A two-stage parameter bounding procedure for Hammerstein models with bounded output errors

V. Cerone, D. Regruto
Dipartimento di Automatica e Informatica, Politecnico di Torino
corso Duca degli Abruzzi 24, 10129 Torino, Italy
vito.cerone@polito.it, diego.regruto@polito.it

Abstract

In this paper we present a two-stage procedure for deriving parameters bounds in Hammerstein models when the output measurement errors are bounded. First, using steady-state input-output data, parameters of the nonlinear part are tightly bounded. Then, for a given input transient sequence we evaluate tight bounds on the unmeasurable inner signal which, together with noisy output measurements are used for bounding the parameters of the linear dynamic block.

1 Introduction

Most physical systems are inherently nonlinear, and, though in some cases they can be represented by linear models over a restricted operating range, only nonlinear representations are adequate for their description. A wide class of nonlinear systems, also called block-oriented systems, can be modeled by interconnected memoryless nonlinear gains and linear subsystems. Nonlinearities may enter the system in different ways: either at the input or at the output end or in the feedback path around a linear model. The configuration we are dealing with in this paper, commonly referred to as Hammerstein model, is shown in Figure 1; it consists of a static nonlinear part $\mathcal{N}$ followed by a linear dynamic system. The identification of such a model relies solely on input-output measurements, while the inner signal x_t, i.e. the output of the nonlinear block, is not assumed to be available.

Identification of the Hammerstein structure has attracted the attention of many authors, as can be seen in [1], [2]. Existing identification procedures can be mainly classified in parametric and nonparametric methods. In the parametric approach, the nonlinearity is usually modeled by a polynomial with a finite and known order and the problem is formulated in terms of linear multi-input single-output parameter estimation. Within this framework, a number of authors give a solution in which existing linear estimation techniques are exploited by proper extension (see, e.g., [3, 4, 5]). Algorithms available in the literature estimate the parameters of the nonlinear block and the linear dynamic part either iteratively or simultaneously. The main problem with iterative procedures is that it is not possible to prove convergence of the estimate under general conditions [6]. On the other hand, as noted by Stoica and Söderström [7], consistent simultaneous estimates are reached under quite unrealistic conditions. Stoica and Söderström [7] proposed a parametric instrumental variable method which, in the presence of either a strictly persistently exciting sequence or a white noise, provides consistent estimates. On the nonparametric side, most of the methods use kernels regression estimates to identify the nonlinearity (see, e.g., [8]).

In all the papers mentioned above, the authors assume that the measurement error η_t is statistically described. However, there are many cases where in practice either a priori statistical hypotheses are seldom satisfied or the errors are better characterized in a deterministic way. Some examples are given by systematic and class errors in measurement equipments, rounding and truncation errors in digital devices. A worthwhile alternative to the stochastic description of measurement errors is the bounded-errors characterization, where uncertainties are assumed to belong to a given set. In the bounding context, all parameter vectors belonging to the *feasible parameter set* (*FPS*), i.e. parameters consistent with the measurements, the error bounds and the assumed model structure, are feasible solutions of the identification problem. The interested reader can find further details on this approach in a number of survey papers (see, e.g., [9, 10]), in the book edited by Milanese *et al.* [11] and the special issues edited by Norton [12, 13].

To our best knowledge, only few contributions can be found which address the identification of Hammerstein models when the measurement error η_t is supposed to be bounded. Belforte and Gay [14] considered an Hammerstein model where the linear block is described by an ARX model. They proposed a solution through the introduction of a linearized augmented Hammerstein model (see, e.g., [4]), whose parameters are identified first using any algorithm available in the parameter bounding literature. From the parameter bounds of such a model, overbounds on both nonlinear and linear block parameters are then derived. Boutayeb and Darouach [15] proposed a recursive estimator which provides a single parameters vector belonging to the feasible parameter region defined on a suitable finite horizon time. Garulli, Giarrè and Zappa [16] considered the identification of low complexity approximate Hammerstein models for a class of nonlinear systems. They proposed a procedure for the computation of the Chebichev conditional center of the *FPS* when the noise is bounded in either ℓ_∞ or ℓ_2 norm.

In this paper we consider the identification of single-input single-output (SISO) Hammerstein models when the nonlinear block can be modeled by a linear combi-

Proceedings of the American Control Conference
Denver, Colorado June 4-6, 2003

nation of a finite and known number of nonlinear static functions, the linear dynamic part is described by an output error model and the output measurement errors are bounded. We present a two-stage identification procedure. First, parameters of the nonlinear block are tightly bounded using input-output data collected from the steady-state response of the system to a set of step inputs with different amplitude. Then, through a dynamic experiment, for all u_t belonging to a given input transient sequence $\{u_t\}$, we compute bounds on the inner signal which, together with noisy output measurements are used for bounding the parameters of the linear part.

