

# Robust feedforward design from data via approximate inverse SM identification<sup>1</sup>

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## Abstract

In this paper the design of the feedforward block of a two degree of freedom controller for reference tracking in SISO systems in the presence of model uncertainty is addressed.

From previous works, it is known that given a set of noisy input-output measurements, an uncertainty model set for the plant can be obtained through Set Membership  $H_\infty$  identification techniques and, on the basis of such a model set, a robust design of the feedback compensator can be performed. In this work the design of the feedforward filter is formulated as a robust model matching problem. An optimal solution is provided in closed form on a finite set of frequencies obtained from a suitable frequency gridding. Then a FIR model, approximation of the optimal solution, is obtained solving a linear programming problem.

The efficiency of the method is tested on two examples, related to two typical patterns of uncertainty, the first one arising in the case of unmodeled dynamics, the second one in the case of parametric uncertainty.

**Keywords** — Two degrees of freedom, robust control, control design from data, feedforward, Set-Membership.

## 1 Introduction

The paper deals with the problem of designing a discrete-time two degrees of freedom control system from experimental data. Two degrees of freedom design is a well known and widely used structure when both reference tracking and disturbance attenuation performances are required (see, e.g., [3], [11]). It is well known that when there is no model uncertainty the design of the feedback controller  $K$  and the feedforward filter  $Q$  (see Fig. 1) can be decoupled without affect-

ing the achievable performances [11]. However that is not true in the presence of modelling error. Since the simultaneous synthesis of  $K$  and  $Q$  might be a difficult problem, the design is usually still performed in two independent stages (see, e.g., [2], [4]), which is also the approach used in this work.

Limebeer *et al.* in [4] design the feedback controller using the  $H_\infty$  loop shaping approach developed by McFarlane and Glover in [5]. Then, taking into account the model uncertainty in the multiplicative input form, they design the feedforward filter using  $H_\infty$  optimization via  $\gamma$ -iteration. Giusto and Paganini in [2] formulate the feedforward filter design in the presence of model uncertainty as a minimization of a linear objective subject to an infinite number of convex constraints. They provide an approximate solution by reducing the original problem to one with a finite number of convex constraints through frequency gridding or optimization over the span of a set of basis functions.

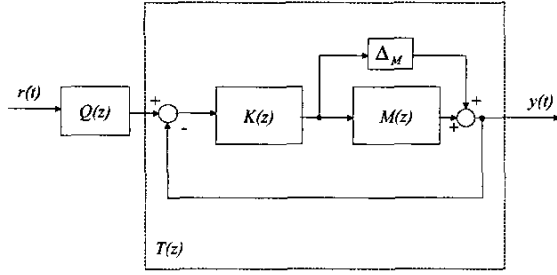
In this work a design procedure for SISO systems based on experimental data is proposed. Given a set of noisy input output measurements, an uncertainty model set for the plant can be obtained through set-membership  $H_\infty$  identification techniques [8], [9]. On the basis of such a model set, the feedback controller which satisfies loop requirements can be designed, for example, as described in [6]. Thus we focus on the design of the feedforward filter.

First, from the identified uncertainty model set of the plant, an uncertainty model set for the closed-loop complementary sensitivity  $T$  is computed, (see, e.g., [7]). Then the design of the feedback block is formulated as an  $H_\infty$  robust model-matching problem solved in two stages. In the first stage the robust-optimal solution  $Q^*(\omega)$  is provided in closed form at each  $\omega$  on a finite set of frequencies  $\omega_k$ ,  $k=1,2,\dots,N$ . Hence a FIR model  $\hat{Q}(z) \in \mathcal{RH}_\infty$  approximating the optimal solution is obtained solving a linear programming problem.

