Interventions and control of equilibria in multi-agent systems modeled as games

Game theory is a mathematical model act to describe behaviors and decisions of rational agents, strategically interacting in a socio-economic system. It is grounded on two fundamental assumptions: every agent seeks to maximize its own utility and its choice also affects the utility of the others.
The aim of the theory is to forecast behaviors and study the emergence of phenomena like cooperation, competition, conflict.

Born and studied during the cold war age, with the fundamental contributions of Von Neumann and Nash, it has been successfully applied in a variety of different fields ranging from economy to strategic politics to finance and also biology.

More recently, in the network age, it has become a broad language to study interactions in networks (e.g. social, economical, infrastructural) and a basic tool for the design of scalable distributed algorithms for multi-agent systems.

One of the key concepts of game theory is that of Nash equilibrium. This is a particular configuration of the system in which no player has any longer an incentive to unilaterally change the action from the one it is currently playing. These are the configurations to which the complex system should eventually converge.

In many interesting games there can be multiple Nash equilibria. This happens for instance in network coordination games where agents can only play two possible actions 0 and 1: to play action 1 is costly and it is only worth it a sufficient number of your friends do the same. An interpretation is that 1 is representing the adoption of a new technology. In this setting, we will typically have that both configurations where all players play 0 or all play 1 are Nash equilibria. While the second one would be overall better for all players, they will not be able to reach it by unilateral moves only starting from the 0 configuration as this is last one is also a Nash equilibrium.

In situations as the one described above it is of great interest to study strategic exogenous interventions in order to push the system from one equilibrium to a more efficient one. Interventions can be modeled in different ways.

A first instance is a modification of the reward function of a subset of players in order to make worth for them to modify their action and possibly push the overall system towards the new equilibrium. This problem naturally leads to optimization problems where we want to trade the efficiency of the action over its cost determined by the amount of the intervention (e.g how many players involved). A preliminary work in this direction is [1] for the case of a network coordination game with binary actions and where the exogenous intervention is simply forcing a subset of players to play action 1.

The natural framework where the results in [1] can be generalized is that of super-modular games, a family of games where the increase in the action of a player is always of benefit to the rest of players. Such games show up quite naturally in economics where interactions have a strategic complementarity effect. Concrete possible research topics in this framework are the following:

(a) study more refined interventions where activation thresholds of a subset of players are modified with possible different levels of intervention;
(b) study super-modular games with finite set of actions of cardinality greater than 2;
(c) study super-modular games with a continuous set of actions;

A second instance of intervention can be considered at the level of the network. How can we modify the topology of the interactions to improve the ability of the system to converge to the desired equilibrium? In this case optimal problems can be formulated where performance is traded against the extent of the rewiring (e.g. number of players involved, number of added links,…). Possible research problem are the following:

(d) for binary network coordination games, determine the minimum number of link additions that, starting from an initial configuration where a certain subset of players is playing action 1, will lead the system to the all 1 configuration.

(e) extend the analysis in (d) to more general super-modular games.

[1] S. Durand, G. Como, F. Fagnani, Controlling network coordination games, ArXiv https://arxiv.org/abs/1912.07859

[2] J. Levin, Supermodular games, https://it.scribd.com/document/72994152/Super-Modular-Games