2 Problem formulation

Consider the SISO discrete-time Hammerstein model depicted in Figure 1, where the nonlinear block maps the input signal u_t into the unmeasurable inner variable x_t through the following nonlinear function

$$x_t = \sum_{k=1}^{n} \gamma_k \psi_k(u_t), \quad t = 1, \ldots, N; \tag{1}$$

where $(\psi_1, \ldots, \psi_n)$ is a known basis of nonlinear functions; N is the length of the input sequence. The linear dynamic part is modeled by a discrete-time system which transforms x_t into the noise-free output w_t according to

$$w_t = \frac{B(q^{-1})}{A(q^{-1})} x_t = G(q^{-1}) x_t, \tag{2}$$

or, equivalently, in terms of a linear difference equation

$$A(q^{-1}) w_t = B(q^{-1}) x_t, \tag{3}$$

where $A(\cdot)$ and $B(\cdot)$ are polynomials in the backward shift operator q^{-1}, $(q^{-1} w_t = w_{t-1})$,

$$A(q^{-1}) = 1 + a_1 q^{-1} + \ldots + a_{na} q^{-na}, \tag{4}$$

$$B(q^{-1}) = b_0 + b_1 q^{-1} + \ldots + b_{nb} q^{-nb}. \tag{5}$$

In line with the work done by a number of authors, we assume that (i) the linear system is asymptotically stable (see, e.g., [7, 8, 17, 18, 19]); (ii) $\sum_{j=0}^{nb} b_j \neq 0$, that is, the steady-state gain is not zero (see, e.g. [17, 18, 19]); (iii) the only *a priori* information needed is an estimate of the process settling-time (see, e.g., [20]). Let y_t be the noise-corrupted measurements of w_t

$$y_t = w_t + \eta_t. \tag{6}$$

Measurements uncertainty is known to range within given bounds $\Delta\eta_t$, i.e.,

$$|\, \eta_t \,| \leq \Delta\eta_t. \tag{7}$$

Unknown parameter vectors $\gamma \in R^n$ and $\theta \in R^p$ are defined, respectively, as

$$\gamma^T = [\gamma_1 \quad \gamma_2 \quad \ldots \gamma_n], \tag{8}$$

$$\theta^T = [a_1 \quad \ldots \quad a_{na} \quad b_0 \quad b_1 \quad \ldots b_{nb}], \tag{9}$$

where $n_a + n_b + 1 = p$. It is easy to show that the parameterization of the structure of Figure 1 is not unique. As a matter of fact, any parameters set $\tilde{b}_j = \alpha^{-1} b_j, j = 1, 2, \ldots, nb$, and $\tilde{\gamma}_k = \alpha\gamma_k, k = 1, 2, \ldots, n$, for some nonzero and finite constant α, provides the same input-output behaviour. Thus, any identification procedure cannot perceive the difference between parameters $\{b_j, \gamma_k\}$ and $\{\alpha^{-1} b_j, \alpha\gamma_k\}$. To get a unique parameterization, a technique widely used in the literature consists of adding the constraints $b_0 = 0$ and $b_1 = 1$. That choice, however, leads to the exclusion of discrete-time linear system with zero input delay and forces the presence of the term with one step input delay, which may not always be the case. Some other authors normalize the coefficients of the nonlinear block assuming $\gamma_1 = 1$. In this work, we assume, without loss of generality, that the steady-state gain of the linear part be one, that is

$$g = \frac{\sum_{j=0}^{nb} b_j}{1 + \sum_{i=1}^{na} a_i} = 1 \tag{10}$$

In this paper we address the problem of deriving bounds on parameters γ and θ consistently with given measurements, error bounds and the assumed model structure. In Section 3, using steady-state input-output data, parameters of the nonlinear part are tightly bounded, while in Section 4, for a given input transient sequence we evaluate bounds on the unmeasurable inner signal which, together with noisy output measurements are used for bounding the parameters of the linear part. A simulated example is reported in Section 5.

