

# Experimental results in vision-based lane keeping for highway vehicles

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## Abstract

In this paper we address the problem of lateral control of vehicles for highway lane keeping using vision system. First we present a suitable simplified model of the plant and show its agreement with typical highway experimental conditions. Then, we formulate the control problem and propose a SISO control strategy based on driver behaviour emulation. The controller design is carried out using classical loop-shaping techniques and taking into account the model uncertainty. The experimental tests, conducted on Italian highways using a FIAT Brava 1600 ELX provided by Centro Ricerche Fiat, showed the good performances of the designed control system.

## 1 Introduction

Intelligent Vehicle Systems (IVS) have recently become an attractive area of research throughout the world. The aim of the research effort is mainly that of enhancing driving safety and reducing driver's workload. In [19] the positive effect of driver assistance systems on driver's physical and mental workload reduction is shown through the measure of some vital reactions such as the electromyographic and the electrodermal activities. In particular, Automated Highway Systems (AHS), extensively studied at the Ohio State University since 1964 ([8]), are receiving renewed attention due to fast developments in hardware/software technology that allow the design of more effective control systems. Since mid-eighties a larger effort is being conducted mainly in the California PATH program. Early attempts of the project were devoted to assess previous knowledge in the field of automatic vehicle control providing the analytical basis for new developments ([18]). In the last years large efforts have been directed to the solution of the highway automatic steering control problem for different type of vehicles and using different control strategies and techniques. Most of the contributions rely on buried magnet or electrified wires placed along the path for the detection of the vehicle lateral position, the so called *look down* sensing scheme. The problem, in the case of passenger cars,

was analyzed in this framework by Patwardhan *et al.* in [17] where they show the fundamental control difficulties of this approach. An interesting alternative approach, that avoids the modification of infrastructures, involves the use of vision sensors placed on the vehicle, the so called *look-ahead* sensing scheme. A comparative study of vision-based control strategies was presented by Košecká *et al.* in [15]. A great deal of remarkable works about application of advanced linear and nonlinear control techniques to the automatic steering control problem were conducted, still in the PATH program, by Tomizuka and coworkers in the case of four wheeled vehicles in [13], heavy-duty vehicles in [22] and tractor-semitrailer combination in [20]. In recent years the problem of steering control has attracted wide interest also outside the PATH program. Relevant contributions in the field of robust steering and vehicle lateral dynamics control were also provided by Ackermann and coworkers (see, e.g., [1], [10]). Preliminary experimental results on robust lateral control conducted by Byrne *et al.* were reported in [3] highlighting several implementation difficulties. Other contributions based on the look-ahead sensing scheme were provided by Hatipoğlu *et al.* in [11], where they use a digital videocamera together with a radar system, and by Broggi *et al.* [2] who used a stereo vision system composed by two videocameras. Significant Japanese contributions to the development of vision-based intelligent vehicles, given since the mid 1970's to early 1990's are surveyed by the paper of Tsugawa [21]. In this paper we address the problem of automatic lane keeping in highway scenario using vision system. First we present the physical plant and derive a suitable simplified model focusing on the accordance between simplification hypotheses and experimental context. Then, we formulate the control problem and show how the original Single Input - Two Outputs (SITO) problem can be reduced to a simpler SISO problem through the feedback of a suitable look-ahead signal. Further we design the controller using classical frequency domain techniques, taking into account the presence of model uncertainties. Finally we present and discuss the experimental results. This research activity was conducted in collaboration with *Centro Ricerche Fiat* that supplied the vehicle equipped with sensors and actuators.

## 2 Plant modeling

In this section we present a simplified mathematical model of the plant. Each simplification hypothesis is discussed in order to show that the proposed model is suitable for both control design and simulation.

**2.1 Hardware description** – The plant to be controlled, provided by Centro Ricerche Fiat, consists of a Fiat Brava 1600 ELX equipped with a vision system and a steering actuator. The vision system is composed by a single CCD videocamera located on the wind shield and image processing algorithms which supply suitable information about the vehicle location on the lane. The steering actuator system is a locally controlled DC brush-less electric motor. Both control and vision algorithms are processed by an INTEL 486 microprocessor based Personal Computer for industrial applications. The sampling time of the whole system is  $T_s = 40$  ms.

