

Vehicle lateral controller design exploiting properties of SITO systems

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Abstract

In this paper the problem of lateral vehicle control in highway experimental conditions is addressed. The vehicle under consideration is equipped with an electric motor acting on the steering angle (the command input) and a vision system providing two measurements (the two outputs): the lateral displacement and the angular orientation of the vehicle with respect to the lane centerline. We show how, exploiting properties of single-input two-outputs systems, our original SITO control problem can be simplified to the design of a SISO controller. Experimental results obtained testing the designed controller on highways are reported.

1 Introduction

Driver assistance systems received a growing attention in the last twenty years due to its acknowledged ability to reduce driver's workload and to enhance driving safety (see, e.g., [14]). Many researches have been done facing a number of problems related to the subject. In the last years remarkable efforts have been focused on the solution of the lateral vehicle dynamics control problem on highway scenery. The objective is to design a steering controller able to keep the vehicle inside the lane on the basis of some measurements provided by the sensors system. Two main approaches have been investigated in the literature, the so-called *look-down* and *look-ahead* sensing schemes. The first approach relies on the measurements of the lateral displacement of the vehicle from the lane centerline provided by buried magnet or electrified wires placed along the road. The problem, in the case of passenger cars, was analyzed in this framework by Patwardhan *et al.* in [13] where they show the fundamental control difficulties of this approach. In the look-ahead sensing scheme, the distance between the vehicle longitudinal axis and the lane centerline measured few meters ahead the vehicle is used for feedback. Since such a quantity is usually provided by vision or radar systems located on the vehicle, this approach avoids the modification of infrastructures. A comparative study of vision-based control strategies was presented by Košeckà *et al.* in [11]. Other contributions based on the look-ahead sensing scheme were given by Hatipoğlu *et al.* in [9], where they use a digital videocamera together with a radar system, and by Broggi *et al.* [3] who used a stereo vision system composed by two videocameras. Significant Japanese contributions to the development of vision-based intelligent vehicles, given from the mid 1970's

till early 1990's are surveyed by Tsugawa [18]. In paper [5], a single-input single-output (SISO) control strategy based on driver behaviour emulation has been proposed and tested along highway paths on a FIAT Brava 1600 ELX provided by Centro Ricerche Fiat; experimental results showed the good performances of the designed control system. Relevant contributions to the application of advanced linear, nonlinear and robust control techniques to the design of lateral dynamics controller were provided by Tomizuka and coworkers (see, e.g., [10], [19], [15], [16]) and by Ackermann and coworkers (see, e.g., [1], [2], [8]).

In this paper we address the lane keeping problem through automatic steering (the steering angle being the input) in highway conditions using a vision system which provides the measurement of two quantities: the lateral vehicle displacement q (first output) and the vehicle orientation m (second output) with respect to a suitable approximation of the centerline of the lane. We focus our attention on the single-input two-output (SITO) structure of the considered system. Recently some attention has been paid to the analysis of properties of the single-input two-output systems, motivated by the large number of control problems which exhibit a SITO structure (see [6], [17] and the references therein). Lu and Tomizuka in [12] faced the problem of vehicle lateral control with combined use of laser scanning radar sensor and rear magnetometers. Using some of the properties introduced in [6], they propose a procedure for designing a two-input single-output (TISO) controller based on the minimization of the effects of the disturbance associated with one channel on the other one and *vice-versa*. In this work, exploiting a result introduced in [6], we show that our original SITO problem can be simplified to the controller design of a SISO control system. Experimental results obtained along highways are presented.

2 Plant description and modeling

The plant to be controlled, provided by Centro Ricerche Fiat, consists of a Fiat Brava 1600 ELX equipped with a vision system and a steering actuator. The vision system is composed by a single CCD video-camera, located on the wind shield, and related image processing algorithms that supply suitable information about the vehicle location on the lane. The steering actuator system is a locally controlled DC brush-less electric motor. Both control and vision algorithms are processed by an INTEL 486 microprocessor based Per-

sonal Computer for industrial applications. The sampling time of the whole system is $T_s = 40$ ms.