## 2 Robust model-matching design

In this section the proposed design procedure is outlined. First, given a set of noisy input-output mea-

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**Figure 1:** Two degree of freedom configuration in the presence of model uncertainty

surements, an uncertainty model set  $\mathcal{M}_M$  is identified through set-membership  $H_\infty$  identification techniques [8], [9]. The obtained model set is an additive frequency shaped model set of the form:

$$\mathcal{M}_M(M_n, W_M) = \{(M_n + \Delta_M) \in \mathcal{S} : |\Delta_M(\omega)| \leq W_M(\omega), \forall \omega \in [0, 2\pi]\} \quad (1)$$

where  $\mathcal{S}$  is the Banach space of causal, single-input single-output, linear time-invariant, discrete-time, BIBO-stable dynamical processes;  $M_n$  is a  $n$ -order real rational model,  $W_M$  is a known function bounding the modelling error, derived from the identification procedure.

Then an uncertainty model set  $\mathcal{M}_T$  for the complementary sensitivity  $T(z)$  can be evaluated from (1) (see, e.g., [7]). Such a set has the following form:

$$\mathcal{M}_T(T_n, W_T) = \{(T_n + \Delta_T) \in \mathcal{S} : |\Delta_T(\omega)| \leq W_T(\omega), \forall \omega \in [0, 2\pi]\} \quad (2)$$

The design of the reference pre-filter  $Q(z)$  is formulated in terms of the following robust  $H_\infty$  model matching problem:

$$Q^*(z) = \arg \min_{Q(z) \in \mathcal{RH}_\infty} \sup_{T \in \mathcal{M}_T} \|Q(z)T(z) - M_r(z)\|_\infty \quad (3)$$

where  $\|\cdot\|_\infty$  is the  $H_\infty$  norm and  $M_r(z)$  is a discrete time reference model. Note that, if reference tracking is desired in a given frequency range, we set  $M_r = 1$  in such a range. Thus, (3) is equivalent to the problem of looking for the optimal worst-case approximation of the inverse of  $T(z)$  over a suitable frequency range specified by the choice of  $M_r(\omega)$ .

We propose a two steps procedure for the solution of the design problem (3). First, a solution is obtained relaxing in (3) the requirement  $Q(z) \in \mathcal{RH}_\infty$ , and looking for optimality at each  $\omega$ , i.e., solving the following problem:

$$Q^*(\omega) = \arg \min_{Q(\omega)} \sup_{T \in \mathcal{M}_T} |Q(\omega)T(\omega) - M_r(\omega)| \quad (4)$$

The following proposition provides the solution of the

optimization problem (4).

**Proposition 1**

For given reference model  $M_r(z)$  and uncertainty model set  $\mathcal{M}_T$ , the solution of problem (4) at fixed  $\omega$ , called robust-optimal filter, is:

$$Q^*(\omega) = \begin{cases} M_r(\omega)/T_n(\omega) & W_T(\omega) \leq |T_n(\omega)| \\ 0 & W_T(\omega) \geq |T_n(\omega)| \end{cases} \quad (5)$$

**Proof of Proposition 1**

Let  $\mathcal{D}(c, r)$  be the disk of centre  $c$  and radius  $r$  in the complex plane, i.e.:

$$\mathcal{D}(c, r) = \{z \in \mathcal{C} : z = c + pe^{j\varphi}, \varphi \in [0, 2\pi], p \in [0, r]\} \quad (6)$$

Denoting the value of the robust-optimal filter at frequency  $\omega$  with  $Q^*(\omega) = |Q^*(\omega)|e^{j\theta}$ , it follows:

$$Q(\omega)\mathcal{D}(c, r) = \{v \in \mathcal{C} : v = Q(\omega)c + |Q(\omega)|pe^{j(\varphi+\theta)}, \varphi \in [0, 2\pi], p \in [0, r]\} = \mathcal{D}(Q(\omega)c, |Q(\omega)|r) \quad (7)$$

