PARAMETERS SET EVALUATION OF WIENER MODELS FROM DATA WITH BOUNDED OUTPUT ERRORS

V. Cerone M. Milanese D. Regruto

Dipartimento di Automatica e Informatica, Politecnico di Torino, corso Duca degli Abruzzi 24, 10129 Torino, Italy — E-mail: vito.cerone@polito.it, mario.milanese@polito.it, diego.regruto@polito.it, Phone: + 39 11 5647064, Fax: + 39 11 5647099

Abstract: In this paper a procedure is presented for deriving parameters bounds in SISO Wiener models when the nonlinear block can be modeled by a polynomial and the output measurement errors are bounded. First, using steady-state input-output data, parameters of the nonlinear block are tightly bounded. Next in order to estimate the parameters of the linear block, the evaluation of the inner unmeasurable signal is considered. No invertibility assumption of the nonlinearity is required. Then, through a suitable design of the identification experiment, bounds on the unmeasurable inner signal are evaluated. Finally, such bounds together with the input sequence are used for bounding the parameters of the linear model.

Keywords: Wiener model, bounded uncertainty, output errors, parameter bounding, linear programming.

1. INTRODUCTION

A wide class of simple nonlinear systems, also called block-oriented systems, can be modeled by interconnected memoryless nonlinear gains and linear subsystems. The configuration considered in this paper, commonly referred to as Wiener model, is shown in Figure 1; it consists of a linear dynamic system followed by static nonlinear block $\mathcal{N}$. The identification of such a model is carried out on the basis of the sequences u_t and y_t, while the inner signal x_t, i.e. the output of the linear block, is not assumed to be available. The Wiener model has been successfully used in a large variety of fields. The identification of Wiener structure has attracted the attention of many authors, as can be seen in Billings (1980), Billings and Fakhouri (1977), Haber and Unbehauen (1990), Bai (2002), Wigren (1998). The main difficulty in the identification of nonlinear block-oriented system is that the internal signals are not available for measurement. As far as Wiener systems are concerned most of contributions assume invertibility of the nonlinearity. As a matter of fact under such an assumption the inner signals can be recovered from the output measurements through inversion of the previously estimated nonlinearity. However, many output nonlinearity encountered in real world problem are non-invertible (see, e.g., Wigren, 1998), thus the invertibility assumption appear to be quite restrictive. Removal of such an hypothesis made the consistent evaluation of the inner signal sequence a difficult task even in the case of exactly known nonlinearity. In all the papers mentioned above, the authors assume that the measurement error η_t is statistically described. A worthwhile alternative to the stochastic description of measurement errors is the bounded-errors characterization, where uncertainties are assumed to belong to a given set. The interested reader can find further details on this approach in a number of survey papers (see, e.g., (Milanese and Vicino, 1991; Walter and Piet-Lahanier, 1990)), in the book edited by Milanese *et al.* (Milanese *et al.*, 1996) and the special issues edited by Norton (Norton(Ed.), 1994; Norton(Ed.), 1995). To the author's best knowledge, no contribution can be found which address the identification of Wiener models when the measurement error η_t is supposed to be bounded. In this paper the identification of single-input single-output (SISO) Wiener models is considered, when the nonlinear block can be modeled by a polynomial, with finite and known order, and when the output measurement errors are bounded. First, using steady-state input-output data, parameters of the nonlinear block are tightly bounded. Next in order to estimate the parameters of the linear block, the evaluation of the inner unmeasurable signal x_t is considered. No invertibility assumption of the nonlinearity is required, thus the inner signal x_t providing a given measured output y_t is not unique even in the uncertainty free case, i.e., the estimated inner sequence $\{x_t\}$ to be used in the evaluation of the feasible parameter set of the linear block, might not be the output of a unique linear systems. In the paper it is shown how such a fundamental problem can be addressed through a suitable design of the identification experiment. Finally, through the designed dynamic experiment, for all y_t belonging to a given output transient sequence $\{y_t\}$, upper and lower bounds on the inner signal x_t are computed. Such bounds, together with the input sequence $\{u_t\}$ are used for bounding the parameters of the linear block.

