

Yaw stability control design through a mixed sensitivity approach

V. Cerone, M. Milanese, D. Regruto

Abstract—In this work a Vehicle Dynamics Control (VDC) system for tracking desired vehicle behavior is developed. A two degrees of freedom control structure is proposed to prevent vehicle skidding during critical maneuvers through the application of differential braking between right and left wheels in order to control yaw motion. The feed-forward filter is a reference generator which compute the desired yaw rate on the basis of the steering angle, while the feedback controller is designed to track the reference as close as possible and to satisfy suitable loop robustness requirements. Mixed-sensitivity minimization techniques are exploited in order to design the loop controller. The performance of the control system is evaluated through Hardware In-the-Loop Simulation (HILS) system both under emergency maneuvers and in non-critical driving conditions, i.e. when the VDC system is not supposed to intervene.

Index Terms—Vehicle Dynamics Control (VDC); Yaw moment control; Hardware in-the-loop simulation (HILS)

I. INTRODUCTION

Active safety systems help prevent accidents by taking control away from the driver temporarily, until the undesired vehicle dynamic behaviour is corrected. One of the most studied active safety system which aims at enhancing the vehicle yaw stability is the Vehicle Dynamics Control system (VDC). Indeed loss of vehicle yaw stability may result from unexpected yaw disturbances like side wind force, tire pressure loss or μ -split braking due to unilaterally different road such as icy or wet pavement. Safe driving requires the driver to react rapidly and properly. The main goal of vehicle yaw stability control systems is to compensate for the driver's inadequacy and generate a control yaw moment through either steering or braking control inputs or both. VDC system directly controls yaw moment by generating differential longitudinal forces on left and right tires, which in turn effectively affect the vehicle lateral motion. The two primary corrective yaw moment generating methods of actuation for VDC systems are compensation using steering commands or using differential wheel braking. Pioneering results on VDC can be found in [1], [2] and [3]. Most of the commercially available VDC systems use differential wheel braking as it is more easily accomplished through already existing ABS hardware (see, e.g., [4]). In [5] a chassis control strategy for improving the limit performance of vehicle motion is proposed. The effects of braking force distribution on a vehicle lateral and longitudinal directions are studied. In [6] an integrated control system of active rear wheel steering and yaw moment control using braking forces is presented.

The authors are with the Dipartimento di Automatica e Informatica, Politecnico di Torino, corso Duca degli Abruzzi 24, 10129 Torino, Italy; e-mail: vito.cerone@polito.it, diego.regruto@polito.it, mario.milanese@polito.it; Tel: +39-011-564 7064; Fax: +39-011-564 7099

The control system was designed using model matching control theory to make the vehicle performance follow a desired dynamics model. An H-infinity yaw moment control using brake torque for improving vehicle performance and stability in high speed driving is described in [7]. In the work [8], a two degrees of freedom steering controller architecture based on a disturbance-observer method is adapted to the vehicle yaw-dynamics problem and shown to robustly improve vehicle yaw dynamics performance. An auxiliary-steering actuation system, a steering controller that only intervenes when necessary, and a velocity-gain scheduled implementation that is tested throughout the range of operation are considered. In [9] the predictive characteristics of the Generalized Predictive Control are exploited in order to derive a yaw stability control algorithm. The control algorithm is based on a linearized vehicle model. A VDC system for improving dynamic stability under critical lateral motions is developed in [10]. The use of yaw moment control is investigated by adjusting the wheel slip ratio for improving handling and stability of vehicle. The purpose of the proposed control system is to make the yaw rate and side slip angle of the vehicle track their corresponding desired values.

In this work a VDC system for tracking desired vehicle behavior is developed. A two degrees of freedom control structure is proposed to prevent vehicle skidding during critical maneuvers through the application of differential braking between right and left wheels in order to control yaw motion. The feed-forward filter is a reference generator which compute the desired yaw rate on the basis of the steering angle, while the feedback controller is designed to track the reference as close as possible and to satisfy suitable loop robustness requirements. The design of the loop controller is formulated as the minimization of a weighted mixed-sensitivity functional performed by means of an iterative procedure which handles approximate model identification and model-based controller design as a joint problem. At each iteration a refined mathematical model of the plant is estimated on the basis of the closed loop experimental data obtained implementing the controller computed at the previous iteration; then, the controller is redesigned, through H_∞ optimization techniques, exploiting the new plant model. Iterations stop when closed-loop performance requirements are met. Thanks to the above approach no accurate modeling of the plant is needed before addressing the design procedure. The iterative procedure intrinsically provides a plant model with the suitable level of accuracy required to design a controller which is guarantee to satisfy closed-loop performance specifications on the actual plant. The performance of the

control system is evaluated through Hardware In-the-Loop Simulation (HILS) system both under emergency maneuvers and in non-critical driving conditions, i.e. when the VDC system is not supposed to intervene.