Now, for a given  $\omega$ , let  $\mathcal{M}_T(\omega) = \{T(\omega) \in \mathcal{C} : T \in \mathcal{M}_T\}$  and let  $\mathcal{M}_{QT}(\omega) = \{Q(\omega)T(\omega) \in \mathcal{C} : T \in \mathcal{M}_T\}$ . Since  $\mathcal{M}_T(\omega) = \mathcal{D}(T_n(\omega), W_T(\omega))$ , it follows that  $\mathcal{M}_{QT}(\omega) = \mathcal{D}(Q(\omega)T_n(\omega), |Q(\omega)|W_T(\omega))$ . Thus, it is easily seen that (4) can be rewritten as:

$$\begin{aligned} Q^*(\omega) &= \arg \min_{Q(\omega)} \{|Q(\omega)T_n(\omega) - M_r(\omega)| + \\ &\quad + W_T(\omega)|Q(\omega)|\} = \\ &= \arg \min_{|Q(\omega)|} \min_{ph(Q(\omega))} \{|Q(\omega)T_n(\omega) - M_r(\omega)| + \\ &\quad + W_T(\omega)|Q(\omega)|\} = \\ &\doteq \arg \min_{|Q(\omega)|} \min_{ph(Q(\omega))} J(|Q(\omega)|, ph(Q(\omega))) \end{aligned} \quad (8)$$

where  $ph(\cdot)$  is the phase of a complex number.

First the minimization over  $ph(Q(\omega))$  is considered:

$$\begin{aligned} ph(Q^*(\omega)) &= \arg \min_{ph(Q(\omega))} \{|Q(\omega)T_n(\omega) - M_r(\omega)| + \\ &\quad + W_T(\omega)|Q(\omega)|\} = \\ &= \arg \min_{ph(Q(\omega))} \{|Q(\omega)T_n(\omega) - M_r(\omega)|\} = \\ &= \arg \min_{ph(Q(\omega))} \{(|Q(\omega)||T_n(\omega)|e^{j(ph(Q)+ph(T_n(\omega)))} + \\ &\quad - |M_r(\omega)|e^{j(ph(M_r))})\} = \\ &= ph(M_r(\omega)) - ph(T_n(\omega)) \end{aligned} \quad (9)$$

Putting together equations (8) and (9) the minimization over  $|Q(\omega)|$  can be dealt with as follows:

$$\begin{aligned} |Q^*(\omega)| &= \arg \min_{|Q(\omega)|} \{|e^{j(ph(M_r))}||Q(\omega)||T_n(\omega)| + \\ &\quad - |M_r(\omega)|\} + W_T(\omega)|Q(\omega)|\} = \\ &= \arg \min_{|Q(\omega)|} \{|e^{j(ph(M_r))}||Q(\omega)||T_n(\omega)| - |M_r(\omega)|\} + \\ &\quad + W_T(\omega)|Q(\omega)|\} = \\ &= \arg \min_{|Q(\omega)|} \{|Q(\omega)||T_n(\omega)| - |M_r(\omega)|\} + \\ &\quad + W_T(\omega)|Q(\omega)|\} = \\ &= \arg \min_{|Q(\omega)|} J(|Q(\omega)|, ph(Q^*(\omega))). \end{aligned} \quad (10)$$

The solution of (10) can be obtained by the following two linear programming problems:

$$\begin{aligned} |Q_1^*(\omega)| &= \arg \min_{|Q(\omega)|} J_1(|Q(\omega)|) \\ \text{s.t. } |Q(\omega)| &\geq \frac{|M_r(\omega)|}{|T_n(\omega)|} \end{aligned} \quad (11)$$

and

$$\begin{aligned} |Q_2^*(\omega)| &= \arg \min_{|Q(\omega)|} J_2(|Q(\omega)|) \\ \text{s.t. } 0 \leq |Q(\omega)| &\leq \frac{|M_r(\omega)|}{|T_n(\omega)|} \end{aligned} \quad (12)$$

where:

$$\begin{aligned} J_1(|Q(\omega)|) &= |Q(\omega)||T_n(\omega)| - |M_r(\omega)| + W_T(\omega)|Q(\omega)| = \\ &= |Q(\omega)|(W_T(\omega) + |T_n(\omega)|) - |M_r(\omega)| \end{aligned} \quad (13)$$

and

$$\begin{aligned} J_2(|Q(\omega)|) &= |M_r(\omega)| - |Q(\omega)||T_n(\omega)| + W_T(\omega)|Q(\omega)| = \\ &= |M_r(\omega)| + |Q(\omega)|(W_T(\omega) - |T_n(\omega)|) \end{aligned} \quad (14)$$

