitate the experimental tests, a software program has been realised. It allows the automatic alignment of the reference system and the storage of the point coordinates of the measurement cycle. To simulate 'normal' operating conditions of a production line in a job-shop, tests have been carried out without the use of an air-conditioning system (no temperature and humidity controls in the measurement room).

### 5.3 Data analysis

A series of statistical tests has been performed on experimental data (autocorrelation test, normality test, ANOVA, *etc.*) (Montgomery, 2001). Variables used for these analyses are  $D_X^{(k)}$ ,  $D_Y^{(k)}$  and  $D_Z^{(k)}$ , as well as  $D^{(k)}$  (under the assumption of normal distribution). It must be highlighted that, since  $D^{(k)}$  incorporates all the contributions coming from the other three variables, it may be interpreted as an aggregate index of the measurement reliability (Franceschini and Settineri, 2000).

Three series of tests (on 26 workpieces) have been carried out using  $\bar{X}$ -R control charts:

- 1 A first test has been focused on determining the natural variability of the measurement process without assignable disturbance causes.
- 2 In a second test, a series of external disturbances has been introduced with the logic of simulating process context perturbations. In particular, after the 10th workpiece measurement, a progressive increase of the room temperature has been induced (climate variation), and, after the 20th cycle, the pressure of the machine pneumon-static guides has been progressively reduced till the automatic shutdown of the system (instrumentation degradation).
- 3 In a third test, a series of mechanical perturbations have been produced by beating the machine structure.

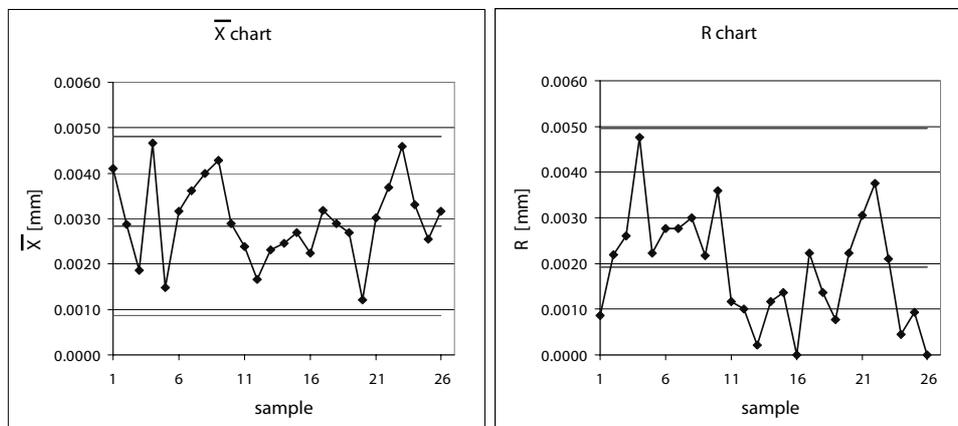
#### 5.3.1 Test (a): determination of the natural variability of the measurement process

Figures 9 and 10 report  $D_1^{(k)}$  variable component control charts for the witness-part and the workpiece (reproducibility of point  $P_1$ ). Control chart limits have been calculated on the basis of the first 20 samples.

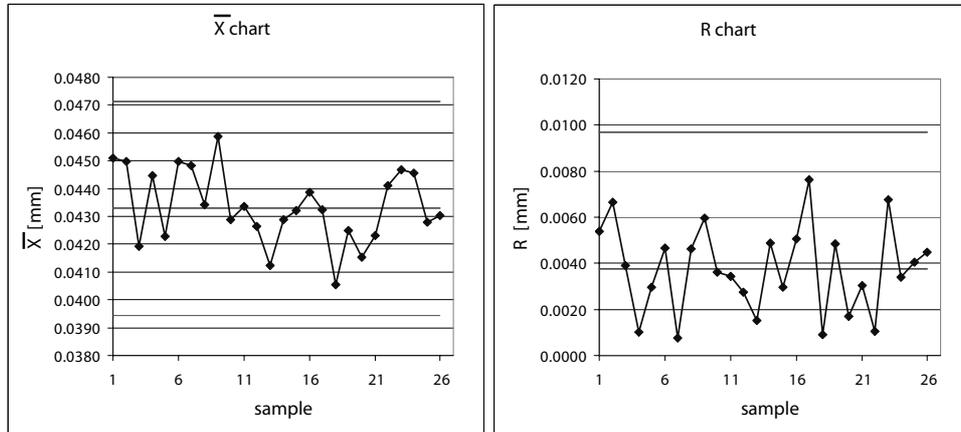
There is no evidence of out-of-controls. The same result has been obtained for the other test-points (*i.e.*, for variable components  $D_2^{(k)}$ ,  $D_3^{(k)}$ ,  $D_4^{(k)}$ , and  $D_5^{(k)}$ ).