3 Assessment of tight bounds on the nonlinear static block parameters

In most physical process we can collect a great deal of data, which often contains steady-state measurements at many different operating conditions. However, usually, only transient data are used in the identification process while steady-state measurements are not explicitly considered. Although data are assumed to be generated by a persistently exciting input, in practice a given plant might only be mildly perturbed around operating conditions, leading to shortage of proper nonlinear information in the transient data. In this work we exploit steady-state operating conditions to bound the parameters of the nonlinear static block. The known input and noise corrupted output sequences are collected from the steady-state response of the system to a set of step inputs with different amplitude. We only assume to have a rough idea of the settling time of the system under consideration, in order to know when steady-state conditions are reached, so that steady-state data can be collected. Indeed, under conditions (i), (ii) and (iii) stated in Section 2, combining equations (1), (3), (6) and (10) in steady-state operating conditions we get the following input-output description

$$\bar{y}_s = \sum_{k=1}^{n} \gamma_k \psi_k(\bar{u}_s) + \bar{\eta}_s, \quad s = 1, \ldots, M \tag{11}$$

where $\bar{u}_s$, $\bar{y}_s$ and $\bar{\eta}_s$ are steady-state values of the known input signal, output observation and measurement error respectively; $M \geq n$ is the length of the steady-state sequences. A block diagram description of equation (11) is depicted in Figure 2. The feasible parameter region of the static nonlinear block is defined as

$$\begin{aligned}\mathcal{D}_\gamma = \{\gamma{\in}R^n{:}\, \bar{y}_s = \sum_{k=1}^{n} \gamma_k \psi_k(\bar{u}_s) + \bar{\eta}_s, \\ \mid \bar{\eta}_s \mid \leq \Delta\bar{\eta}_s;\ \ s = 1, \ldots, M\},\end{aligned} \tag{12}$$

where $\{\Delta\bar{\eta}_s\}$ is the sequence of bounds on measurements uncertainty. We further note that the description (1) adopted for the nonlinear block provides a linearly parameterized model. From definition (12) it can be seen that $\mathcal{D}_\gamma$ is exactly described by the following constraints in the n-dimensional parameter space

$$\bar{\varphi}_s^{\mathrm{T}}\gamma \leq \bar{y}_s + \Delta\bar{\eta}_s \tag{13}$$

$$\bar{\varphi}_s^{\mathrm{T}}\gamma \geq \bar{y}_s - \Delta\bar{\eta}_s \tag{14}$$

where

$$\bar{\varphi}_s = [\psi_1(\bar{u}_s)\quad \psi_2(\bar{u}_s)\quad \psi_3(\bar{u}_s) \ldots \psi_n(\bar{u}_s)]^{\mathrm{T}} \tag{15}$$

for $s = 1, 2, \ldots, M$. The above exact description of $\mathcal{D}_\gamma$ will be used in the next section when deriving tight bounds on the unmeasurable inner signal x_t. Since $\mathcal{D}_\gamma$ is a convex polytope, whose shape may result quite complex for increasing n and M, an outer bound to it such as an ellipsoid or a box is often computed. In this paper we consider an orthotope-outer bounding set $\mathcal{B}_\gamma$ containing $\mathcal{D}_\gamma$

$$\begin{aligned}\mathcal{B}_\gamma = \{\gamma \in R^n : \gamma_j = \gamma_j^c + \delta\gamma_j, \\ \mid \delta\gamma_j \mid \leq \Delta\gamma_j/2, j = 1, \ldots, n\},\end{aligned} \tag{16}$$

where

$$\gamma_j{}^c = \frac{\gamma_j^{min} + \gamma_j^{max}}{2}, \tag{17}$$

$$\Delta\gamma_j = \mid \gamma_j^{max} - \gamma_j^{min} \mid, \tag{18}$$

and

$$\gamma_j^{min} = \min_{\gamma\in\mathcal{D}_\gamma} \gamma_j, \quad \gamma_j^{max} = \max_{\gamma\in\mathcal{D}_\gamma} \gamma_j. \tag{19}$$

The set $\mathcal{B}_\gamma$ as defined in (16) is a tight orthotope outer-bound on the exact feasible parameter region $\mathcal{D}_\gamma$ and its evaluation requires the solution of $2n$ linear programming problems with n variables and $2M$ constraints.

4 Bounding the parameters of the linear dynamic model

In the second stage of our procedure we evaluate bounds on the parameters of the linear dynamic block. Given the feasible parameter set $\mathcal{D}_\gamma$, for any given input sequence $\{u_t\} \in R^N$, bounds on the inner unmeasurable signal x_t can be evaluated through the following expressions

$$x_t^{min} = \min_{\gamma\in\mathcal{D}_\gamma} \varphi_t^{\mathrm{T}}\gamma, \quad x_t^{max} = \max_{\gamma\in\mathcal{D}_\gamma} \varphi_t^{\mathrm{T}}\gamma \tag{20}$$

where $\varphi_t = [\psi_1(u_t)\quad \psi_2(u_t)\quad \psi_3(u_t) \ldots \psi_n(u_t)]^{\mathrm{T}}$, $t = 1, 2, \ldots, N$. Computation of bounds in equation (20) requires the solution of $2N$ LP problems with n variables and $2M$ constraints. If we define the following quantities