In fact:

$$\begin{aligned} \min_{|Q(\omega)|} J(|Q(\omega)|, ph(Q^*(\omega))) &= \\ = \min\{J_1(|Q_1^*(\omega)|), J_2(|Q_2^*(\omega)|)\} \end{aligned} \quad (15)$$

The solutions of problems (11) and (12) are respectively:

$$|Q_1^*(\omega)| = \frac{|M_r(\omega)|}{|T_n(\omega)|} \quad (16)$$

which provides the minimum:

$$J_1(|Q_1^*(\omega)|) = \frac{|M_r(\omega)|W_T(\omega)}{|T_n(\omega)|} \quad (17)$$

and

$$|Q_2^*(\omega)| = \begin{cases} |M_r(\omega)|/|T_n(\omega)| & W_T(\omega) \leq |T_n(\omega)| \\ 0 & W_T(\omega) \geq |T_n(\omega)| \end{cases} \quad (18)$$

which provides the minimum:

$$J_2(|Q_2^*(\omega)|) = \begin{cases} \frac{|M_r(\omega)|W_T(\omega)}{|T_n(\omega)|} & W_T(\omega) \leq |T_n(\omega)| \\ |M_r(\omega)| & W_T(\omega) \geq |T_n(\omega)| \end{cases} \quad (19)$$

Finally from (15) the following results is obtained:

$$\begin{aligned} J(|Q^*(\omega)|, ph(Q^*(\omega))) &= \\ = \min\{J_1(|Q_1^*(\omega)|), J_2(|Q_2^*(\omega)|)\} &= \\ = \begin{cases} \frac{|M_r(\omega)|W_T(\omega)}{|T_n(\omega)|} & W_T(\omega) \leq |T_n(\omega)| \\ |M_r(\omega)| & W_T(\omega) \geq |T_n(\omega)| \end{cases} \end{aligned} \quad (20)$$

which is achieved with the filter  $Q^*(\omega)$  in (5). ■

Proposition 1 shows that at all frequencies at which the relative uncertainty  $\frac{W_T(\omega)}{|T_n(\omega)|}$  is less than 1, the robust-optimal solution  $Q^*(\omega)$  equals the solution of the problem in the uncertainty-free case, i.e., the nominal solution  $Q_n(\omega) = M_r(\omega)/T_n(\omega)$ . At frequencies at which  $\frac{W_T(\omega)}{|T_n(\omega)|} \geq 1$  the optimum is achieved turning off the controller.

The robust-optimal  $Q^*(\omega)$  can be computed for a set of frequencies  $\omega_k, k=1,2,\dots,N$  which suitably grids the interval  $[0, \pi]$  and a rational stable approximation  $\hat{Q}(z) \in \mathcal{RH}_\infty$  of  $Q^*(z)$  can be obtained through standard interpolation/approximation techniques like, e.g., exact or approximated Nevanlinna-Pick interpolation [1]. However, numerical problems may arise in Nevanlinna-Pick interpolation for values of  $N$  larger than  $50 \div 100$ . Thus, an alternative approach is considered in this paper. A finite impulse response (FIR) filter  $\hat{Q}_\nu(z)$  of order  $\nu$ , described by its impulse response  $h^{\hat{Q}_\nu} = [h_0^{\hat{Q}_\nu}, h_1^{\hat{Q}_\nu}, \dots, h_\nu^{\hat{Q}_\nu}]^T$ , is computed solving the following problem:

$$h^{\hat{Q}_\nu} = \arg \min_{h^{\hat{Q}_\nu} \in \text{FIR}_\nu^z} \|W(y - F_N h^{\hat{Q}_\nu})\|_\infty \quad (21)$$