2. PROBLEM FORMULATION

Consider the SISO discrete-time Wiener model shown in Figure 1, where the linear dynamic block, which is modeled by a discrete-time system, maps the input signal u_t into the unmeasurable inner variable x_t according to

$$x_t = \frac{B(q^{-1})}{A(q^{-1})}u_t = G(q^{-1})u_t, \tag{1}$$

or, equivalently, in terms of a linear difference equation

$$A(q^{-1})x_t = B(q^{-1})u_t, \tag{2}$$

where $A(\cdot)$ and $B(\cdot)$ are polynomials in the backward shift operator q^{-1}, $(q^{-1}w_t = w_{t-1})$,

$$A(q^{-1}) = 1 + a_1q^{-1} + \ldots + a_{na}q^{-na}, \tag{3}$$

$$B(q^{-1}) = b_0 + b_1q^{-1} + \ldots + b_{nb}q^{-nb}. \tag{4}$$

The nonlinear block transforms x_t into the noise-free output w_t through the following polynomial function

$$w_t = \sum_{k=1}^{n} \gamma_k x_t^k, \quad t = 1, \ldots, N; \tag{5}$$

whose order n is taken to be finite and a-priori known; N is the length of the input sequence. In line with the work done by a number of authors, it is assumed that (i) the linear system is asymptotically stable (see, e.g. Stoica and Söderström, 1982; Krzyżak, 1993; Lang, 1993; Sun *et al.*, 1999); (ii) $\sum_{j=0}^{nb} b_j \neq 0$, that is, the steady-state gain is not zero (see, e.g. Lang, 1993; Sun *et al.*, 1999); (iii) the only *a priori* information needed is an estimate of the process settling-time (see, e.g. Kalafatis *et al.*, 1997). Let y_t be the noise-corrupted measurements of w_t

$$y_t = w_t + \eta_t. \tag{6}$$

Measurements uncertainty is known to range within given bounds $\Delta\eta_t$, i.e.,

$$\mid \eta_t \mid \leq \Delta\eta_t. \tag{7}$$

Unknown parameter vectors $\gamma \in R^n$ and $\theta \in R^p$ are defined, respectively, as

$$\gamma^T = [\gamma_1 \quad \gamma_2 \quad \ldots \gamma_n], \tag{8}$$

$$\theta^T = [a_1 \quad \ldots \quad a_{na} \quad b_0 \quad b_1 \quad \ldots b_{nb}], \tag{9}$$

where $n_a + n_b + 1 = p$. It is easy to show that the parameterization of the structure of Figure 1 is not unique. As a matter of fact, any parameters set $\tilde{b}_j = \alpha^{-1}b_j, j = 1, 2, \ldots, nb$, and $\tilde{\gamma}_k = \alpha\gamma_k, k = 1, 2, \ldots, n$, for some nonzero and finite constant α, provides the same input-output behaviour. Thus, any identification procedure cannot perceive the difference between parameters $\{b_j, \gamma_k\}$ and $\{\alpha^{-1}b_j, \alpha\gamma_k\}$. In this work, it is assumed, without loss of generality, that the steady-state gain of the linear part be one, that is

$$g = \frac{\sum_{j=0}^{nb} b_j}{1 + \sum_{i=1}^{na} a_i} = 1 \tag{10}$$

In this paper the problem of deriving bounds on parameters γ and θ consistently with given measurements, error bounds and the assumed model structure is addressed.

3. ASSESSMENT OF TIGHT BOUNDS ON THE NONLINEAR STATIC BLOCK PARAMETERS

In this work we exploit steady-state operating conditions to bound the parameters of the nonlinear static block. The known input and noise corrupted output sequences are collected from the steady-state response of the system to a set of step inputs with different amplitude. It is only assumed that a rough information on the settling time of the system under consideration is available, in order to know when steady-state conditions are reached, so that steady-state data can be collected. Indeed, combining equations (5), (2), (6) and (10) in steady-state operating conditions the following input-output description is obtained.