**2.2 Vehicle lateral dynamics** – Vehicle dynamics can be accurately described by a 6 degrees of freedom nonlinear model obtained writing Newton - Euler equilibrium equations of forces and torques. However, for the aim of this paper, a simplified model describing only lateral dynamics seems to be more suitable. In this section the well known and widely used 2 degrees of freedom linear single track model is considered (see, e.g., [14]) since it gives a simple and sufficiently accurate description of the vehicle lateral dynamics when highway experimental conditions are considered. The state space equations of the linear single track model, written in the vehicle local frame and assuming the longitudinal velocity  $v_x$  as a parameter, are:

$$\begin{bmatrix} \dot{v}_y \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \frac{a_1}{v_x} & \frac{a_2 - a_5 v_x^2}{a_5 v_x} \\ \frac{a_3}{v_x} & \frac{a_4}{v_x} \end{bmatrix} \begin{bmatrix} v_y \\ \psi \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \delta_v \quad (1)$$

where  $v_y$  is the lateral velocity,  $\psi$  is the yaw angle,  $\delta_v$  is the steering-wheel angle and the coefficients values are  $\{a_1, a_2, a_3, a_4, a_5, b_1, b_2\} = \{-127.24, 82536, 43.44, -148.36, 1226, 0.0475, 0.0317\}$ . Equation (1) is the result of three simplification hypotheses. First the vehicle is reduced to a rigid body moving on a plane. Then the obtained model is approximated with a nonlinear single track one decoupling lateral and longitudinal dynamics. Finally, equation (1) is deduced linearizing the single track model and considering  $v_x$  as a parameter. The rigid body assumption implies that the wheel suspension dynamics are completely neglected, that is no pitch, roll and heave motions are considered; consequently only small steering angles and approximately constant longitudinal velocity are admissible under this hypothesis. The reduction to a single track model describing solely lateral dynamics is possible only assuming symmetry of the vehicle with respect to the longitudinal plane containing  $x$  and  $z$  axes and neglecting the rolling friction. In addition to the small steering angles hypothesis, linearization requires also small side slip angle to describe the tires with a linear model. The above analysis shows that the mathematical model expressed by equation (1) is able to adequately describe the vehicle behaviour along paths characterized by flat road with large radius curves and when the vehicle velocity is approximately

constant. Thus we can conclude that the linear single track model is a simplified model that properly fits our experimental conditions. Only in presence of a vehicle equipped with soft suspension systems it could be useful to extend the model in order to include roll dynamics as shown by Feng *et al.* in [6].

**2.3 Vision system** – The vision system supplies the value for two parameters, namely  $m$  and  $q$ , which characterize the linear approximation of the centerline of the lane (see Figure 1) through the following equation written in the vehicle local frame:

$$y = \tan(m)x + q \quad (2)$$

where  $m$  is the angle between the linear approximation of the centerline of the lane and the longitudinal axis of the vehicle, while  $q$  is the distance, measured along the  $y$  axis, between the above mentioned linear approximation and the vehicle center of gravity. Recalling the assumption of small steering angle, we note that  $\tan(m) \approx m$ , thus the interaction between the vehicle motion and the vision system can be approximately expressed by the following two differential equations:

$$\dot{q}(t) = v_x(t)m(t) - v_y(t) - v_x(t)K_L(t)L \quad (3)$$

$$\dot{m}(t) = v_x(t)K_L(t) - \dot{\psi}(t) \quad (4)$$

where  $L$  is the distance, along the longitudinal axis, from the vehicle center of gravity to a suitable point on the road between 3 and 20 meters ahead the vehicle (see Figure 2), the so called look-ahead distance;  $K_L(t)$  is road curvature, i.e., the inverse of the instantaneous curve radius measured at the look-ahead point. A quite similar mathematical description of the interaction between the vehicle and the vision system is also presented in [14] and [15] where  $\varepsilon_L$  and  $y_L$  are introduced instead of  $m$  and  $q$ ;  $\varepsilon_L$  is the angle between the vehicle longitudinal axis and the centerline tangent at the look-ahead distance  $L$ , while  $y_L$  is the distance between vehicle longitudinal axis and the tangent point (see Figure 2). It is relevant to note how deduction of equations (3) and (4) is based only on the approximations suggested by the assumption of the single track model. Hence, equations (3) and (4) are simplified relations coherent with our experimental conditions. Combining equations (3) and (4) with equations (1) we obtain the following state space model for the vehicle equipped with the vision system:

$$\begin{bmatrix} \dot{v}_y \\ \dot{\psi} \\ \dot{q} \\ \dot{m} \end{bmatrix} = \begin{bmatrix} a_1/v_x & \frac{-a_5 v_x^2 + a_2}{a_5 v_x} & 0 & 0 \\ a_3/v_x & a_4/v_x & 0 & 0 \\ -1 & 0 & 0 & v_x \\ 0 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_y \\ \psi \\ q \\ m \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ 0 \\ 0 \end{bmatrix} \delta_v + \begin{bmatrix} 0 \\ 0 \\ -Lv_x \\ v_x \end{bmatrix} K_L \quad (5)$$

where the command input is the steering-wheel angle  $\delta_v$  and the road curvature  $K_L$  is a disturbance input.

**2.4 The steering actuator** – In the vast literature on vehicle lateral control little attention is devoted to the problem of accurate modeling and control of

the actuator. However, as stated by Hingwe *et al.* in [12], the steering system is a critical component of the vehicle lateral control system for AHS (Automated Highway System). As a matter of fact lateral control design carried out disregarding this aspect can lead to bad results due to phase and time lag introduced by the actuator dynamics. The steering actuator used here is a DC brush-less electric motor equipped with a local control loop. The local controller was designed by the researchers of *Centro Ricerche Fiat* in a previous work ([4]). This closed loop actuator is modeled, for our purpose, by the following two transfer functions:

$$\frac{\delta_v}{\theta} = \frac{0.4537z + 0.3509}{z^2 - 0.2344z + 0.03907} \quad (6)$$

$$\frac{V_a}{\theta} = \frac{0.4636(z^2 - 1.306z + 0.4639)}{z^2 - 0.2344z + 0.03907} \quad (7)$$

where  $V_a$  is the voltage motor command and  $\theta$  is the reference steering signal provided by the lateral dynamics controller to be designed.

**2.5 Uncertainty characterization** – In the previous subsections a simplified nominal model for the overall plant has been presented. Most of the contributors in the field of steering control based their analysis and design on the use of nominal reduced order models similar to (5), sometimes considering the effect of the longitudinal velocity variations (see for example [7]). Only recently, works have been presented where the attention is focused on model uncertainty (see, e.g., [1], [3], [10], [22]). Looking at the proposed model it can be noted that some unmodeled linear and nonlinear dynamics are present in the actual plant like for example roll, yaw and heave effects neglected by the single track model, steering gear backlash or actuator voltage command saturation. Moreover, the system has a time varying nature due to parametric dependence by longitudinal velocity  $v_x$ , and some parameters of the simplified model can't be exactly known. Thus, all these sources of uncertainty should be taken into account in the controller design. However, it can be noted that unmodeled dynamics are little excited along path with large radius curve and slow steering action and highway longitudinal velocity variations are typically slow. Thus, all considered we choose to design a fixed controller that robustly stabilize the plant in the face of the following parameters uncertainty. The vehicle mass  $m_v$  can vary from the nominal value of 1226 kg to 1626 kg corresponding to the mass of the vehicle with 5 passengers of 80 kg on board, the inertial moment  $I_\psi$  can vary coherently from 1900 kgm<sup>2</sup> to 2520 kgm<sup>2</sup>, the cornering stiffness coefficients  $c_f$  and  $c_r$  range respectively in [51000, 69000] N/rad and in [81600, 110400] N/rad and, finally, also the velocity is handled as an uncertain parameter whose value belongs the range [60, 130] km/h.

### 3 Analysis and design of the control loop

**3.1 Performance specifications and control strategy** – The general task of the control problem addressed in this work can be described by the following set of specifications:

$$|q(t)| \leq 20 \text{ cm} \quad (8)$$

$$|v_y(t)| \leq 1.5 \text{ m/s} \quad (9)$$

$$V_a(t) \leq 3 \text{ V} \quad (10)$$

$$|a_L(t) - a_C(t)| \leq 0.3G \approx 3.3 \text{ m/s}^2 \quad (11)$$

where  $V_a$  is the voltage command of the electric motor,  $a_L$  is the actual lateral acceleration and  $a_C$  is the theoretic value of lateral acceleration needed to perform the bend. Inequality (8) is a performance specification about the vehicle position error with respect to the centerline of the lane, (10) is about actuator saturation while both (9) and (11) are about passengers comfort as required by C.R.F. All specifications must be satisfied when  $v_x$  takes values in the range [60, 130] km/h and the road curvature satisfies the constraints  $K_L \leq 0.001 \text{ m}^{-1}$ . We have formulated the control problem in terms of zero-regulation of the distance between the vehicle and the centerline, in presence of the disturbance input  $K_L$ . As a matter of fact this formulation involves adequate use of both  $q$  and  $m$ . From this point of view the problem can be considered as a SITO control problem in which the control task must be achieved acting on the steering command on the basis of the measurements provided by the vision system. Thus, a key point is how to efficiently use these signals. For the aim of designing a control system able to perform the automatic lane keeping and providing good ride comfort, a possible strategy is to emulate the common driver behaviour. Land and Lee in [16] showed experimentally that the human driver directs his gaze to the tangent point of the curve one or two seconds before the car enters the bend, i.e., the driver evaluates the correct steering action to apply on the basis of the distance between the lane and the longitudinal axis of the car at a look-ahead point. From the control theory point of view this result is confirmed by the analysis of Košecá *et al.* presented in [14]. Using the measurement of  $y_L$  provided by their vision system as feedback output, they showed that as the look-ahead distance increases, the closed-loop poles damping increases too. This fact highlights the need of visual information at the look-ahead point for smoothly performing the turn, while feedback based on position error measured at center of gravity CG leads to bad ride comfort. Thus, all these considerations suggest a suitable way of using measurements of  $q$  and  $m$ . These quantities can, in fact, be used to obtain a feedback output approximating the distance between the lane and the longitudinal axis of the car at a look-ahead point, through the following linear relation:

$$y_{fb} = q + mL \quad (12)$$

where  $y_{fb}$  is the distance, measured at the look-ahead point, between the longitudinal axis of the vehicle and the linear approximation of the centerline of the lane (see Figure 1). It is important to remark that we use  $y_{fb}$  as feedback signal while specification (8) is defined on the output  $q$  which is more directly related to the actual vehicle position on the lane. From this point of view there is a trade off in the choice of the look-ahead distance value. In our experimental tests we noted that increasing  $L$  improves comfort performances though it tends to cause large lateral position error at center of gravity CG when a curve is approached. The experimental results presented in this paper were obtained with  $L = 11.5 \text{ m}$ .

It is possible to show ([5]) that the SISO strategy adopted in this paper can also be derived using proprieties of single-input, two-output feedback systems ([9]).

**3.2 Controller design and Robustness issue** – The controller design was carried out in the frequency domain using classical loop-shaping techniques taking into account the uncertainty characterization given in subsection 2.5. The control structure adopted is a one degree of freedom structure. The achievement of the performances specifications (8), (9), (10), (11) was optimized through the fine tuning of the controller on the basis of time domain simulations. Two different six-order controllers, both fulfilling all specifications in the simulation stage, are presented in this work. The first one,  $C_1(z) = n_1(z)/d_1(z)$ , was designed for the aim of obtaining better lane keeping precision, while the second one,  $C_2(z) = n_2(z)/d_2(z)$ , was designed trying to optimize the ride comfort. Thus we experimentally compared two different driving behaviours, looking for the best trade-off between small position error and ride comfort. The numerical values of the controllers parameters, in descending powers of  $z$ , are:  $n_1 = \{-7.844, 30.82, -47.37, 35.51, -13.24, 2.388, -0.2273\}$ ,  $d_1 = \{1, -4.92, 10.06, -10.96, 6.703, -2.181, 0.2949\}$ ,  $n_2 = \{-7.387, 29.03, -44.6, 33.43, -12.46, 2.2, -0.2133\}$ ,  $d_2 = \{1, -4.937, 10.13, -11.07, 6.794, -2.218, 0.3008\}$ . Uncertainty regions of the shaped open loop, depicted in Figure 3 for the case of  $C_1$  and in Figure 4 for the case of  $C_2$ , show that both the controllers ensure robust stability.

**3.3 Experimental results** – In this section we report the experimental results obtained testing the controlled system along italian highways. Figures 5, 7 show the lateral offset  $q$  and the steering angle  $\delta_v$  respectively, measured when the controller  $C_1$  was implemented. The test track is a curve with radius  $R \approx 1000$  m followed by a straight section. The velocity was kept approximately constant at 100 km/h. In Figures 6, 8 the same quantities are depicted when  $C_2$  was used. Both the controllers satisfy the voltage command constraint.