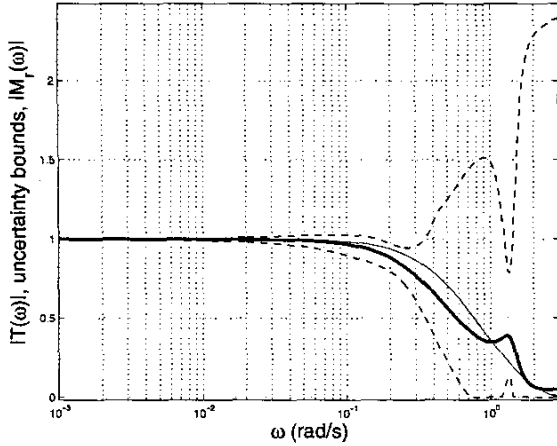
where  $W$  is a weighting matrix accounting for the desired tolerance at different frequencies and:

$$\begin{aligned} y_k &= [\text{Re}(Q^*(\omega_k)) \quad \Im m(Q^*(\omega_k))] \in \mathbb{R}^{1 \times 2} \\ y &= [y_1 \quad \dots \quad y_N]^T \in \mathbb{R}^{2N \times 1} \\ F_N &= [\Omega^T(\omega_1) \quad \dots \quad \Omega^T(\omega_N)]^T \in \mathbb{R}^{2N \times (\nu+1)} \\ \Omega(\omega_k) &= \begin{bmatrix} \text{Re}(\Psi(\omega_k)) \\ \Im m(\Psi(\omega_k)) \end{bmatrix} \in \mathbb{R}^{2 \times (\nu+1)} \\ \Psi(\omega_k) &= [1 \quad e^{-j\omega_k} \quad e^{-j2\omega_k} \quad \dots \quad e^{-j\nu\omega_k}] \in \mathbb{C}^{1 \times (\nu+1)} \\ \text{FIR}_\nu^z &= \{Q_\nu(z) = \sum_{k=1}^{\nu} h_k^{\hat{Q}_\nu} z^{-k} : \|\hat{Q}_\nu(z)\|_\infty \leq \gamma\} \end{aligned} \quad (22)$$

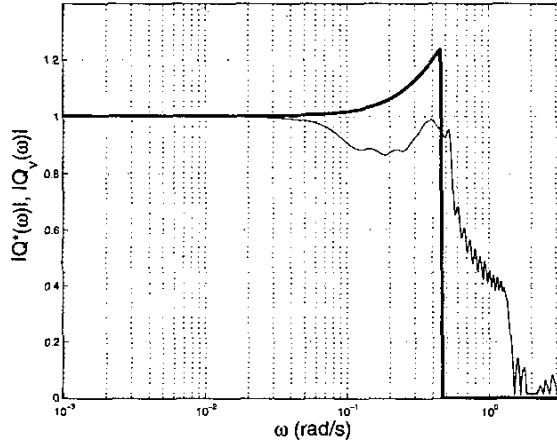
Note that in (22) a constraint is added on the transfer function derivative of the approximating FIR. Indeed, if large values of  $\nu$  are required for obtaining reasonable approximation, problem (21) may become ill-posed for  $\gamma = \infty$ . Well-posed solutions can be obtained by suitably choosing the "regularization" parameter  $\gamma$  [10], which imposes a certain degree of smoothness of the intersample magnitude of the approximating FIR. The solution of (21) can be efficiently derived by means of linear programming techniques for quite large values of  $\nu$  and  $N$ .