$$\bar{y}_s = \sum_{k=1}^{n} \gamma_k \bar{u}_s^k + \eta_s, \quad s = 1, \ldots, M \tag{11}$$

where $\bar{u}_s$ and $\bar{y}_s$ are steady-state values of the known input signal and output observation respectively, while η_s is the measurement error; $M \geq n$ is the length of the steady-state sequences. A block diagram description of equation (11) is depicted in Figure 2. Thus, the feasible parameter region of the static nonlinear block is defined as

$$\mathcal{D}_\gamma = \{\gamma \in R^n : \bar{y}_s = \sum_{k=1}^{n} \gamma_k \bar{u}_s^k + \eta_s, \; \mid \eta_s \mid \leq \Delta\eta_s; \; s = 1, \ldots, M\}. \tag{12}$$

where $\{\Delta\eta_s\}$ is the sequence of bounds on measurements uncertainty. From the definition of $\mathcal{D}_\gamma$ one gets

$$\bar{\varphi}_s^{\mathrm{T}}\gamma \leq \bar{y}_s + \Delta\eta_s \tag{13}$$

$$\bar{\varphi}_s^{\mathrm{T}}\gamma \geq \bar{y}_s - \Delta\eta_s \tag{14}$$

where

$$\bar{\varphi}_s = \begin{bmatrix}\bar{u}_s & \bar{u}_s^2 & \bar{u}_s^3 \dots \bar{u}_s^n\end{bmatrix}^{\mathrm{T}} \tag{15}$$

for $s = 1, 2, \dots, M$. The above exact description of $\mathcal{D}_\gamma$ will be used in the next section when deriving tight bounds on the unmeasurable inner signal x_t. Since $\mathcal{D}_\gamma$ is a convex polytope, whose shape may result quite complex for increasing n and M, an outer bound to it such as an ellipsoid or a box is often computed. In this paper we consider an orthotope-outer bounding set $\mathcal{B}_\gamma$ containing $\mathcal{D}_\gamma$

$$\begin{aligned}\mathcal{B}_\gamma = \{\gamma \in R^n : \gamma_j = \gamma_j^c + \delta\gamma_j, \\ \mid \delta\gamma_j \mid \leq \Delta\gamma_j/2, j = 1, \dots, n\},\end{aligned} \tag{16}$$

where

$$\gamma_j{}^c = \frac{\gamma_j^{min} + \gamma_j^{max}}{2}, \tag{17}$$

$$\Delta\gamma_j = \mid \gamma_j^{max} - \gamma_j^{min} \mid, \tag{18}$$

and

$$\gamma_j^{min} = \min_{\gamma \in \mathcal{D}_\gamma} \gamma_j, \quad \gamma_j^{max} = \max_{\gamma \in \mathcal{D}_\gamma} \gamma_j. \tag{19}$$

The evaluation of $\mathcal{B}_\gamma$ requires the solution of $2n$ linear programming problems with n variables and $2M$ constraints.

4. EVALUATION OF FEASIBLE INTERVALS FOR OUTPUT MEASUREMENTS AND INNER SIGNALS

In the previous section it has been shown how the nonlinear block can be characterized using steady-state data. In order to estimate the parameters of the linear model, one should first evaluate the inner signal x_t from the output records y_t via the polynomial nonlinearity. Unfortunately, one must consider the fact that nonlinearity (5) is, in general, noninvertible, which means that, given the measured output y_t, the inner signal x_t cannot be evaluated uniquely even in the case of exactly known polynomial and noise free measurements. As a matter of fact, considering (5) at a single time t one gets

$$p_t(x_t, w_t) = 0 \tag{20}$$

where

$$p_t(x_t, w_t) = w_t - \sum_{k=1}^{n} \gamma_k x_t^k. \tag{21}$$

belongs to the following family of polynomials

$$\mathcal{P}_t = \{p_t(x_t, w_t) : w_t \in R\} \tag{22}$$

For given w_t and exactly known γ_k, $k = 1, 2, \dots, n$, equation (20) shows, in general, n different solutions in the unknown x_t. Nonuniqueness, unfortunately, is responsible of nonconsistent inner signal estimates, i.e., estimated $\{x_t\}$ might not be the output of a unique linear system. The main idea for overcoming this problem is to design the input sequence $\{u_t\}$ which will force an output sequence $\{w_t\}$ so that polynomial (21) will show either only one real root when n is odd or two real roots when n is even in the case of exactly known parameters of the nonlinear block and noiseless measurements. In the case of uncertain polynomial parameters the following family of polynomials can be defined

$$\Pi_t = \{p_t(x_t, w_t, \gamma) : w_t \in R,\ \gamma \in \mathcal{D}_\gamma\} \tag{23}$$

where

$$p_t(x_t, w_t, \gamma) = w_t - \sum_{k=1}^{n} \gamma_k x_t^k. \tag{24}$$

It is assumed that all polynomials in Π_t have degree equal to n. In this case, in order to evaluate the inner signal x_t one has to find the real roots of the uncertain polynomial (24). The *real spectral set* of the family of polynomial Π_t, is defined as

$$\begin{aligned}\mathcal{S}_{\mathrm{R}}(\Pi_t) = \{x_t \in R : p_t(x_t, w_t, \gamma) = 0, \\ \text{for some } \gamma \in \mathcal{D}_\gamma, w_t \in R\}\end{aligned} \tag{25}$$