Comparing Figure 9 with Figure 10, some differences appear in the mean and limit values. The reason can be ascribed to the different CMM probe contact conditions. This behaviour suggests using witness-parts as similar to the measured object as possible.

**Figure 9**  $\bar{X}$ -R control charts of  $D_1^{(k)}$  variable component (point  $P_1$ ) for the first reproducibility test on the *witness-part*. The measurements have been performed in the absence of any assignable perturbation causes



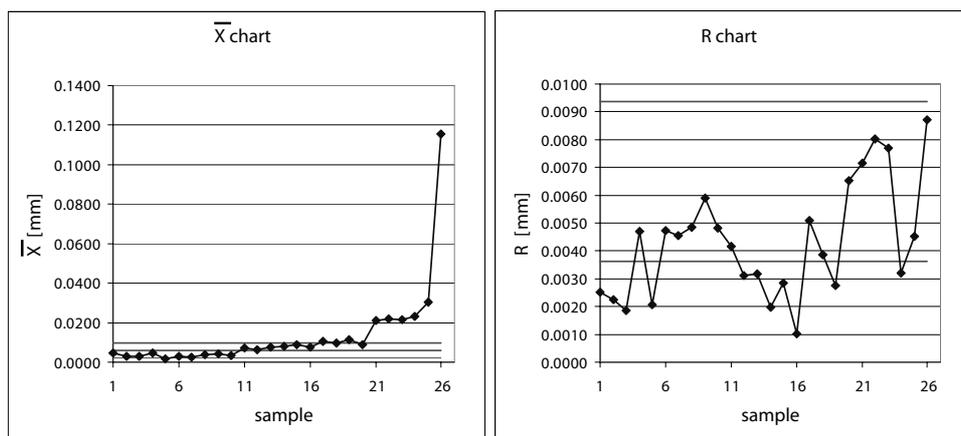
**Figure 10**  $\bar{X}$  – R control charts of  $D_1^{(k)}$  variable component (point  $P_1$ ) for the first reproducibility test on the *workpiece*. The measurements have been performed in the absence of any assignable perturbation causes



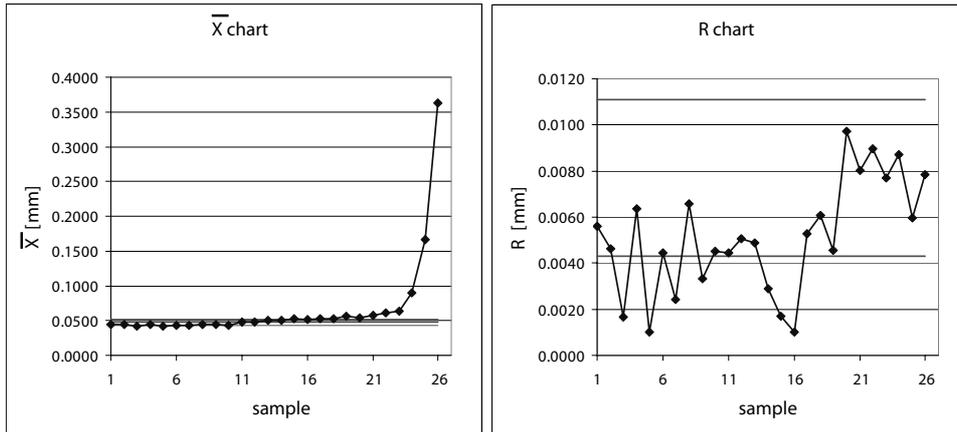
5.3.2 Test (b): thermal perturbation and instrument degradation

Figures 11 and 12 report  $D_1^{(k)}$  variable component control charts for the witness-part and the workpiece after a thermal perturbation and a subsystem perturbation (reproducibility of point  $P_1$ ). The two figures show a series of out-of-control points. A trend in both charts appears at the beginning of the progressive increase of the room temperature. An analogous behaviour has been observed for the other test-points (variable components  $D_2^{(k)}, D_3^{(k)}, D_4^{(k)},$  and  $D_5^{(k)}$ ).