The mathematical modeling of such a plant was discussed in detail in paper [5] in which a simplified model, able to describe the vehicle behaviour in highway experimental conditions, was presented. The equations of such a model are here recalled for self-consistency of the paper. The interaction between vehicle lateral dynamics and vision system can be modeled by the following state space equations parameterized by the longitudinal velocity v_x :

$$\begin{bmatrix} \dot{v}_y \\ \dot{\psi} \\ \dot{q} \\ \dot{m} \end{bmatrix} = \begin{bmatrix} -a_1 & -m_v v_x^2 + a_2 & 0 & 0 \\ \frac{m_v v_x}{a_3} & \frac{m_v v_x}{-a_4} & 0 & 0 \\ I_\psi v_x & I_\psi v_x & 0 & v_x \\ -1 & 0 & 0 & v_x \\ 0 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_y \\ \psi \\ q \\ m \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ 0 \\ 0 \end{bmatrix} \delta_v + \begin{bmatrix} 0 \\ 0 \\ -L v_x \\ v_x \end{bmatrix} K_L \quad (1)$$

where L is the distance, along the longitudinal axis, from the vehicle center of gravity to a suitable point in the road between 3 and 20 meters ahead the vehicle (see Figure 1), the so called look-ahead distance; $K_L(t)$ is the inverse of the instantaneous curve radius (road curvature) at that look-ahead point; δ_v is the steering-wheel angle; m and q are the measurements supplied by the vision system about the vehicle location on the lane as shown in Figure 2. The meaning of the symbols involved in the equations is:

m_v : vehicle mass;

v_x : longitudinal component of center of gravity (CG) velocity;

v_y : transverse component of CG velocity;

ψ : vehicle yaw angle;

I_ψ : inertial vehicle moment around center of gravity;

δ_v : steering-wheel angle;

k : front wheels angle/steering-wheel angle ratio, expressed in rad/degrees;

$\delta_f = \delta_v k$: front wheels angle;

c_r : cornering stiffness of front tires;

c_f : cornering stiffness of rear tires;

l_r : distance between the rear axle and the center of gravity;

l_f : distance between the front axle and the center of gravity;

$l = l_r + l_f$: wheelbase;

$a_1 = c_f + c_r$;

$a_2 = c_r l_r - c_f l_f$;

$a_3 = -l_f c_f + l_r c_r$;

$a_4 = l_f^2 c_f + l_r^2 c_r$;

$b_1 = \frac{c_f}{m} k$;

$b_2 = \frac{l_f c_f}{I_\psi} k$.

The nominal values and the uncertainty intervals of the parameters are summarized in Table 1.

Table 1: Values of model parameters

Parameter	Nominal	Uncertainty
m_v (kg)	1226	[1226, 1626]
I_ψ (kgm ²)	1900	[1900, 2520]
c_r (N/rad)	96000	[81600, 110400]
c_f (N/rad)	60000	[51000, 69000]
l_r (m)	1,506	–
l_f (m)	1,034	–
v_x (km/h)	–	[60, 130]
k (rad/degrees)	$\pi/(180 \cdot 18)$	–

The closed loop steering actuator, designed by researchers of *Centro Ricerche Fiat* in a previous work [4], is described, for our purpose, by the following two transfer functions:

$$\frac{\delta_v}{\theta} = \frac{0.4537z + 0.3509}{z^2 - 0.2344z + 0.03907} \quad (2)$$

$$\frac{V_a}{\theta} = \frac{0.4636(z^2 - 1.306z + 0.4639)}{z^2 - 0.2344z + 0.03907} \quad (3)$$

where V_a is the voltage motor command and θ is the reference steering signal provided by the lateral controller to be designed.