$$x_t^c = \frac{x_t^{min} + x_t^{max}}{2} \tag{21}$$

$$\Delta x_t = \frac{x_t^{max} - x_t^{min}}{2} \tag{22}$$

a compact description of x_t in terms of its central value x_t^c and its perturbation δx_t is as follows

$$x_t = x_t^c + \delta x_t \tag{23}$$

$$\mid \delta x_t \mid \leq \Delta x_t. \tag{24}$$

We can formulate the identification of the linear model in terms of the noisy output sequence $\{y_t\}$ and the uncertain inner sequence $\{x_t\}$ as shown in Figure 3. Such a formulation is commonly referred to as an errors-in-variables problem (EIV), i.e. a parameter estimation problem in a linear-in-parameter model where the output and some or all the explanatory variables are uncertain. As a matter of fact, combining equations (3), (4), (5), (6), (23) we get

$$y_t {=} {-}\sum_{i=1}^{na}(y_{t-i} - \eta_{t-i})a_i {+} \sum_{j=0}^{nb}(x_{t-j}^c + \delta x_{t-j})b_j {+} \eta_t. \tag{25}$$

The feasible parameter region for the linear system is defined as

$$\begin{aligned}\mathcal{D}_\theta = \{\theta{\in}R^p{:}\, A(q^{-1})[y_t - \eta_t] = B(q^{-1})[x_t^c + \delta x_t]; \\ g = 1; \mid \eta_t \mid \leq \Delta\eta_t; \mid \delta x_t \mid \leq \Delta x_t; t = 1, \ldots, N\}.\end{aligned} \tag{26}$$

From equation (25) it can be seen that consecutive regressions are related deterministically by uncertain output samples and uncertain input samples (dynamic EIV) giving rise to possible nonlinear exact parameter bounds, which could be not easily and exactly computed [21]. On the other end, when the uncertain variables appearing in successive regressions are supposed to vary independently (static EIV) exact parameter bounds are piecewise linear and, thus, as shown in [21], can be more conveniently handled than the FPS of dynamic EIV. That motivates the use, in this paper, of results from the static EIV [22] which, however, will lead to outer approximations of the exact FPS. Thus, in this work, a polytopic outer approximation $\mathcal{D}'_\theta$ of the exact FPS $\mathcal{D}_\theta$, i.e. $\mathcal{D}'_\theta \supset \mathcal{D}_\theta$, will be presented, together with an orthotope-outer bounding set $\mathcal{B}_\theta$ of $\mathcal{D}'_\theta$. When we apply results from [22] to our problem we get the following description of the feasible parameter set $\mathcal{D}'_\theta$ at the single time t

$$(\phi_t - \Delta\phi_t)^{\mathrm{T}}\theta \leq y_t + \Delta\eta_t \tag{27}$$

$$(\phi_t + \Delta\phi_t)^{\mathrm{T}}\theta \geq y_t - \Delta\eta_t \tag{28}$$

where

$$\phi_t^{\mathrm{T}} = \left[-y_{t-1} \ldots - y_{t-na}\ x_t^c\ x_{t-1}^c \ldots x_{t-nb}^c\right] \tag{29}$$

$$\begin{aligned}\Delta\phi_t^{\mathrm{T}} = [\Delta\eta_{t-1}\mathrm{sgn}(a_1)\ \ \ldots\ \ \Delta\eta_{t-na}\mathrm{sgn}(a_{na}) \\ \Delta x_t\mathrm{sgn}(b_0)\ \ \Delta x_{t-1}\mathrm{sgn}(b_1)\ \ \ldots\ \ \Delta x_{t-nb}\mathrm{sgn}(b_{nb})]\end{aligned} \tag{30}$$

Proceedings of the American Control Conference
Denver, Colorado June 4-6, 2003

A further significant reduction of $\mathcal{D}'_\theta$ is obtained adding the constraint about the steady-state gain, equation (10), written in the following form

$$[1 \quad \ldots \quad 1 \quad -1 \quad -1 \quad \ldots \quad -1]\,\theta = -1 \qquad (31)$$

The orthotope-outer bounding set $\mathcal{B}_\theta$ is defined as

$$\begin{aligned}\mathcal{B}_\theta = \{\theta \in R^p : \theta_j = \theta_j^c + \delta\theta_j,\\ \mid \delta\theta_j \mid \leq \Delta\theta_j/2, j = 1,\ldots,p\},\end{aligned} \qquad (32)$$

where

$$\theta_j^c = \frac{\theta_j^{min} + \theta_j^{max}}{2}, \qquad (33)$$

$$\Delta\theta_j = \mid \theta_j^{max} - \theta_j^{min} \mid, \qquad (34)$$

and

$$\theta_j^{min} = \min_{\theta \in \mathcal{D}'_\theta} \theta_j, \quad \theta_j^{max} = \max_{\theta \in \mathcal{D}'_\theta} \theta_j. \qquad (35)$$

Parameter vectors γ^c and θ^c are Chebishev centers in the ℓ_∞ norm of $\mathcal{D}_\gamma$ and $\mathcal{D}'_\theta$ respectively and are commonly referred to as central estimates.