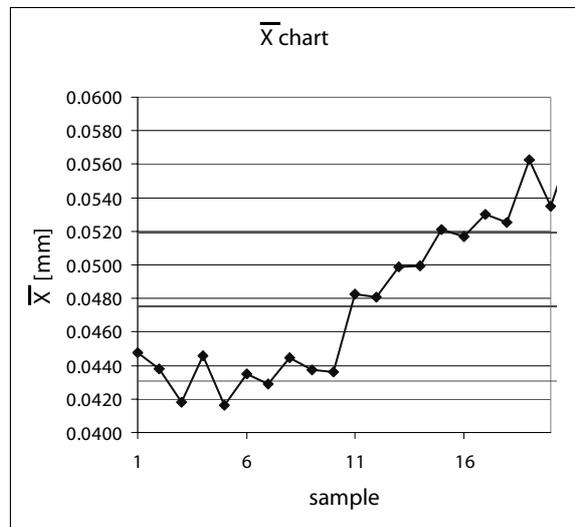
**Figure 11**  $\bar{X}$  – R control charts of  $D_1^{(k)}$  variable component (point  $P_1$ ) for the second reproducibility test on the witness-part. After the 10th workpiece measurement, a progressive increase of the room temperature has been induced, and after the 20th cycle, the pressure of the machine pneumon-static guides has been progressively reduced till the automatic shutdown of the system.



**Figure 12**  $\bar{X}$  – R control charts of  $D_1^{(k)}$  variable component (point  $P_1$ ) for the second reproducibility test on the *workpiece*. After the 10th *workpiece* measurement, a progressive increase of the room temperature has been induced, and after the 20th cycle, the pressure of the machine pneumon-static guides has been progressively reduced till the automatic shutdown of the system. The effect of the pressure variation is strongly evidenced by the appearing of some out-of-control points (control limits are shadowed by the graph scale)



**Figure 13** Detail of  $\bar{X}$  chart in Figure 12



After the 20th cycle, the charts catch the pressure variation effect (see the out-of-controls appearing after the 20th cycle), but not the perturbation owing to the temperature (this effect is shadowed by pressure effect, which is higher).

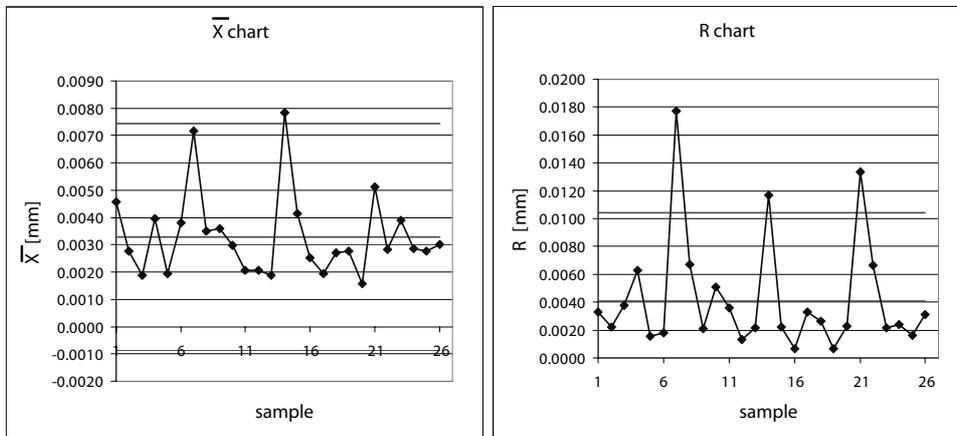
It must be highlighted that a ‘position effect’ of the touched points can be evidenced by the comparison of the two control charts (see Figures 11 and 12). The introduced perturbation generates the same variation in the two charts; nevertheless many differences appear. This discrepancy turns out because, while the witness-part is close to

the natural axes origin (the pressure effect is weaker), the workpiece is located closer to the opposite corner (a small reduction of guide pressure is strongly undergone). The same results have been obtained for  $D_2^{(k)}$ ,  $D_3^{(k)}$ ,  $D_4^{(k)}$ , and  $D_5^{(k)}$  components.

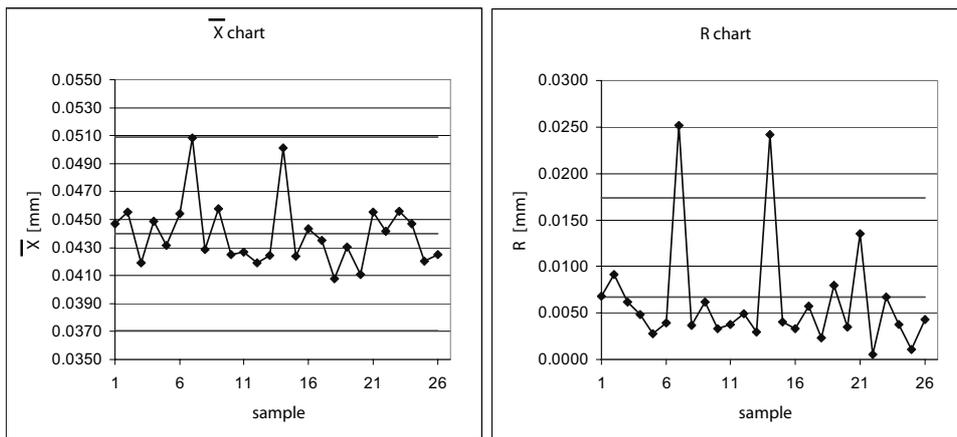
### 5.3.3 Test (c): mechanical perturbation