### 3 Examples

Two examples are reported, related to two typical patterns of the uncertainty on the complementary



**Figure 2:** Example 1 - Nominal closed loop  $|T_n|$  (solid), uncertainty bounds  $|T_n| \pm W_T$  (dashed) and reference model  $|M_r|$  (thin)



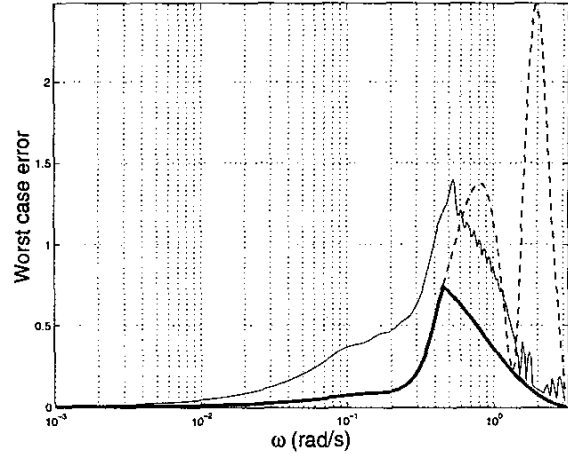
**Figure 3:** Example 1 - Magnitude of the optimal filter  $|Q^*|$  (solid) and of the approximating filter  $|Q_v|$  (thin)

sensitivity. In Example 1 large uncertainty ( $\geq 100\%$ ) is present at high frequencies. Thus the robust-optimal filter  $Q^*(\omega)$  is given by the nominal one  $Q_n(\omega)$  up to the frequency at which uncertainty becomes large ( $\geq 100\%$ ), and zero from that frequency on. This typically happens when the uncertainty is due to unmodelled dynamics. In Example 2 the uncertainty is large at middle frequencies, so that the robust-optimal filter is zero in such a range. This may happen in the case of parametric uncertainty.

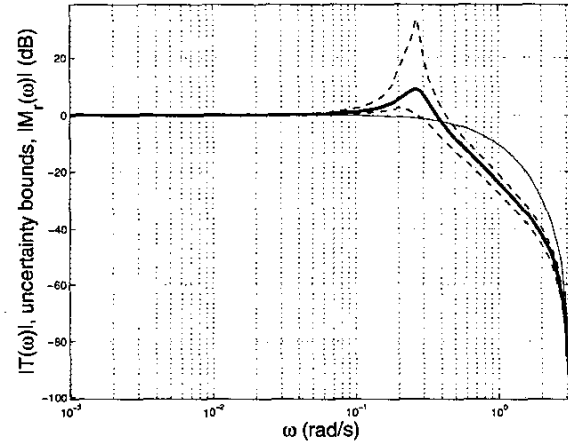
**Example 1:** In this example the following nonminimum-phase continuous-time system is considered:

$$P(s) = \frac{-0.0609s^4 - 0.4871s^3 - 0.4871s^2 + 1.9482s + 2.9224}{s^4 + 0.4311s^3 + 2.6764s^2 + 0.4384s + 0.1412}$$

On the basis of a set of input-output experimental measurements obtained with  $T_s = 1$  s as sampling time, the



**Figure 4:** Example 1 - Worst case errors: optimal filter  $Q^*$  (solid), approximating filter  $Q_v$  (thin) and nominal filter  $Q_n$  (dashed)



**Figure 5:** Example 2 - Nominal closed loop  $|T_n|$  (solid), uncertainty bounds  $|T_n| \pm W_T$  (dashed) and reference model  $|M_r|$  (thin)

identification of model set  $\mathcal{M}_M$  for  $P$  and the subsequent design of the loop controller  $K(z)$  guaranteeing robust stability and disturbance attenuation specification, was presented in [6]. Using the tools described in [7] we have computed a model set  $\mathcal{M}_T$  of the form (2) for the complementary sensitivity function. The magnitude of the nominal model:

$$T_n(z) = \frac{-0.01341z^5 + 0.1592z^4 + 0.1726z^3 + 0.05013z - 0.01474}{z^5 - 0.5571z^4 - 0.1064z^3 + 0.4718z^2 - 0.6957z + 0.2775}$$

together with the corresponding uncertainty region are depicted in Figure 2. The following reference model was considered:

$$M_r(z) = \frac{-0.008735z^3 + 0.09939z^2 + 0.166z + 0.05788}{z^3 - 0.8858z^2 + 0.2139z - 0.01353}$$