4.1 Uncertain polynomial nonlinearity — Here it is assumed that $\gamma \in \mathcal{D}_\gamma$ and the problem of evaluating the inner signal x_t in terms of roots of the uncertain polynomial (24) is addressed. One shall look for conditions under which each polynomial $p_t(x_t, w_t, \gamma) \in \Pi_t$ shows either only one real root when n is odd or two real roots when n is even.

Proposition 1. Each polynomial $p_t(x_t, w_t, \gamma) \in \Pi_t$, shows either only one real root when n is odd or two real roots when n is even if and only if

$$w_t > \bar{w} \text{ or } w_t < \underline{w} \text{ when } n \text{ is odd} \tag{26}$$

or

$$\operatorname{sign}(\gamma_n) w_t > \frac{1 + \operatorname{sign}(\gamma_n)}{2}\bar{w} - \frac{1 - \operatorname{sign}(\gamma_n)}{2}\underline{w} \quad \text{when } n \text{ is even} \tag{27}$$

where

$$\bar{w} = \max_{x_t \in \Upsilon_t} \max_{\gamma \in \mathcal{D}_\gamma} \sum_{k=1}^{n} \gamma_k x_t^k \tag{28}$$

$$\underline{w} = \min_{x_t \in \Upsilon_t} \min_{\gamma \in \mathcal{D}_\gamma} \sum_{k=1}^{n} \gamma_k x_t^k \tag{29}$$

$$\Upsilon_t = \left\{x_t \in R\colon \frac{d}{dx_t} \sum_{k=1}^{n} \gamma_k x_t^k = 0, \text{for some } \gamma \in \mathcal{D}_\gamma\right\} \tag{30}$$

Moreover, under conditions (26) and (27) the real roots of polynomial (24) satisfy

$$x_t \in]\bar{x}, +\infty] \tag{31}$$

$$x_t \in [-\infty, \underline{x}[\tag{32}$$

$$\begin{aligned}\bar{x} = \max\{x_t \in R : \frac{1 + \operatorname{sign}(\gamma_n)}{2}\bar{w} + \\ \frac{1 - \operatorname{sign}(\gamma_n)}{2}\underline{w} - \sum_{k=1}^{n} \gamma_k x_t^k = 0, \\ \text{for some } \gamma \in \mathcal{D}_\gamma\}\end{aligned} \tag{33}$$

$$\begin{aligned}\underline{x} = \min\{x_t \in R : \frac{1 + (-1)^n \operatorname{sign}(\gamma_n)}{2}\bar{w} + \\ \frac{1 - (-1)^n \operatorname{sign}(\gamma_n)}{2}\underline{w} - \sum_{k=1}^{n} \gamma_k x_t^k = 0, \\ \text{for some } \gamma \in \mathcal{D}_\gamma\}.\end{aligned} \tag{34}$$

The proof of Proposition 1 can be found in (Cerone *et al.*, 2002).

The computational aspects of quantities and sets involved in Proposition 1 are now discussed.

Computation of Υ_t — First consider the set defined by equation (30), i.e. the set of real valued x_t for which the uncertain polynomial shows stationary points (relative maxima, relative minima or points of inflexion). The first derivative of the uncertain polynomial is still an uncertain polynomial, namely

$$p_t\,'(x_t, \gamma) = -\frac{d}{dx_t}\sum_{k=1}^{n}\gamma_k x_t^k = -\sum_{k=1}^{n} k\gamma_k x_t^{k-1} \quad (35)$$

which, clearly, shows nonlinear relations in the unknown x_t and the uncertain γ. A one-dimensional gridding procedure for finding the roots of (35) is proposed. It is noticed that for a given $x_t \in R$, in order to find the real spectral set of polynomial (35) one must solve a set of $2M$ linear inequalities (i.e., $\gamma \in \mathcal{D}_\gamma$) and one linear equality (i.e., $\sum_{k=1}^{n} k\gamma_k x_t^{k-1} = 0$) in the unknown $\gamma \in R^n$.