Figures 14 and 15 report  $D_1^{(k)}$  variable component control charts for the witness-part and the workpiece when a series of mechanical perturbations have been produced by beating the machine structure (reproducibility of point  $P_1$ ).

**Figure 14**  $\bar{X}$  - R control charts of  $D_1^{(k)}$  variable component (point  $P_1$ ) for the third reproducibility test on the *witness-part*. A series of mechanical perturbations has been produced by beating the machine structure



**Figure 15**  $\bar{X}$  - R control charts of  $D_1^{(k)}$  variable component (point  $P_1$ ) for the third reproducibility test on the *workpiece*. A series of mechanical perturbations has been produced by beating the machine structure

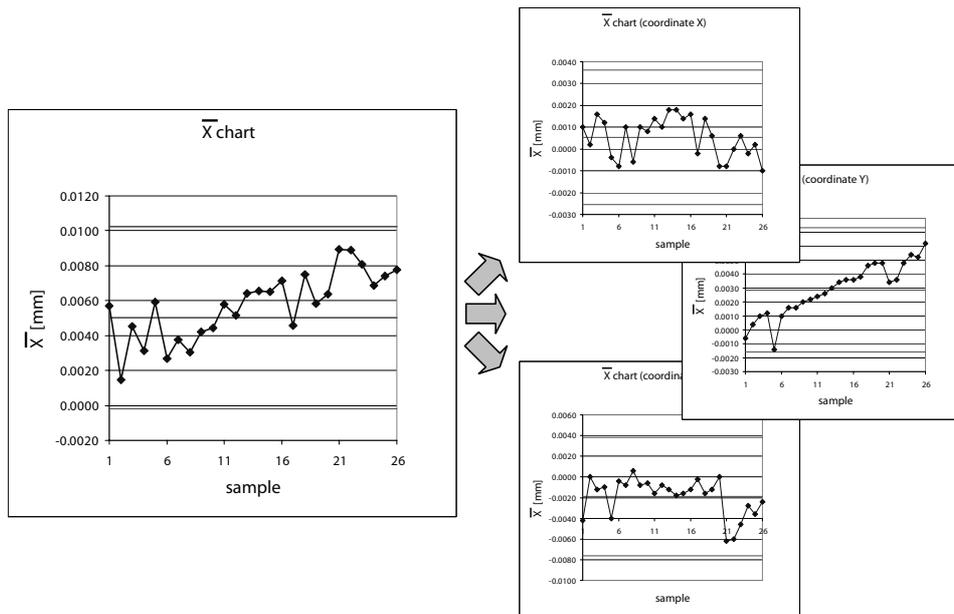


Both figures show the effect of the introduced perturbation: three out-of-controls (corresponding to the mechanical perturbations) appear in both R-charts. The  $\bar{X}$ -chart presents some difficulty in detecting this effect that is due to the averaging of perturbed measurements with the others (not perturbed) in the sample.

### 5.4 Splitting $\bar{X}$ -R charts in single components

The main characteristic of the proposed approach is related to the possibility of splitting the aggregated values control charts in many detailed charts, referring to each single measured variable ( $D_X^{(k)}$ ,  $D_Y^{(k)}$ , and  $D_Z^{(k)}$ ). Hence, any assignable cause can be isolated and analysed. For example, an  $\bar{X}$ -R control chart evidencing the appearing of some out-of-controls can be split into three different charts representing the distance components along the three measuring axes (see Figure 16). In this way, it is possible to verify if the cause can be assigned to some specific coordinate reproducibility fault ( $D_X^{(k)}$ ,  $D_Y^{(k)}$  or  $D_Z^{(k)}$ ).

**Figure 16** The aggregated  $\bar{X}$  chart of  $D_1^{(k)}$  variable component is split into three different charts representing the distance components along the three measuring axes (X, Y, Z). The figure evidences that the trend in the aggregated chart is due to Y variable.



## 6 Conclusion

The paper presents a set of criteria for the taxonomy of different methods applicable to an online performance verification of three-axes CMMs.