II. PLANT DESCRIPTION

The plant to be controlled is an Hardware-in-the-loop test-bench built by the Vehicle Dynamics Research Team of the Mechanical Engineering Department of Politecnico di Torino ([11], [12]). Such a test-bench, shown in Fig. 2, consists of the whole braking system of a FIAT passenger car properly interfaced, through a personal computer and a suitable dSPACE® board, with a real-time vehicle dynamics simulator. More precisely the components of the system are the vacuum booster, the Tandem Master Cylinder, the wheels calipers with their mechanical supports, and a hydraulic VDC unit with twelve PWM-controlled solenoid valves used to regulate the oil pressure in the brakes chambers. The hydraulic circuit of the test-bench consists of an electric motor, a gear pump, an accumulator, some pressure limiter/reducer valves and a proportional valve. Four pressure sensors are used to measure the four wheels calipers pressures which are the inputs of the vehicle model. From such measurements, wheel braking torques are easily computed exploiting standard physical equations (see, e.g., [13]). The real-time nonlinear model used on the test-bench to simulate the vehicle dynamics has eight degrees of freedom. Four degrees (lateral, longitudinal, yaw and roll motions) account for the vehicle body dynamics, while four degrees are used to describe the wheels rotational motions. Tyres were modeled using Pacejka Magic Formula [14]. The test-bench also includes a low-level open loop controller of the hydraulic actuator. Such controller suitably regulates both the PWM voltage of the solenoid valves and the hydraulic pump on/off signal in order to actuate the desired pressure signal provided by the high-level VDC controller to be designed. A set of different manoeuvres can be performed by means of a suitable driver simulator included in the model. In paper [12] a validation of the overall vehicle model was performed: simulated and experimental results were compared on a suitable range of manoeuvres including steering angle steps, ramps and frequency sweep. A detailed description of the test-bench can be found in the papers by Sorniotti [11] and Sorniotti and Velardocchia [12].

III. CONTROL OBJECTIVE AND PERFORMANCE REQUIREMENTS

The problem we are dealing with in this paper is the design of a closed loop control system able to prevent vehicle skidding during critical manoeuvres through the application of differential braking between right and left wheels in order to control yaw motion. More specifically, the control problem is defined with reference to the following two specific tests: (M1) *Step Steer test*: the test starts with a vehicle traveling at the constant speed of 100 km/h. The driver turns the steering wheel at the speed of 250 degrees/second until a specified target angle is reached. Then, the target angle is

held. Different steering target angles have to be considered from 50 degrees to 110 degrees. This maneuver is used to define desired behaviour of the controlled vehicle in critical driving conditions.

(M2) *Slow Ramp Steer test*: the test is performed at the constant speed of 100 km/h. The driver slowly increases the steering wheel angle (15 degrees/second) from 0 to 130 degrees. This maneuver is used to define desired behaviour of the controlled vehicle in non-critical driving conditions.

The controlled vehicle has to satisfy the following performance specifications, derived from qualitative considerations on vehicle yaw stability, when a Step Steer test is run:

(S1) time between first and second peak of yaw rate less than 1 second;

(S2) amplitude difference between first and second peak of yaw rate less than 15 degrees/second;

(S3) side-slip angle first peak amplitude less than 7 degrees;

(S4) rise time of the controlled vehicle similar to the one of the passive vehicle;

(S5) steady-state behaviour of the controlled vehicle similar to the one of the passive vehicle;

(S6) difference between controlled and passive vehicle velocities less than 5 km/h.

All specifications (S1) - (S6) have to be satisfied for each value of the step amplitude between 50 and 110 degrees.

As far as the Ramp Steer test is considered, the controller is required not to act on the braking system during such a maneuver in order to avoid perturbation of the behaviour of the car during non-critical driving conditions.