The magnitude of the reference model  $M_r$  is shown in Figure 2. The robust-optimal filter  $Q^*(\omega)$  has been computed on a set of 500 logarithmically equally spaced frequencies in the interval  $[0, \pi]$ . Finally, a FIR filter of

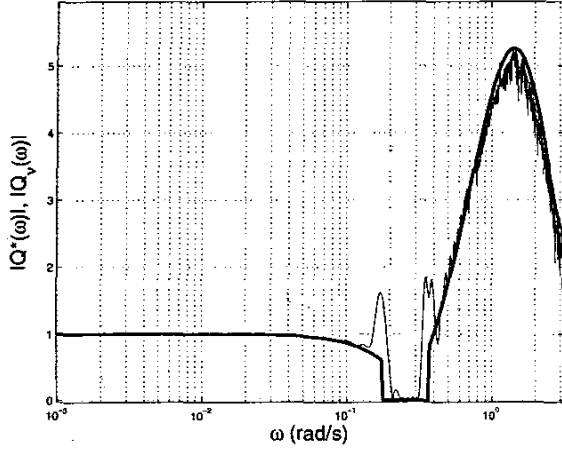


Figure 6: Example 2 - Magnitude of the optimal filter  $|Q^*(\omega)|$  (solid) and of the approximating filter  $|Q_\nu(\omega)|$  (thin)

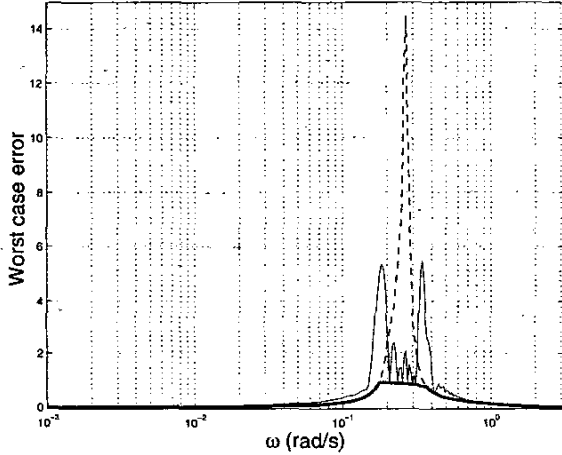


Figure 7: Example 2 - Worst case errors: optimal filter  $Q^*$  (solid), approximating filter  $Q_\nu$  (thin) and nominal filter  $Q_n$  (dashed)

order  $\nu = 100$  was computed solving problem (21) with suitable choices of parameters  $W$  and  $\gamma$ . Figure 3 shows the magnitude of the robust-optimal filter  $Q^*(\omega)$  and of the approximating filter  $Q_\nu(\omega)$ . Figure 4 illustrates the worst case model matching error:

$$\begin{aligned} \sup_{T \in \mathcal{M}_T} |Q(\omega)T(\omega) - M_r(\omega)| &= \\ &= |Q(\omega)T_n(\omega) - M_r(\omega)| + W_T(\omega)|Q(\omega)| \end{aligned}$$

when both the nominal filter  $Q_n = M_r/T_n$  and the filter  $Q_\nu(\omega)$  are considered.

**Example 2:** In this example the model set  $\mathcal{M}_T$  for the closed loop system has the nominal model

$$T_n(z) = \frac{0.01754z^2 + 0.03509z + 0.01754}{z^2 - 1.836z + 0.9064}$$

The magnitude of  $T_n$  together with the corresponding uncertainty region are depicted in Figure 5. The following reference model was considered:

$$M_r = \frac{0.05194z^3 + 0.1306z^2 + 0.1053z + 0.0267}{z^3 - 0.8858z^2 + 0.2139z - 0.01353}$$

The magnitude of the reference model  $M_r$  is illustrated in Figure 5. Figure 6 shows the magnitude of the optimal filter  $Q^*(\omega)$ , computed over 500 logarithmically equally spaced frequencies in the interval  $[0, \pi]$ , and the magnitude of the approximating filter  $Q_\nu(\omega)$  of order 200. Figure 7 shows the worst case error when both the nominal filter  $Q_n = M_r/T_n$  and the filter  $Q_\nu(\omega)$  are considered.