#### 4 Discussion

The experimental results of the previous section show that both the controllers satisfy the specifications about position error with respect to the centerline of the lane and actuator saturation, while specifications about comfort quality cannot be evaluated from data because lateral acceleration and lateral velocity sensors are not available on the vehicle. On the other hand, it is difficult to relate comfort quality to any physical quantity characterizing the vehicle motion, due to the difficulties in describing the comfort criterion optimized by a human driver. Thus we followed a different approach for optimizing comfort performances, which consists of testing different solutions with the help of one or more expert test drivers. Comparing the two controllers proposed in this paper, test drivers of Centro Ricerche Fiat expressed a better appraisal about the controller  $C_2$  than about the controller  $C_1$  showing how excessive precision in following the centerline of the lane leads to a less comfortable driving behaviour because of its disagreement with the typical human driving behaviour. The test drivers appraisal is confirmed by data shown in Figure 7 and Figure 8

which highlight the different control action performed by the two controllers. Indeed, the steering-wheel angle and the lateral offset obtained with controller  $C_2$  are smoother than those obtained with controller  $C_1$ .

#### 5 Conclusion

In this paper the problem of highway lane keeping through automatic steering was addressed. First a simplified mathematical model of the vehicle was presented showing its agreements with the typical highway experimental conditions. Then, two controllers designed using classical loop-shaping techniques were presented. Uncertainty regions depicted on the Nichols chart show that those controllers robustly stabilize the system in presence of the considered model uncertainty. Experimental tests, conducted on italian highways using a FIAT Brava 1600 ELX provided by Centro Ricerche Fiat, showed the fulfillment of the performance specifications about lane keeping error and actuator saturation. Moreover, comparison between the two controllers, show that the one that keeps the lane with better precision, leads to worse ride comfort, highlighting how trade off between small position error and good ride comfort could be arranged only on the basis of the driver's perceptions.

#### 6 Acknowledgements

This research was supported by Centro Ricerche Fiat (CRF), under the contract "Analisi e sintesi di nuove architetture di controllo laterale per il mantenimento della corsia di marcia di un autoveicolo". The authors are grateful to Ing. S. Campo of Centro Ricerche Fiat and to Prof. M. Milanese of Politecnico di Torino for the fruitful discussions during the developments of the project and their comments on several aspects of this paper.

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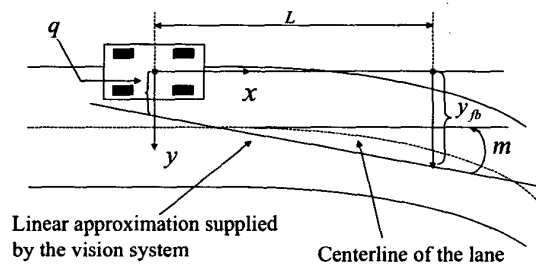


Figure 1: Centerline linear approximation supplied by the vision system

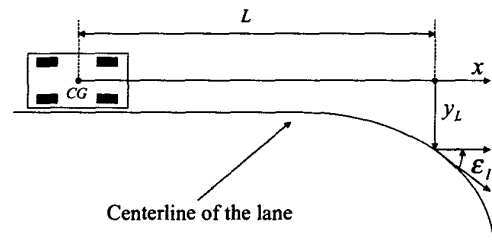


Figure 2: Look-ahead distance

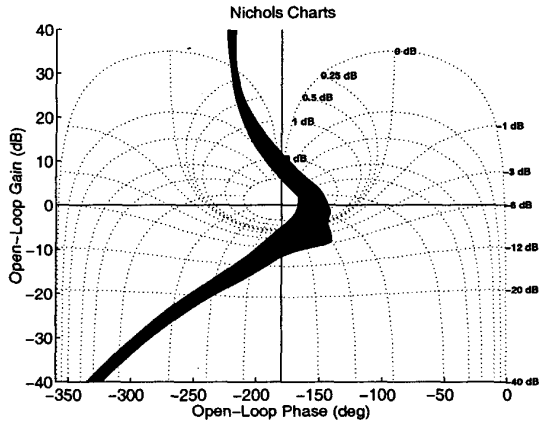


Figure 3: Uncertainty region of the open loop shaped plant with controller  $C_1$