IV. CONTROL STRUCTURE AND STRATEGY

A. Control structure

In order to meet the performance requirements specified in Section III, we propose the two degrees of freedom control structure of Fig. 1 where controller C_2 is a reference generator which compute the desired yaw rate on the basis of the steering angle, and C_1 is the feedback controller designed to track the reference as well as possible while satisfying suitable loop robustness requirements. The output signal of controller C_1 is the plant command input $p(t)$. The absolute value $|p(t)|$ is the actual reference oil pressure to be actuated in the brakes chamber while the sign of $p(t)$ is used, together with the sign of the steering angle $\delta(t)$, to select the wheels brakes which has to be activated. More specifically a brakes calipers selector acts on the hydraulic circuit to apply the pressure $|p(t)|$ to either the right or the left wheels brakes according to the following rule: left wheels brakes are activated if either $\delta(t), p(t) > 0$ or $\delta(t), p(t) < 0$; right wheels brakes are activated otherwise. The vehicle is modeled by $G_s(s)$, i.e. the transfer function between the steering angle δ and the yaw rate $\dot{\psi}$, and by $G_p(s)$ which is a linear model of the relation between the desired pressure $p(t)$ and the yaw rate including the controlled actuator and the calipers selector. How to obtain model $G_s(s)$ and $G_p(s)$ will be discussed in Section IV-D and IV-E.

B. Design of the reference generator C_2

In order to properly design the reference generator C_2 it is first required to define the main properties of the desired yaw rate to be generated. Analysis of performance specifications leads to the choice of a yaw rate reference signal which has to preserve the steady state behaviour (specification (S5)) and the speed of response (specification (S4)) of the uncontrolled vehicle while significantly reducing the oscillation during transients (specifications (S1) - (S3)). Besides, the yaw rate reference signal generated when the driver performs a Slow Ramp Steer test should be as close as possible to the yaw rate of the uncontrolled vehicle in order to guarantee that the control system will not act on the braking system. The idea exploited in this work to meet all such requirements is to compute the reference yaw rate $\dot{\psi}_r$ through the:

$$\dot{\psi}_r(t) = f(t) * g(t) = f(t) * (h^{-1}(\delta(t), v_x(t))/v_x(t)) \quad (1)$$

where $f(t)$ is the impulse response of a low pass filter with transfer function $F(s) = 10/(s + 10)$, $*$ is the convolution integral and the function $h(a_y, v_x(t))$ is the so called understeering curve of the uncontrolled vehicle which, for any fixed velocity $v_x(t)$, relates the lateral acceleration $a_y(t)$ and the steering angle $\delta(t)$ at steady state. The understeering curve $\delta(t) = h(a_y(t), v_x(t))$ of the vehicle considered in this paper is depicted in Fig. 3 for the case $v_x = 100$ km/h. Since it is easy to show that at constant longitudinal velocity v_x , the lateral acceleration $a_y(t)$ and the yaw rate $\dot{\psi}$ at steady state satisfy equation $a_y(t) = v_x \dot{\psi}(t)$ (see, e.g., equation (8.26) of [13]), it turns out that function $g(t)$ is, for any approximately constant velocity v_x , a good approximation of the static mapping which relates the steering angle $\delta(t)$ and the yaw rate $\dot{\psi}(t)$ of the passive vehicle at steady state. Thus, the proposed reference yaw rate, which at steady state equals $g(t)$, preserve the steady state behaviour of the vehicle (specification (S5)). The filter $F(s)$ was introduced to properly set the rise time of the controlled vehicle in order to obtain the same speed of response of the uncontrolled one (specification (S4)). Besides, since $\dot{\psi}_r$ is the output of a static function filtered through a linear system with a real pole, the reference yaw rate will not show any oscillation when a Step Steer test is performed (specification (S1) - (S3)). Finally, one must consider that the understeering curve $h(a_y, v_x(t))$ is usually obtained experimentally performing a Slow Ramp Steer test. Thus, at least in principle, the proposed yaw rate reference will equal the actual yaw rate of the uncontrolled vehicle when a Slow Ramp Steer test is performed, intrinsically avoiding the actuation of the braking system.