In both examples, FIR filters are obtained, giving quite good approximations of the robust-optimal filters  $Q^*$  and providing significant reductions of the worst case model matching errors with respect to nominal filters.

#### 4 Conclusion

In this paper a simple procedure for the design of the feedforward filter of a two degree of freedom controller for SISO systems in the presence of model uncertainty, has been presented.

Given an uncertainty model set for the complementary sensitivity function  $T(z)$ , the problem has been formulated as a robust  $H_\infty$  model matching problem. First the robust-optimal solution  $Q^*(\omega)$  is provided in closed form on a finite set of frequencies  $\omega_k, k=1,2,\dots,N$ , obtained from a suitable gridding of the interval  $[0, \pi]$ . Then a  $\nu$ -order FIR model  $Q_\nu(z)$  approximating  $Q^*$  is obtained solving a linear programming problem. The approach appears to be computationally efficient and, what's more, it explicitly highlights that the feedforward filter should be designed as near as possible to the nominal one in the frequency ranges where the uncertainty of the complementary sensitivity function is less than 100%, and as near as possible to zero for frequencies where the uncertainty is greater than 100%. The efficiency of the method is tested on two examples, related to two typical patterns of uncertainty, the first one arising in the case of unmodeled dynamics, the second one in the case of parametric uncertainty. The extension of this approach to general MIMO systems seems not to be straightforward, while the case of square systems is at present under investigation.

#### References

- [1] J. Chen and G. Gu, *Control-Oriented System Identification: An  $H_\infty$  approach*. New York, John Wiley and Son, Inc., 2000.
- [2] A. Giusto and F. Paganini, "Robust Synthesis

of Feedforward Compensators”, *IEEE Transaction on Automatic Control*, Vol 44, no. 8, pp. 1578-1582, 1999.

[3] I. Horowitz, *Synthesis of Feedback Systems*, New York: Academic Press, 1963.

[4] D.J.N. Limebeer, E.M. Kasenally and J.D. Perkins, “On the design of Robust Two Degree of Freedom Controllers”, *Automatica*, Vol 29, no. 1, pp. 157-168, 1993.

[5] D.C. McFarlane and K. Glover, “A loop-shaping design procedure using  $H_\infty$  synthesis”, *IEEE Transaction on Automatic Control*, Vol 37, no. 6, pp. 759 -769, 1992.

[6] S. Malan, M. Milanese, D. Regruto, M. Taragna, “Robust control from data via uncertainty model sets identification”, in *Proc. of the 40th IEEE Conference on Decision and Control*, (Orlando, FL), pp. 2686-2691, 2001.

[7] S. Malan, M. Milanese, M. Taragna, “MATLAB tools for SM  $H_\infty$  identification and guaranteed control performances computation”, in *Proc. of the 41th IEEE Conference on Decision and Control*, (Las Vegas, NV), 2002.

[8] M. Milanese and M. Taragna, “Suboptimality evaluation of approximated models in  $H_\infty$  identification”, in *Proc. of the 38th IEEE Conference on Decision and Control*, (Phoenix, AZ), pp. 1494-1499, 1999.

[9] M. Milanese and M. Taragna, “Set Membership identification for  $H_\infty$  robust control design”, in *Proc. of 12th IFAC Symposium on System Identification SYSID 2000*, (Santa Barbara, CA), 2000.

[10] A. N. Tikhonov and V. Y. Arsenin, *Solution of Ill-posed Problems*. Winston, Washington DC, 1977.

[11] M. Vidyasagar, *Control system synthesis*, Cambridge, MA: The MIT Press, 1985.