Computation of $\bar{w}$ *and* $\underline{w}$ — Next equations (28) and (29) which define two nonlinear programming problems are considered. However, when x_t is given, they simplify to linear programs. Thus, to compute $\bar{w}$ and $\underline{w}$, for each value of $x_t \in \Upsilon_t$ the solution of two linear programming problems with n variables and $2M$ constraints is required. A one-dimensional gridding procedure is used in order to carry out the optimization over a finite number of $x_t \in \Upsilon_t$.

Computation of $\bar{x}$ *and* $\underline{x}$ — Finally, equation (33) and equation (34) are considered. In order to simplify the discussion, odd order polynomial with $\gamma_n > 0$ are first considered. In this case one gets

$$\bar{x} = \max\{x_t \in R : \bar{w} - \sum_{k=1}^{n}\gamma_k x_t^k = 0, \text{ for some } \gamma \in \mathcal{D}_\gamma\} \quad (36)$$

$$\underline{x} = \min\{x_t \in R : \underline{w} - \sum_{k=1}^{n}\gamma_k x_t^k = 0, \text{ for some } \gamma \in \mathcal{D}_\gamma\} \quad (37)$$

Equations (36) and (37) show nonlinear relations in the unknown x_t and the uncertain γ. In this case a search procedure is proposed for finding the roots of uncertain polynomials involved in the above equations. It is noticed that for a given $x_t \in R$, in order to solve each one of the above equations one must solve a set of $2M$ linear inequalities (i.e., $\gamma \in \mathcal{D}_\gamma$) and one linear equality (i.e., $\sum_{k=1}^{n}\gamma_k x_t^k = 0$) in the unknown $\gamma \in R^n$. It is also seen that the search can be started from the unique real root of one nominal polynomial obtained, e.g., setting $\gamma = \gamma^c$; next, only right side of the nominal root of equation (36) and only left side of the nominal root of equation (37) are explored in order to find a suitable approximation of $\bar{x}$ and $\underline{x}$ respectively. Analogous considerations can be made in all other cases ($\gamma_n > 0$, $\gamma_n < 0$, n odd, n even).

4.2 Input sequence design — One is left with the problem of the choice of the input sequence $\{u_t\}$. In order to drive the inner signal $\{x_t\}$ into the desired interval, the input signal $\{u_t\}$ should contain a DC component u_{DC} (offset) and a dynamic exciting signal $\{u_{td}\}$ whose amplitudes should be chosen in such a way that $x_t = x_{DC} + x_{td}$ satisfies either (31) $\forall t$ or (32) $\forall t$. Since the steady-state gain of the linear subsystem is constrained to be one, the amplitudes of the DC components in $u_t = u_{DC} + u_{td}$ and x_t are the same, i.e., $u_{DC} = x_{DC}$. Guidelines for the design of the dynamic exciting signal $\{u_{td}\}$ are provided by the following two propositions.

Proposition 2. For given $u_{DC} \geq \overline{x}$, the sequence $\{x_t\}$ satisfies (31) if and only if:

$$\|\{u_{td}\}\|_\infty \leq \frac{|u_{DC} - \overline{x}|}{\|g\|_1} \quad (38)$$

where g and $\|g\|_1$ are, respectively, the impulse response and the ℓ_1 norm of the linear block; $\|\cdot\|_\infty$ is the ℓ_∞ norm of a sequence.

Proposition 3. For given $u_{DC} \leq \underline{x}$, the sequence $\{x_t\}$ satisfies (32) if and only if:

$$\|\{u_{td}\}\|_\infty \leq \frac{|u_{DC} - \underline{x}|}{\|g\|_1} \quad (39)$$

Propositions 2 and 3 are straightly derived from the definition of ℓ_∞-norm/ℓ_∞-norm system gain which equals the ℓ_1-norm of g.

Remark 2 — Note that Proposition 2 and Proposition 3 give necessary and sufficient conditions for $\{w_t\}$ to satisfy either inequality (26) or (27).