3 Control problem formulation

The problem we are dealing with in this paper is the design of an controller able to keep the vehicle inside the lane along typical highway paths. Thus, the objective of the control problem is to keep the value of the lateral displacement of the vehicle with respect to the centerline of the lane within a prescribed tolerance, i.e., $|q(t)| \leq \gamma$, where $\gamma = 0.2$ m. Moreover, the passenger comfort must be taken into account. As a matter of fact, although specifications are explicitly given only on the value of $|q|$, the minimization of such a quantity is also strictly related to the regulation of the orientation of the vehicle measured by the value of m . Thus, the addressed design control problem requires the rejection of the effect of the disturbance K_L on both the two measured outputs q and m through the design of a TISO controller, as depicted in Figure 3, where $y_1 = q$, $y_2 = m$ and $d = K_L$. Such a formulation fits in the framework of the single-input two-output (SITO) systems, whose properties have been extensively studied in recent years (see, e.g., [6], [17], [20] and the references therein).

4 Some properties of SITO systems

In this section we recall some definitions and results introduced by Freudenberg and Middleton in [6] and

discuss their application to our problem. All the symbols and the quantities used in this section are referred to the general block diagram shown in Figure 3. First the concept of alignment angles are recalled:

Definition 1 [6]:

The *plant-disturbance alignment angle* is defined as

$$\phi_{pd}(\omega) \doteq \arccos \left(\frac{|P^H(j\omega)P_d(j\omega)|}{\|P(j\omega)\| \|P_d(j\omega)\|} \right) \quad (4)$$

where $P(s) = [P_1(s) P_2(s)]^T$, $P_d(s) = [P_{d_1}(s) P_{d_2}(s)]^T$, $(\cdot)^H$ stands for Hermitian adjoint and $\|\cdot\|$ is the euclidean norm. ■

Definition 2 [6]:

The *plant-controller alignment angle* is defined as

$$\phi_{pc}(\omega) \doteq \arccos \left(\frac{|C(j\omega)P(j\omega)|}{\|C(j\omega)\| \|P(j\omega)\|} \right) \quad (5)$$

where $C(s) = [C_1(s) C_2(s)]$. ■

Next define the input and the output loop gain as $L_I = CP$ and $L_O = PC$ respectively and the input and output sensitivity as $S_I = 1/(1 + L_I)$ and $S_O = L_O(I + L_O)^{-1}$, where I is the identity matrix. Then the closed loop transfer function of interest in our disturbance attenuation problem is:

$$\frac{y(s)}{d(s)} = S_O(s)P_d(s) \quad (6)$$

where $y(s) = [y_1(s) y_2(s)]^T = [q(s) m(s)]^T$ is the Laplace transform of the output and $d(s) = K_L(s)$ is the Laplace transform of the disturbance input. The following proposition, taken from [6], states the dependence of transfer function (6) upon the plant-disturbance $\phi_{pd}(\omega)$ and the plant-controller $\phi_{pc}(\omega)$ alignment angles:

Proposition 1 [6]:

$$S_{lw}(\omega) \leq \frac{\|S_O(j\omega)P_d(j\omega)\|}{\|P_d(j\omega)\|} \leq S_{up}(\omega) \quad (7)$$

where:

$$S_{lw}(\omega) = \{ \sin^2 \phi_{pd}(j\omega) + | \cos \phi_{pd}(j\omega) | S_I(j\omega) - \sin \phi_{pd}(j\omega) | T_I(j\omega) | \tan \phi_{pc}(j\omega) |^2 \}^{\frac{1}{2}} \quad (8)$$

and

$$S_{up}(\omega) = \{ \sin^2 \phi_{pd}(j\omega) + (\cos \phi_{pd}(j\omega) | S_I(j\omega) | + \sin \phi_{pd}(j\omega) | T_I(j\omega) | \tan \phi_{pc}(j\omega) |^2)^{\frac{1}{2}} \} \quad (9)$$

■

Remark 1: In paper [6] a working assumption on the stability of transfer matrix $P_d(s)$ is made. As a matter of fact such an assumption can be removed in the case considered in this paper where both $P(s)$ and $P_d(s)$ share the same poles in the origin of the s -plane. This can be easily seen by looking at the proof of (7) (see Proof of Proposition 9 in [6]) where a number of products of the form $|S_I| \|P_d\|$ arise. Indeed, such products are well defined in $s = 0$.