5 A simulated example

In this section we illustrate the proposed parameter bounding procedure through a numerical example. The system considered here is characterized by (1), (3) and (6) with: $\gamma_1 = 1$, $\gamma_2 = 1$, $\gamma_3 = 1$, $\psi_1(u_t) = u_t$; $\psi_2(u_t) = u_t^2$; $\psi_3(u_t) = u_t^3$; $A(q^{-1}) = (1 - 1.1q^{-1} + 0.28q^{-2})$ and $B(q^{-1}) = (0.1q^{-1} + 0.08q^{-2})$. Thus, the true parameter vectors are $\theta_\gamma = [\gamma_1\ \gamma_2\ \gamma_3]^{\mathrm{T}} = [1\ 1\ 1]^{\mathrm{T}}$ and $\theta = [a_1\ a_2\ b_1\ b_2]^{\mathrm{T}} = [-1.1\ 0.28\ 0.1\ 0.08]^{\mathrm{T}}$. Two different structures of measurement errors are considered: relative and absolute error. From the simulated transient sequence $\{w_t, \eta_t\}$ and steady-state data $\{\bar{w}_s, \bar{\eta}_s\}$, the signal to noise ratios SNR and $\overline{SNR}$ are evaluated, respectively, through

$$SNR = 10\log\left\{\sum_{t=1}^{N} w_t^2 \Big/ \sum_{t=1}^{N} \eta_t^2\right\} \qquad (36)$$

$$\overline{SNR} = 10\log\left\{\sum_{s=1}^{M} \bar{w}_s^2 \Big/ \sum_{s=1}^{M} \bar{\eta}_s^2\right\} \qquad (37)$$

Relative errors — First, bounded relative output errors have been considered for simulating the collection of both steady state and transient data. More precisely, we assumed $\eta_t = \epsilon_t^y y_t$, $\mid \epsilon_t^y \mid \leq \Delta\epsilon_t^y$, $\bar{\eta}_s = \bar{\epsilon}_s^y \bar{y}_s$, $\mid \bar{\epsilon}_s^y \mid \leq \Delta\bar{\epsilon}_s^y$; where $\{\epsilon_t^y\}$ and $\{\bar{\epsilon}_s^y\}$ are random sequences belonging to the uniform distributions $U[-\Delta\epsilon_t^y, +\Delta\epsilon_t^y]$ and $U[-\Delta\bar{\epsilon}_s^y, +\Delta\bar{\epsilon}_s^y]$ respectively. Bounds on steady-state and transient output measurement errors were supposed to have the same value, i.e., $\Delta\epsilon_t^y = \Delta\bar{\epsilon}_s^y \triangleq \Delta\epsilon^y$. Five different values of uncertainty bounds were considered: $\Delta\epsilon^y = 0.1\%,\ 1\%, 10\%, 20\%$. For a given $\Delta\epsilon^y$, the length of steady-state and the transient data are $M = 10$ and $N = [100, 1000]$ respectively. The steady-state input sequence $\{\bar{u}_s\}$ belongs to the interval $[-2, +2]$, while the transient input sequence $\{u_t\}$ belongs to the uniform distribution $U[-2, +2]$. Results about the nonlinear block are reported in Table I, where the reader can find parameter central estimates (γ_j^c), and parameter bounds $(\gamma_j^{min}, \gamma_j^{max})$ against varying measurements uncertainty $(\Delta\epsilon^y)$ and signal to noise ratio $\overline{SNR}$. Table III shows numerical results obtained for the linear system. There we can see parameter central estimates (θ_j^c) and parameter bounds $(\theta_j^{min}, \theta_j^{max})$ against varying measurements uncertainty $(\Delta\epsilon^y)$, signal to noise ratio SNR, and number of transient samples (N). For low noise level $(\Delta\epsilon^y = 0.1\%)$ and for all N, the central estimates of both the nonlinear static block and the linear model are consistent with the true parameters. For higher noise level $(\Delta\epsilon^y \geq 1\%)$, both γ^c and θ^c give satisfactory estimation of the true parameters. As the number of observations increases (from $N = 100$ to $N = 1000$), the width of all parameter uncertainty intervals $[\gamma_j^{min}, \gamma_j^{max}]$ and $[\theta_j^{min}, \theta_j^{max}]$ decreases unsurprisingly. Finally, one of the aspects to be evaluated about algorithms is the computing time since it provides a measure of their efficiency. Simulations introduced in this paper were developed on a Pentium II, 300 MHz personal computer. The CPU time required to run the complete simulation was about 240 seconds. One session on steady-state data processing (n=3, M=10) took about 0.2 seconds, while processing transient data (p=4, N=1000) required about 40 seconds.