Remark 3 — Note that even if only a rough upper bound g_{up} of $\|g\|_1$ is known, inequality (38) is satisfied by choosing an input dynamic exciting sequence $\{u_{td}\}$ such that

$$\|\{u_{td}\}\|_\infty \leq \frac{|u_{DC} - \bar{x}|}{g_{up}} \quad (40)$$

while, (39) is satisfied by choosing an input dynamic exciting sequence $\{u_{td}\}$ such that

$$\|\{u_{td}\}\|_\infty \leq \frac{|u_{DC} - \underline{x}|}{g_{up}} \quad (41)$$

Otherwise, when no a priori information on the ℓ_1-norm of the linear systems is available, inequalities (38) and (39) can be indirectly satisfied varying the amplitude of the sequence $\{u_{td}\}$ by trial and error until inequalities (26) and (27) are met by the output sequence $\{w_t\}$, i.e., until the measured output sequence $\{y_t\}$ satisfies the following inequalities $\forall t$:

$$(y_t - \Delta\eta_t) > \bar{w} \text{ or } (y_t + \Delta\eta_t) < \underline{w}, \text{ when } n \text{ is odd} \quad (42)$$

or

$$\mathrm{sign}(\gamma_n)(y_t - \mathrm{sign}(\gamma_n)\Delta\eta_t) > \frac{1 + \mathrm{sign}(\gamma_n)}{2}\bar{w} + - \frac{1 - \mathrm{sign}(\gamma_n)}{2}\underline{w}, \text{ when } n \text{ is even} \quad (43)$$

5. EVALUATION OF BOUNDS ON THE UNMEASURABLE INNER SIGNAL.

Given the polynomial nonlinearity as characterized in Section 3 and a feasible sequence of measured outputs $\{y_t\}$ as characterized in Section 4, in this section it is shown how upper and lower bounds on the unmeasurable inner signal x_t are evaluated. Such bounds, together with the input sequence u_t will be used to bound the parameters of the linear dynamic block in Section 6. Combining equations (5), (6) and (7) one obtains

$$\left| y_t - \sum_{k=1}^{n} \gamma_k x_t^k \right| \leq \Delta\eta_t, \quad t = 1, 2, \ldots, N \tag{44}$$

Given the output measurement y_t, its uncertainty bounds $\Delta\eta_t$ and the feasible parameter set $\mathcal{D}_\gamma$, upper and lower bounds on the unmeasurable inner signal x_t are defined as

$$x_t^{max} = \max \left\{ x_t \in R : y_t - \sum_{k=1}^{n} \gamma_k x_t^k \leq \Delta\eta_t, \right. \\ \left. y_t - \sum_{k=1}^{n} \gamma_k x_t^k \geq -\Delta\eta_t, \ \text{for some } \gamma \in \mathcal{D}_\gamma \right\} \tag{45}$$

$$x_t^{min} = \min \left\{ x_t \in R : y_t - \sum_{k=1}^{n} \gamma_k x_t^k \leq \Delta\eta_t, \right. \\ \left. y_t - \sum_{k=1}^{n} \gamma_k x_t^k \geq -\Delta\eta_t, \ \text{for some } \gamma \in \mathcal{D}_\gamma \right\} \tag{46}$$

Equations (45) and (46) show nonlinear relations in the unknown x_t and the uncertain γ. The computation of x_t^{max} and x_t^{min} can be carried out through a one dimensional searching procedure. It is noticed that for a given $x_t \in R$, in order to solve each one of the above equation one must solve a set of $2M+2$ linear inequalities. It is also seen that the search can be started from the unique real root of one nominal polynomial obtained, e.g., setting $\gamma = \gamma^c$ and $\Delta\eta = 0$; next, right side and left side of that root are explored to find x_t^{max} and x_t^{min} respectively. If the following quantities are defined

$$x_t^c = \frac{x_t^{min} + x_t^{max}}{2} \tag{47}$$

$$\Delta x_t = \frac{x_t^{max} - x_t^{min}}{2} \tag{48}$$

a compact description of x_t in terms of x_t^c and δx_t is as follows

$$x_t = x_t^c + \delta x_t \tag{49}$$

$$|\ \delta x_t \ | \leq \Delta x_t. \tag{50}$$

6. BOUNDING THE PARAMETERS OF THE LINEAR DYNAMIC MODEL

In this section bounds on the parameters of the linear dynamic block are evaluated. The identification of the linear block can be formulated in the frame of output error models, i.e., in terms of the known input sequence $\{u_t\}$ and the uncertain inner sequence $\{x_t\}$ as shown in Figure 3. Combining equation (2), (3), (4) and (49) one gets

$$x_t^c = -\sum_{i=1}^{na} (x_{t-i}^c - \delta x_{t-i}) a_i + \sum_{j=0}^{nb} u_{t-j} b_j + \delta x_t. \tag{51}$$