C. Design of controller $C_1(s)$: problem formulation

The feedback controller C_1 must be designed to track, as close as possible, the desired yaw rate provided by controller C_2 while guaranteeing satisfactory loop robustness margins. From the block diagram of Fig. 1 it is easily seen that the transfer function between the steering angle $\delta(t)$ and the

tracking error $e(t) = \dot{\psi}_r - \dot{\psi}$ is given by:

$$\frac{E(s, C_1)}{\Delta(s)} = S(s, C_1)(\bar{C}_2(s) - G_s(s)) \quad (2)$$

where $E(s, C_1)$ and $\Delta(s)$ are the Laplace transforms of the signals $e(t, C_1)$ and $\delta(t)$ respectively, $S(s) = 1/(1 + C_1(s)G_p(s))$ is the sensitivity function and $\bar{C}_2(s)$ is a linear transfer function which approximates the nonlinear reference generator C_2 .

Equation (2) shows that the tracking error $e(t)$ can be reduced by properly shaping the frequency response of the sensitivity function $S(s)$. More precisely, our objective is to design a controller $C_1(s)$ such that the transfer function between the steering angle and the tracking error satisfies the following inequality:

$$\left| \frac{E(j\omega, C_1)}{\Delta(j\omega)} \right| \leq |W^{-1}(j\omega)|, \quad \forall \omega \quad (3)$$

where $W(s)$ is a rational function which properly embed tracking performance requirements. Exploiting equations (2) and (3) and the definition of H_∞ norm of a single-input single-output system, the control problem can be formulated as the following H_∞ sensitivity minimization problem:

$$C_1^*(s) = \arg \min_{C_1 \in \mathcal{C}} \|S(s, C_1)W_S(s)\|_\infty \quad (4)$$

where \mathcal{C} is the class of controllers which stabilizes the plant and $|W_S(j\omega)| \geq |W(j\omega)(\bar{C}_2(j\omega) - G_s(j\omega))|$, $\forall \omega$.

As well known the solution of a pure sensitivity minimization problem can lead to the design of a control system with quite a large bandwidth which can cause instability problems when the controller is applied to the real plant. In order to limit the control system bandwidth, controller $C_1(s)$ is actually computed solving the following mixed-sensitivity problem:

$$\begin{aligned} C_1^*(s) &= \arg \min_{C_1 \in \mathcal{C}} J(C_1, G_p) \\ &= \arg \min_{C_1 \in \mathcal{C}} \left\| \begin{array}{c} S(C_1, G_p)W_S(s) \\ T(C_1, G_p)W_T(s) \end{array} \right\|_\infty \end{aligned} \quad (5)$$

where $T(C_1, G_p) = C_1 G_p / (1 + C_1 G_p)$ is the complementary sensitivity function and where $W_T(s)$ is a rational function which properly embed bandwidth requirements.

D. Design of controller $C_1(s)$: selection of weighting functions $W_S(s)$ and $W_T(s)$

The following structure has been assumed for the weighting function $W_S(s)$:

$$W_S(s) = \frac{s^2/\omega_n^2 + 1.414s/\omega_n + 1}{\alpha(s/z + 1)s} \quad (6)$$

where the pole at the origin has been included to guarantee robust tracking of constant signal and parameters z , ω_n and α must be selected in order to impose a lower bound on the control system bandwidth and to satisfy the inequality $|W_S(j\omega)| \geq |W(j\omega)(\bar{C}_2(j\omega) - G_s(j\omega))|$, $\forall \omega$. The values $z = 0.8$, $\alpha = 0.12$ and $\omega_n = 3$ have been selected. The following weighting function $W_T(s) = (s/10 + 1)/1.2$ has been chosen in order to impose a proper upper bound on the

control system bandwidth. The transfer function $G_s(s) = 0.82307(s + 1.885)/(s^2 + 2.916s + 10.13)$ has been obtained from the standard linear bicycle model with mass $m = 1678$ kg, moment of inertia $I = 3070$ kg m², distance from the front axle to the center of gravity (CG) $l_f = 1.15$ m, distance from the rear axle to CG $l_r = 1.55$ m, and front and rear tires cornering stiffness coefficients $c_f = 28648$ N/rad and $c_r = 37425$ N/rad respectively. Note that the value of c_r and c_f have been tuned in order to match experimental yaw rate response of the passive vehicle to a Step Steer test of 110 degrees. The linear model $\bar{C}_2(s) = 1.636/(s + 10)$ of the reference generator has been obtained by approximating the understeering curve at velocity $v_x = 100$ km/h with a constant gain.