Remark 2: The ratio $\alpha(\omega) = \frac{\|S_O(j\omega)P_d(j\omega)\|}{\|P_d(j\omega)\|}$ is the disturbance attenuation coefficient provided by the closed loop. Large attenuation is obtained when $\alpha(\omega) \approx 0$ while no attenuation is provided when $\alpha(\omega) = 1$. Then, the goal of the disturbance attenuation problem can be specified through the assignment of desired values of $\alpha(\omega)$.

Remark 3: From Proposition 1 we observe that at frequencies where $\phi_{pd}(\omega) = 0$ and $\phi_{pc}(\omega) \neq \pi/2$ the bounds $S_{lw}(\omega)$ and $S_{up}(\omega)$ shrink to $|S_I|$. When such a case occurs, the coefficient $\alpha(\omega)$ is well approximated by $|S_I|$, therefore the SITO disturbance attenuation problem we are dealing with can be simplified to an equivalent SISO disturbance attenuation one which can be addressed through the frequency shaping of $|S_I|$.

5 Controller design

Since, as shown in previous sections, we are dealing with a disturbance attenuation problem, the first step of the design is to identify which class of disturbances $K_L(t)$ may arise from the experimental conditions of interest. Italian legislation on roads building states that straight sections must be connected to bends through a given class of geometric curves [7]. Using the equations of such curves we can evaluate the corresponding curvature variation $K_L(t)$ when the longitudinal velocities v_x is in the range [60, 130] km/h and the radii of the bends are greater than 500 m. Such an analysis highlights that the worst-case disturbance $K_L(t)$ is a step of size $1/500 = 0.002 \text{ m}^{-1}$ which correspond to a straight section directly connected to a circle with constant radius of 500 m. Now, from Figure 4, which shows the plant-disturbance alignment angle for a set of longitudinal velocity $v_x \in [60, 130]$ km/h, we can see that $\phi_{pd}(\omega) \approx 0$ in the frequency range $[0, 0.7]$ rad/s. Thus, as long as the designed controller C be such that $\phi_{pc}(\omega) \neq \pi/2$ for $\omega \in [0, 0.7]$ (condition which has to be verified when the controller design is completed), thanks to *Remark 3*, the attenuation of the disturbance K_L can be performed minimizing the magnitude of the SISO sensitivity $|S_I(\omega)|$ for $\omega \in [0, 0.7]$ rad/s. From Figure 5, which shows the magnitude of such a worst case disturbance in the frequency domain, we see that $|K_L(\omega)| < -50 \text{ dB} \quad \forall \omega \geq 0.7 \text{ rad/s}$, i.e., over the frequency range where we cannot rely on the shaping of the SISO sensitivity $|S_I(\omega)|$, the magnitude of the worst case disturbance can be considered negli-

ble. It must be noted that the minimization of the SISO sensitivity $|S_I(\omega)|$ is only related to the shaping of $|L_I| = |C_1P_1 + C_2P_2|$ without any constraint on the choice of the internal structure of the controller. Thus we have some degrees of freedom to assess the passenger comfort as well as the stability margins.