Absolute errors — Next, bounded absolute output errors have been considered for simulating the collection of both steady state data, $\{\bar{u}_s, \bar{y}_s\}$, and transient sequence $\{u_t, y_t\}$. Here we assumed $\mid \eta_t \mid \leq \Delta\eta_t$ and $\mid \bar{\eta}_s \mid \leq \Delta\bar{\eta}_s$ where η_t and $\bar{\eta}_s$, are random sequences belonging to the uniform distributions $U[-\Delta\eta_t, +\Delta\eta_t]$ and $U[-\Delta\bar{\eta}_s, +\Delta\bar{\eta}_s]$ respectively. Bounds on steady-state and transient output measurement errors were supposed to have the same value, i.e., $\Delta\eta_t = \Delta\bar{\eta}_s \triangleq \Delta\eta$, and were chosen in such a way as to simulate five different values of signal to noise ratio at the output, namely 60 dB, 50 dB, 30 dB and 20 dB. For a given $\Delta\eta$, the length of steady-state and the transient data are $M = 10$ and $N = [100, 1000]$ respectively. The steady-state input sequence $\{\bar{u}_s\}$ belongs to the interval $[-2, +2]$, while the transient input sequence $\{u_t\}$ belongs to the uniform distribution $U[-2, +2]$. Results about the nonlinear block are reported in Table II. Table IV shows numerical results obtained for the linear system. For low noise level ($SNR = 60$ dB) and for all N, the central estimates of both the nonlinear static block and the linear model are consistent with the true parameters. For higher noise level ($SNR \leq 40$ dB), both γ^c and θ^c give satisfactory estimation of the true parameters. As the number of observations increases (from $N = 100$ to $N = 1000$), the width of all parameter uncertainty intervals $[\gamma_j^{min}, \gamma_j^{max}]$ and $[\theta_j^{min}, \theta_j^{max}]$ decreases, as expected. The CPU time required to run the complete simulation in the presence of absolute output errors was about 240 seconds. One session on steady-state data processing (n=3, M=10) took about 0.2 seconds, while processing transient data (p=4, N=1000) required about 40 seconds.

6 Conclusions

A two-stage parameter bounding procedure in SISO Hammerstein models for systems with bounded output errors has been outlined. First, using steady-state input-output data, parameters of the nonlinear block, which was assumed to be modeled by a linear combination of a finite and known number of nonlinear static functions, have been tightly bounded. Then, for a given input transient sequence we have computed bounds on the unmeasurable inner signal which, together with output noisy measurements have been used for overbounding the parameters of the linear part. The numerical example showed the effectiveness of the proposed procedure.

Acknowledgments

This research was partly supported by the italian Ministry of Universities and Research in Science and Technology (MURST), under the plan "Robustness techniques for control of uncertain systems".

References

[1] S.A. Billings, "Identification of nonlinear systems — a survey," *IEE Proc. Part D*, vol. 127, no. 6, pp. 272–285, 1980.

[2] R. Haber and H. Unbehauen, "Structure identification of nonlinear dynamic systems – a survey on input/uotput approaches," *Automatica*, vol. 26, no. 4, pp. 651–677, 1990.

[3] K.S. Narenda and P.G. Gallman, "An iterative method for the identification of nonlinear systems using a Hammerstein model," *IEEE Trans. Automatic Control*, vol. AC-11, pp. 546–550, 1966.

[4] F.H.I. Chang and R. Luus, "A noniterative method for identification using Hammerstein model," *IEEE Trans. Automatic Control*, vol. AC-16, pp. 464–468, 1971.

[5] T.C. Hsia, "A multi stage least squares method for identifying Hammerstein model nonlinear systems," in *Proc. of IEEE Conference on Decision and Control*, 1976, pp. 934–938.

[6] P. Stoica, "On the convergence of an iterative algorithm used for Hammerstein system identification," *IEEE Trans. Automatic Control*, vol. AC-26, no. 4, pp. 967–969, 1981.

[7] P. Stoica and T. Söderström, "Instrumental-variable methods for identification of Hammerstein systems," *Int. J. Control*, vol. 35, no. 3, pp. 459–476, 1982.

[8] A. Krzyżak, "Identification of nonlinear block-oriented systems by the recursive kernel estimate," *Int. J. Franklin Inst.*, vol. 330, no. 3, pp. 605–627, 1993.