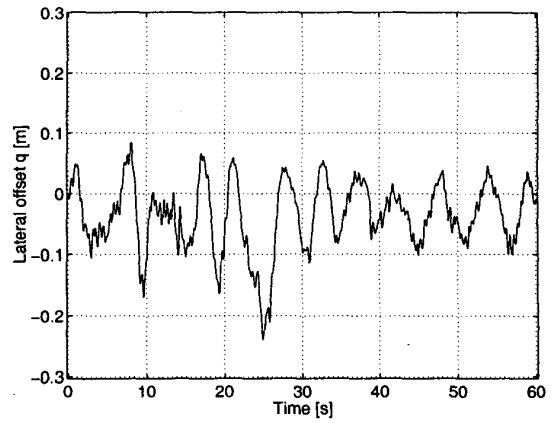


Figure 6: Lateral offset  $q$  with controller  $C_2$

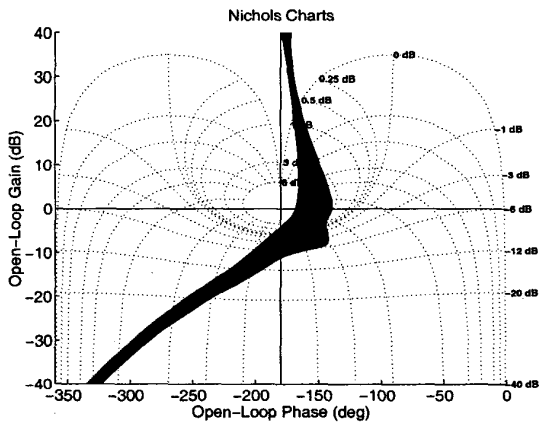


Figure 4: Uncertainty region of the open loop shaped plant with controller  $C_2$

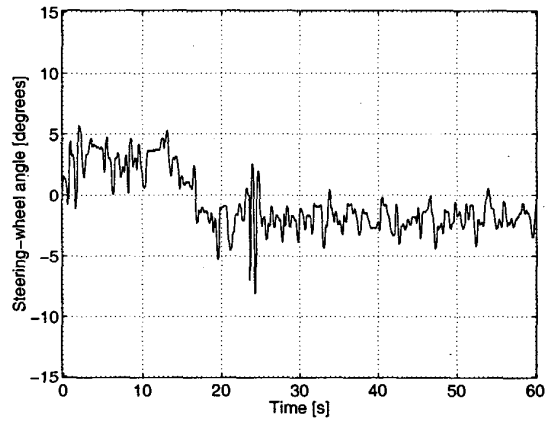


Figure 7: Steering angle  $\delta_v$  with controller  $C_1$

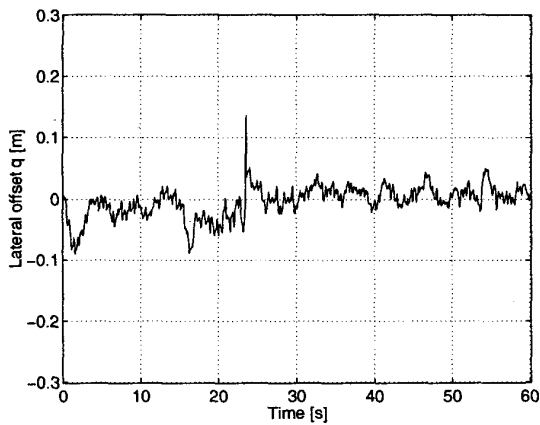


Figure 5: Lateral offset  $q$  with controller  $C_1$

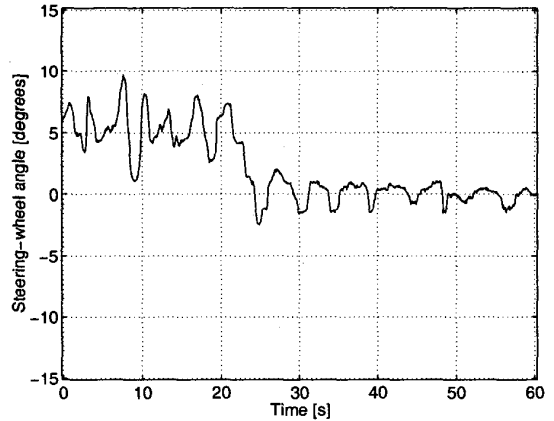


Figure 8: Steering angle  $\delta_v$  with controller  $C_2$