In paper [5] we have discussed how a good ride comfort can be obtained through the emulation of the common driver behaviour and we have shown that such an approach leads to the design of a SISO controller $\tilde{C}(s)$ of a feedback control systems where the feedback signal is $y_{fb} = q + mL$ (see Figure 7) which is the distance, measured at the look-ahead, between the longitudinal axis and the linear approximation of the centerline of the lane (see Figure 1). Comparing Figure 3 and Figure 7 it can be seen that such a control strategy is equivalent to the choice $C_1(s) = \tilde{C}(s)$ and $C_2(s) = L\tilde{C}(s)$ for the internal structure of the TISO controller $C(s)$, thus the shaping of $|L_I| = |\tilde{C}(P_1 + LP_2)|$ is reduced to the design of the single SISO controller \tilde{C} . From equation (5) it is easy to show that the required condition $\phi_{pc}(\omega) \neq \pi/2$ is implied by $\frac{C_2(j\omega)}{C_1(j\omega)} \neq -\frac{P_1(j\omega)}{P_2(j\omega)}$ which can be easily satisfied by a suitable choice of the value of the look-ahead distance L , being $\frac{C_2(j\omega)}{C_1(j\omega)} = L$.

The design of the SISO controller \tilde{C} was carried out in the frequency domain through classical loop shaping techniques and taking into account the presence of parametric uncertainty. The transfer function of the designed controller is $\tilde{C}(z) = n(z)/d(z)$ where the numerical values of the controllers parameters, in descending powers of z , are: $n = \{-7.844, 30.82, -47.37, 35.51, -13.24, 2.388, -0.2273\}$, $d = \{1, -4.92, 10.06, -10.96, 6.703, -2.181, 0.2949\}$. Figure 6 shows the magnitude of the input sensitivity $|S_I|$ together with the upper and the lower bounds S_{up} and S_{lw} of the attenuation coefficient $\alpha(\omega)$ when $v_x = 130$ km/h. Figure 8 illustrates the plant-controller alignment angle $\phi_{pc}(\omega)$ for a set of longitudinal velocity v_x belonging to the interval $[60, 130]$ km/h and taking $L = 11.5$ m. From such figures we can see that, as expected, the bounds S_{up} and S_{lw} shrink to $|S_I|$ for $\omega \in [0, 0.7]$ rad/s and the plant-controller alignment angle is $\phi_{pc}(\omega) \neq \frac{\pi}{2} \forall \omega$.

6 Experimental results

In this section we report the experimental results obtained testing the controlled vehicle. The test track is a curve with radius $R \approx 600$ m followed by a straight section along an Italian highway. The velocity was kept approximately constant at 110 km/h. From Figure 9, which shows the lateral offset q , we see that the designed SISO controller satisfies the specification about the position error with respect to the centerline of the lane.

7 Conclusions

In this paper we addressed the problem of lateral vehicle control in highway experimental conditions. The control system under consideration exhibit a single-input two-output (SITO) structure where the steering angle is the input while the two outputs are the lat-

eral displacement and the angular orientation of the vehicle referred to the centerline of the lane. By exploiting some properties of SITO systems, the given control problem is simplified to the design of a single-input single-output (SISO) controller. Experimental tests along highway paths using a FIAT Brava 1600 ELX provided by Centro Ricerche Fiat, showed the fulfillment of given specifications.

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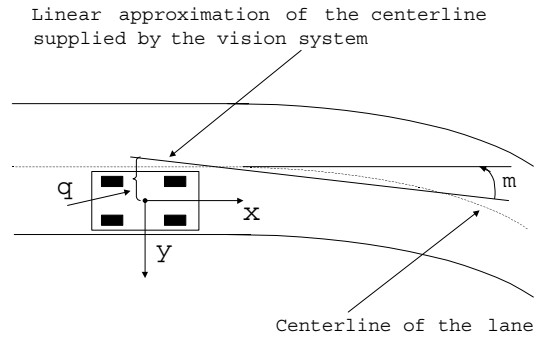