The feasible parameter region for the linear system is defined as

$$\mathcal{D}_\theta = \{\theta \in R^p : A(q^{-1})[x_t^c - \delta x_t] = B(q^{-1}) u_t; \\ g = 1; |\ \delta x_t \ | \leq \Delta x_t; t = 1, \ldots, N\}. \tag{52}$$

Due to serial dependence between x_t samples at different time, exact parameter bounds for model (51) are no longer linear (Veres and Norton, 1991). Thus, in this paper, a polytopic outer approximation $\mathcal{D}'_\theta$ of the exact *FPR* $\mathcal{D}_\theta$, i.e. $\mathcal{D}'_\theta \supset \mathcal{D}_\theta$, will be presented, together with an orthotope-outer bounding set $\mathcal{B}_\theta$ of $\mathcal{D}'_\theta$, which provides parameters uncertainties intervals.

The dynamic model (51) with bounds on the inner signal uncertainty δx_t (50) fits in the framework of the bounded output error model outlined by Clement and Gentil (1988) and in the more general framework of the bounded-errors-in-variables model (see, e.g. Cerone, 1993; Veres and Norton, 1991). Indeed, the feasible parameter region $\mathcal{D}'_\theta$ can be described by

$$(\phi_t - \Delta\phi_t)^{\mathrm{T}} \theta \leq y_t + \Delta\eta_t \tag{53}$$

$$(\phi_t + \Delta\phi_t)^{\mathrm{T}} \theta \geq y_t - \Delta\eta_t \tag{54}$$

where

$$\phi_t^{\mathrm{T}} = \left[-x_{t-1}^c \ldots - x_{t-na}^c \ u_t \ u_{t-1} \ldots u_{t-nb} \right] \tag{55}$$

$$\Delta\phi_t^{\mathrm{T}} = [\Delta x_{t-1} \mathrm{sgn}(a_1) \ \ldots \ \Delta x_{t-na} \mathrm{sgn}(a_{na}) \\ 0 \ 0 \ \ldots \ 0] \tag{56}$$

A further significant reduction of $\mathcal{D}'_\theta$ is obtained adding the constraint about the steady-state gain. As a matter of fact, equation (10), which can be written in the following form

$$[1 \ \ldots \ 1 \ -1 \ -1 \ \ldots \ -1] \, \theta = -1 \tag{57}$$

forces the feasible parameter region to belong to the hyperplane described by (57).

The orthotope-outer bounding set $\mathcal{B}_\theta$ is defined as

$$\mathcal{B}_\theta = \{\theta \in R^p : \theta_j = \theta_j^c + \delta\theta_j, \\ |\ \delta\theta_j \ | \leq \Delta\theta_j/2, j = 1, \ldots, p\}, \tag{58}$$

where

$$\theta_j^c = \frac{\theta_j^{min} + \theta_j^{max}}{2}, \tag{59}$$

$$\Delta\theta_j = |\ \theta_j^{max} - \theta_j^{min} \ |, \tag{60}$$

and

$$\theta_j^{min} = \min_{\theta \in \mathcal{D}'_\theta} \theta_j, \quad \theta_j^{max} = \max_{\theta \in \mathcal{D}'_\theta} \theta_j. \tag{61}$$

Parameter vectors γ^c and θ^c are Chebishev centers in the ℓ_∞ norm of $\mathcal{D}_\gamma$ and $\mathcal{D}'_\theta$ respectively and are commonly referred to as central estimates.