[9] M. Milanese and A. Vicino, "Optimal estimation theory for dynamic sistems with set membership uncertainty: an overview," *Automatica*, vol. 27(6), pp. 997–1009, 1991.

[10] E. Walter and H. Piet-Lahanier, "Estimation of parameter bounds from bounded-error data: a survey," *Mathematics and Computers in simulation*, vol. 32, pp. 449–468, 1990.

[11] M. Milanese, J. Norton, H. Piet-Lahanier, and E. Walter, Eds., *Bounding approaches to system identification*, Plenum Press, New York, 1996.

[12] J.P. Norton(Ed.), "Special issue on bounded-error estimation," *Int. J. of Adapt. Control & Sign. Proces.*, vol. 8, no. 1, 1994.

[13] J.P. Norton(Ed.), "Special issue on bounded-error estimation," *Int. J. of Adapt. Control & Sign. Proces.*, vol. 9, no. 1, 1995.

[14] G. Belforte and P. Gay, "Hammerstein model identification with set membership errors," in *Proc. of IEEE Conference on Decision and Control*, 1999, pp. 592–597.

[15] M. Boutayeb and M. Darouach, "Identification of Hammerstein model in the presence of bounded disturbances," in *Proc. of IEEE Int. Conference on Industrial Technology*, 2000, pp. 590–594.

[16] A. Garulli, L. Giarrè, and G. Zappa, "Identification of approximated Hammerstein models in a worst-case setting," *IEEE Trans. Automatic Control*, vol. 47, no. 12, pp. 2046–2050, 2002.

[17] Z.Q. Lang, "Controller design oriented model identification method for Hammerstein system," *Automatica*, vol. 29, no. 3, pp. 767–771, 1993.

[18] Z.Q. Lang, "A nonparametric polynomial identification algorithm for the Hammerstein system," *IEEE Trans. Automatic Control*, vol. 42, no. 10, pp. 1435–1441, 1997.

[19] L. Sun, W. Liu, and A. Sano, "Identification of a dynamical system with input nonlinearity," *IEE Proc. Part D*, vol. 146, no. 1, pp. 41–51, 1999.

[20] A.D. Kalafatis, L. Wang, and W.R. Cluett, "Identification of Wiener-type nonlinear systems in a noisy enviroment," *Int. J. Control*, vol. 66, no. 6, pp. 923–941, 1997.

[21] S.M. Veres and J.P. Norton, "Parameter-bounding algorithms for linear errors in variables models," in *Proc. of IFAC/IFORS Symposium on Identification and System Parameter Estimation*, 1991, pp. 1038–1043.

[22] V. Cerone, "Feasible parameter set for linear models with bounded errors in all variable," *Automatica*, vol. 29, no. 6, pp. 1551–1555, 1993.

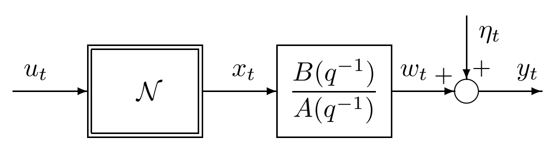

Figure 1: Single-input single-output Hammerstein model.

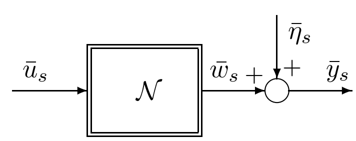

Figure 2: Steady-state behaviour of the Hammerstein model when $g = 1$.

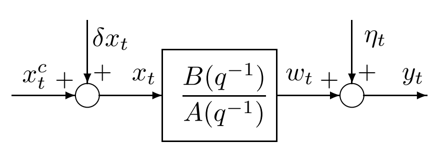

Figure 3: Errors-in-variables set-up for bounding the parameters of the linear system.

Proceedings of the American Control Conference
Denver, Colorado June 4-6, 2003

Table I: Relative Error — Nonlinear block parameter central estimates (γ_j^c) and parameter bounds (γ_j^{min}, γ_j^{max}) against varying measurements uncertainty ($\Delta\epsilon^y$) and signal to noise ratio ($\overline{SNR}$).

$\Delta\epsilon^y$ (%)	$\overline{SNR}$ (dB)	True Value	γ_j^{min}	γ_j^c	γ_j^{max}
0.1	61.4	1.000	1.000	1.000	1.001
		1.000	0.999	1.000	1.001
		1.000	0.998	0.999	1.000
1	43.8	1.000	0.990	0.997	1.005
		1.000	0.994	1.001	1.008
		1.000	0.994	1.000	1.007
10	25.7	1.000	0.952	1.059	1.166
		1.000	0.816	0.950	1.084
		1.000	0.799	0.919	1.040
20	17.6	1.000	0.893	0.997	1.102
		1.000	0.883	0.994	1.105
		1.000	0.827	0.961	1.095

Table II: Absolute Error — Nonlinear block parameter central estimates (γ_j^c) and parameter bounds (γ_j^{min}, γ_j^{max}) against signal to noise ratio ($\overline{SNR}$).