Figure 2: Centerline linear approximation supplied by the vision system

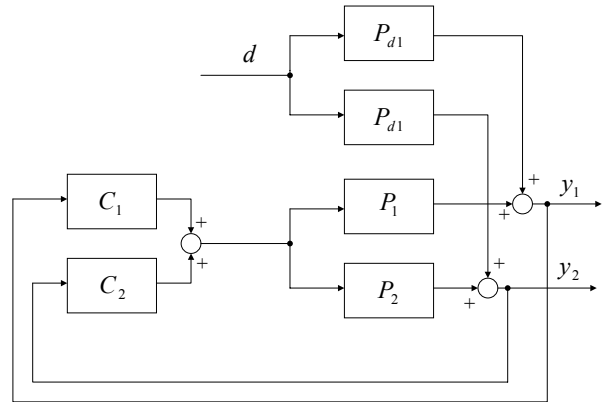


Figure 3: Block diagram of SITO feedback systems

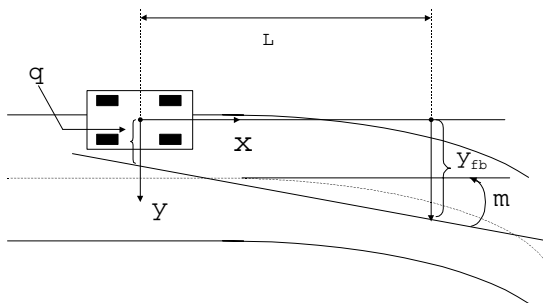


Figure 1: Feedback quantity y_{fb}

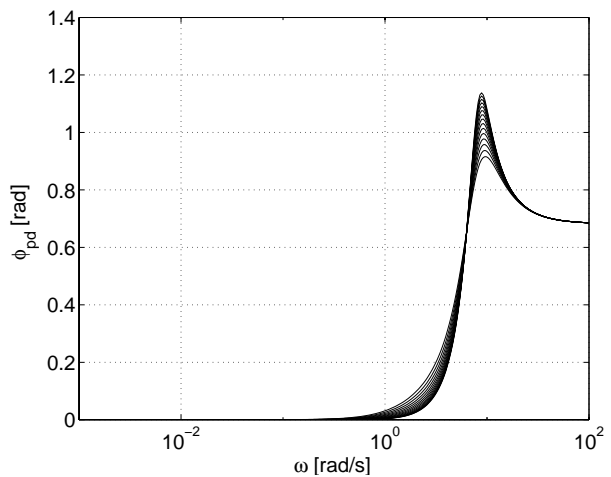


Figure 4: Alignment angle $\phi_{pd}(\omega)$

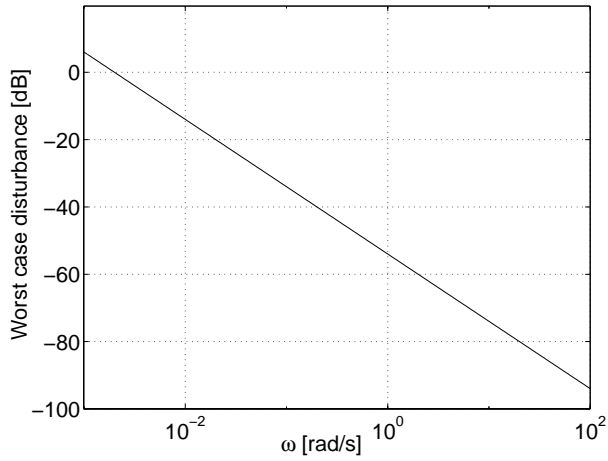


Figure 5: Magnitude of the worst case curvature disturbance $|K_L(\omega)|_{worst}$