7. CONCLUSIONS

In this paper the identification of SISO Wiener models has been considered when the nonlinear block can be modeled by a polynomial, with finite and known order, and when the output measurements are corrupted by unknown but bounded noise. First, using steady-state input-output data, parameters of the nonlinear block have been tightly bounded. Next in order to estimate the parameter of the linear block, the evaluation of the inner unmeasurable signal has been considered under the hypothesis of non-invertibility of the nonlinear block. Conditions under which the inner signal can be consistently estimated by input-output data have been established and, on the basis of such conditions, the design of a suitable input sequence has been outlined. Then, through a dynamic experiment, upper and lower bounds on the inner signal have been computed. Finally, such bounds, together with the input sequence, have been used for bounding the parameters of the linear block.

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9. REFERENCES

Bai, E.W. (2002). A blind approach to the Hammerstein - Wiener model identification. *Automatica* **38**, 967–979.

Billings, S.A. (1980). Identification of nonlinear systems — a survey. *IEE Proc. Part D* **127**(6), 272–285.

Billings, S.A. and S.Y. Fakhouri (1977). Identification of nonlinear systems using Wiener model. *Electron. Lett.* **17**, 502–504.

Cerone, V. (1993). Feasible parameter set for linear models with bounded errors in all variable. *Automatica* **29**(6), 1551–1555.

Cerone, V. M. Milanese and D. Regruto (2002). Parameters bounds for Wiener models. *Internal Report DAI0215.*

Clement, T. and S. Gentil (1988). Recursive membership set estimation for armax models: an output-error approach. In: *Proc. 12th IMACS Congress on Scientific Computation.* pp. 484–486.

Haber, R. and H. Unbehauen (1990). Structure identification of nonlinear dynamic systems – a survey on input/uotput approaches. *Automatica* **26**(4), 651–677.

Kalafatis, A.D. L. Wang and W.R. Cluett (1997). Identification of Wiener-type nonlinear systems in a noisy enviroment. *Int. J. Control* **66**(6), 923–941.

Krzyżak, A. (1993). Identification of nonlinear block-oriented systems by the recursive kernel estimate. *Int. J. Franklin Inst.* **330**(3), 605–627.

Lang, Z.Q. (1993). Controller design oriented model identification method for Hammerstein system. *Automatica* **29**(3), 767–771.

Milanese, M. and A. Vicino (1991). Optimal estimation theory for dynamic sistems with set membership uncertainty: an overview. *Automatica* **27(6)**, 997–1009.

Milanese, M. Norton, J. Piet-Lahanier, H. and Walter, E. Eds.) (1996). *Bounding approaches to system identification.* Plenum Press. New York.

Norton(Ed.), J.P. (1994). Special issue on bounded-error estimation. *Int. J. of Adaptive Control & Signal Processing.*

Norton(Ed.), J.P. (1995). Special issue on bounded-error estimation. *Int. J. of Adaptive Control & Signal Processing.*

Stoica, P. and T. Söderström (1982). Instrumental-variable methods for identification of hammerstein systems. *Int. J. Control* **35**(3), 459–476.

Sun, L. W. Liu and A. Sano (1999). Identification of a dynamical system with input nonlinearity. *IEE Proc. Part D* **146**(1), 41–51.

Veres, S.M. and J.P. Norton (1991). Parameter-bounding algorithms for linear errors in variables models. In: *Proc. of IFAC/IFORS Symposium on Identification and System Parameter Estimation.* pp. 1038–1043.

Walter, E. and H. Piet-Lahanier (1990). Estimation of parameter bounds from bounded-error data: a survey. *Mathematics and Computers in simulation* **32**, 449–468.

Wigren, T. (1998). Output error convergence of adaptive filters with compensation for output nonlinearities. *IEEE Trans. Automatic. Control* **43**(7), 975–978.

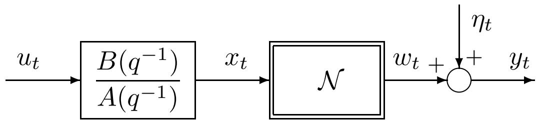

Fig. 1. Single-input single-output Wiener model.

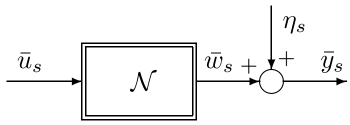

Fig. 2. Steady-state behaviour of the Wiener model when $g = 1$.

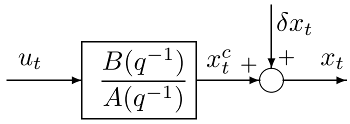

Fig. 3. Output error set-up for bounding the parameters of the linear system.