Two approaches have been defined: the ‘physical’ redundancy, based on a replication of the measurement equipment, and the ‘analytical’ redundancy, based on the use of some specific mathematical models. Some mixed strategies can be built exploiting the combined use of suitable mathematical models and external references (reference gauges or oppositely characterised workpieces).

Focusing attention on the analytical and mixed approaches, we showed that two typologies of analytical redundancy models can be identified: intrinsic and extrinsic.

We also demonstrated that different analytical models may be more or less able to evidence the appearance of ‘incongruence’ in a CMM performance, depending on their mathematical structure (invariant, variant and strictly variant).

Furthermore, we showed that, in spite of a great deal of standards for offline CMM performance verification, online techniques standardisation is still lacking. This paper could be a first cue for launching a new proposal for a standardised approach.

On the basis of the proposed taxonomy, we presented an organic approach to define, case by case, the most appropriate online verification method.

In the end, an experimental investigation based on the combined use of a purely analytical redundancy and a mixed approach is described and analysed. The use of multiple control charts allows a simple and immediate monitoring of the machine performance.

Future developments will concern the analysis and implementation of automated procedures for general approaches to online CMM performance verification, as well as the application of different monitoring tools, such as CUSUM or EWMA control charts.

## References

- Bosch, J.A. (1995) *Coordinate Measuring Machines and Systems*, New York, USA: Marcel Dekker.
- Cauchick-Miguel, P., King, T. and Davis, J. (1996) ‘CMM verification: a survey’, *Measurement*, Vol. 17, No. 1, pp.1–16.
- Clarke, D.W. (1995) ‘Sensor, actuator, and loop validation’, *IEEE Control Systems Magazine*, Vol. 15, pp.39–45.
- de Aquino Silva, J.B. and Burdekin, M. (2002) ‘A modular space frame for assessing the performance of Coordinate Measuring Machines (CMMs)’, *Precision Engineering*, Vol. 26, pp.37–48.
- Franceschini, F. and Settineri, L. (2000) ‘Control charts for the on-line diagnostics of CMM performances’, *International Journal of Computer Integrated Manufacturing*, Vol. 13, No. 2, pp.148–156.
- Franceschini, F., Galetto, M. and Settineri, L. (2002) ‘On-line diagnostic tools for CMM performance’, *International Journal of Advanced Manufacturing Technology*, Vol. 19, pp.125–130.
- Frank, P.M. (1990) ‘Fault diagnosis in dynamic systems using analytical and knowledge-based redundancy – a survey and some new results’, *Automatica*, Vol. 26, pp.459–474.
- Gertler, J.J. (1998) *Fault Detection and Diagnosis in Engineering Systems*, New York, USA: Marcel Dekker.
- Henry, M.P. and Clarke, D.W. (1992) ‘The self-validating sensor: rationale, definitions and examples’, *Control Engineering Practice*, Vol. 1, pp.585–610.
- Isermann, R. (1984) ‘Process fault detection based on modeling and estimation methods – a survey’, *Automatica*, Vol. 20, pp.387–404.

- Isermann, R. (1993) 'Fault diagnosis of machines via parameter estimation and knowledge processing – tutorial paper', *Automatica*, Vol. 29, pp.815–835.
- ISO 10012–2 (1997) *Quality Assurance for Measuring Equipment – Part 2: Guidelines for Control of Measurement Processes*.
- ISO 10360–2 (2001) *Geometrical Product Specifications (GPS) – Acceptance and Reverification Tests for Coordinate Measuring Machines (CMM) – Part 2: CMMs Used for Measuring Size*.
- ISO/TS 15530–6 (2000) 'Geometrical Product Specifications (GPS) – Coordinate Measuring Machines (CMM): techniques for determining the uncertainty of measurements. Part 6: uncertainty assessment using un-calibrated workpieces', *Working Draft*, September, Version 18.
- Montgomery, D.C. (2001) *Introduction to Statistical Quality Control*, 4th ed., New York, USA: John Wiley & Sons.
- Oksman, J. (1993) 'Définition et obtention en temps réel de la crédibilité d'une mesure', *6e Congrès International de Métrologie*, Lille, 19–21 October, pp.224–229.
- Reznik, L.K. and Solopchenko, G.N. (1985) 'Use of a priori information on functional relations between measured quantities for improving accuracy of measurement', *Measurement*, Vol. 3, No. 3, pp.98–106.
- Savio, E., Hansen, H.N. and De Chiffre, L. (2002) 'Approaches to the calibration of freeform artifacts on coordinate measuring machines', *Annals of CIRP 51/1*, San Sebastian, Spain, pp.433–436.