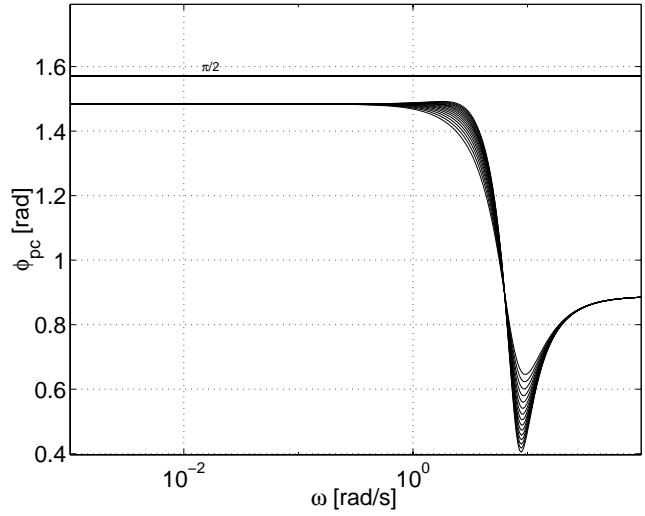


Figure 8: Alignment angle $\phi_{pc}(\omega)$

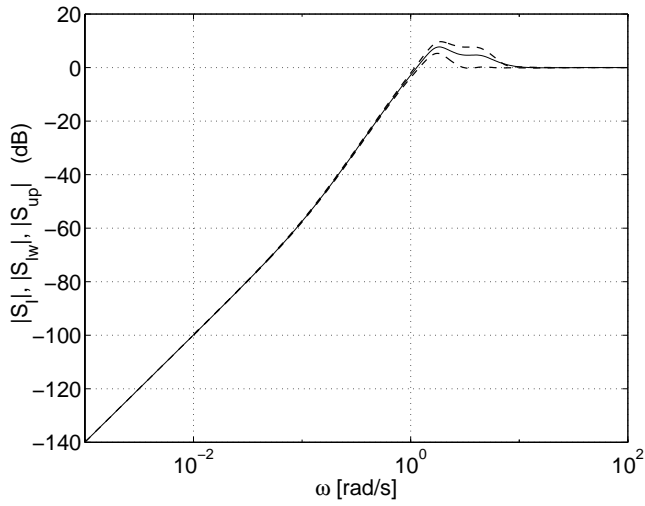


Figure 6: $|S_I|$ (solid) and bounds S_{up} and S_{lw} (dashed)

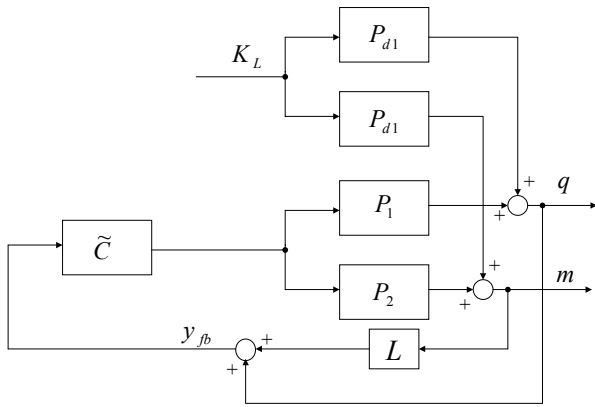


Figure 7: Block diagram of the SISO control strategy used in [5]

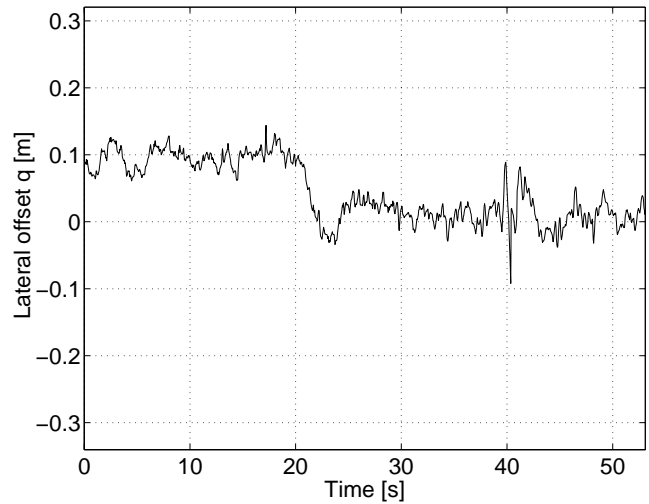


Figure 9: Lateral offset q