$\overline{SNR}$ (dB)	True Value	γ_j^{min}	γ_j^c	γ_j^{max}
58.9	1.000	0.996	1.001	1.006
	1.000	0.996	0.998	1.000
	1.000	0.997	1.000	1.002
49.3	1.000	0.989	1.016	1.042
	1.000	0.990	0.996	1.002
	1.000	0.986	0.996	1.006
31.7	1.000	0.914	1.085	1.257
	1.000	0.933	1.003	1.074
	1.000	0.906	0.979	1.052
19.7	1.000	0.822	1.344	1.865
	1.000	0.729	0.899	1.069
	1.000	0.734	0.945	1.156

Table III: Relative Error — Linear system parameter central estimates (θ_j^c) and parameter bounds (θ_j^{min}, θ_j^{max}) against varying measurements uncertainty ($\Delta\epsilon^y$), signal to noise ratio (SNR) and number of transient samples (N).

$\Delta\epsilon^y$ (%)	SNR (dB)	N	True Value	θ_j^{min}	θ_j^c	θ_j^{max}
0.1	64.8	100	-1.100	-1.101	-1.100	-1.098
			0.280	0.278	0.280	0.281
			0.100	0.100	0.100	0.100
			0.080	0.080	0.080	0.080
	64.9	1000	-1.100	-1.100	-1.100	-1.100
			0.280	0.280	0.280	0.280
			0.100	0.100	0.100	0.100
			0.080	0.080	0.080	0.080
1	44.6	100	-1.100	-1.120	-1.103	-1.086
			0.280	0.268	0.282	0.297
			0.100	0.099	0.100	0.101
			0.080	0.076	0.079	0.082
	44.9	1000	-1.100	-1.103	-1.099	-1.096
			0.280	0.276	0.279	0.283
			0.100	0.100	0.100	0.101
			0.080	0.079	0.080	0.081
10	24.7	100	-1.100	-1.255	-1.135	-1.014
			0.280	0.208	0.315	0.422
			0.100	0.090	0.106	0.122
			0.080	0.062	0.083	0.104
	24.8	1000	-1.100	-1.167	-1.123	-1.078
			0.280	0.255	0.302	0.350
			0.100	0.097	0.107	0.117
			0.080	0.069	0.081	0.092
20	18.8	100	-1.100	-1.549	-1.181	-0.812
			0.280	0.052	0.370	0.688
			0.100	0.075	0.101	0.128
			0.080	0.027	0.079	0.131
	18.8	1000	-1.100	-1.178	-1.103	-1.027
			0.280	0.209	0.284	0.360
			0.100	0.093	0.105	0.116
			0.080	0.066	0.081	0.095

Table IV: Absolute Error — Linear system parameter central estimates (θ_j^c) and parameter bounds (θ_j^{min}, θ_j^{max}) against signal to noise ratio (SNR) and number of transient samples (N).

SNR (dB)	N	True Value	θ_j^{min}	θ_j^c	θ_j^{max}
60.0	100	-1.100	-1.104	-1.100	-1.096
		0.280	0.277	0.280	0.284
		0.100	0.100	0.100	0.100
		0.080	0.079	0.080	0.080
59.9	1000	-1.100	-1.102	-1.100	-1.098
		0.280	0.278	0.280	0.282
		0.100	0.100	0.100	0.100
		0.080	0.080	0.080	0.080
47.8	100	-1.100	-1.114	-1.099	-1.084
		0.280	0.264	0.278	0.293
		0.100	0.099	0.101	0.102
		0.080	0.078	0.080	0.083
50.0	1000	-1.100	-1.108	-1.101	-1.094
		0.280	0.274	0.281	0.287
		0.100	0.099	0.100	0.101
		0.080	0.079	0.080	0.081
24.8	100	-1.100	-1.210	-1.096	-0.983
		0.280	0.175	0.281	0.387
		0.100	0.094	0.103	0.112
		0.080	0.066	0.084	0.102
29.7	1000	-1.100	-1.168	-1.096	-1.024
		0.280	0.210	0.274	0.338
		0.100	0.094	0.099	0.104
		0.080	0.068	0.081	0.094
13.8	100	-1.100	-1.821	-1.366	-0.911
		0.280	0.077	0.526	0.975
		0.100	0.037	0.092	0.147
		0.080	0.001	0.067	0.133
20.2	1000	-1.100	-1.514	-1.195	-0.877
		0.280	0.074	0.358	0.643
		0.100	0.084	0.105	0.126
		0.080	0.020	0.069	0.119