E. Design of controller $C_1(s)$: iterative approach

The computation of the controller as solution of problem (5) is based on a model $G_p(s)$ which is an approximate description of the true unknown plant. Thus, the performance of the actual control system will depend on such a model and the problem of a how to properly select the transfer function $G_p(s)$ arises. As pointed out in papers [15], [16] approximate model identification and model based controller design have to be treated as a joint problem in order to guarantee a reasonable degree of robustness. To be more precise, let us use the symbols \tilde{G}_p to indicate the unknown true plant. The actual problem to be solved can be formulated as:

$$\begin{aligned} C_1^*(s) &= \arg \min_{C_1 \in \mathcal{C}} J(C_1, \tilde{G}_p) \\ &= \arg \min_{C_1 \in \mathcal{C}} \left\| \begin{array}{l} S(C_1, \tilde{G}_p)W_S(s) \\ T(C_1, \tilde{G}_p)W_T(s) \end{array} \right\|_{\infty} \end{aligned} \quad (7)$$

where $J(C_1, \tilde{G}_p)$ is the mixed sensitivity functional of the actual control system. Since the true plant \tilde{G}_p is not known, problem (7) cannot be exactly solved. However, according to papers [15], [16], we can consider the following triangle inequality:

$$\begin{aligned} \|J(C_1, \tilde{G}_p)\|_{\infty} &\leq \|J(C_1, G_p)\|_{\infty} \\ &+ \|J(C_1, \tilde{G}_p) - J(C_1, G_p)\|_{\infty} \end{aligned} \quad (8)$$

Exploiting equation (8) the control problem can be reformulated as the following minimization problem:

$$\begin{aligned} (C_1^*(s), G_p^*(s)) &= \arg \min_{C_1, G_p} \{ \|J(C_1, G_p)\|_{\infty} \\ &+ \|J(C_1, \tilde{G}_p) - J(C_1, G_p)\|_{\infty} \} \end{aligned} \quad (9)$$

In order to solve problem (9), an iterative approach has been proposed in papers [15], [16]. At the i -th iteration the plant model $G_p^i(s)$ and the feedback controller $C_1^i(s)$ are computed according to:

$$G_p^i(s) = \arg \min_{G_p} \|J(C_1^{i-1}, \tilde{G}_p) - J(C_1^{i-1}, G_p)\|_{\infty} \quad (10)$$

and

$$C_1^i(s) = \arg \min_{C_1} \|J(C_1, G_p^i)\|_{\infty} \quad (11)$$

Problem (11) can be solved by means of standard H_{∞} control technique.

Problem (10) is equivalent to an H_{∞} closed-loop identification problem and can be solved exploiting the approach proposed in [17] which is based on the Dual-Youla Parameterization of all plants which are stabilized by a given controller. The identification is performed exploiting the experimental closed loop data obtained implementing controller C_1^{i-1} .

After three iterations the following results have been obtained:

$$G_p^3(s) = 0.006441 \frac{(s + 17.4)}{((s + 7.745)(s + 1.203))} \quad (12)$$

and

$$C_1^3(s) = 12046354 \frac{(s + 7.745)(s + 1.657)(s + 1.203)}{s(s + 6579)(s + 17.82)(s + 0.8)} \quad (13)$$

Controller C_1^3 largely satisfies the performance specifications (S1) - (S6) as shown by the experimental results presented in Section V.

F. Implementation issues: controller activation threshold

As discussed in Section IV-B, in principle the yaw rate reference signal should be equal to the actual yaw rate of the uncontrolled vehicle when a Slow Ramp Steer test (test M2) is performed, intrinsically avoiding the actuation of the braking system. Since in practice the difference between the two signals is not exactly zero, it was necessary to introduce a threshold on the error signal $e(t)$: the control system is activated when the absolute value of the error is greater than 2 degrees/sec.

V. EXPERIMENTAL RESULTS

In this section we report the experimental results obtained testing the controlled system on the HIL test-bench. A Step Steer test (test M1) of 110 degrees on a dry road was performed. The behaviour of the vehicle during such a manouvre is highly nonlinear since, as shown by Fig. 10, the lateral acceleration approaches its saturation limit (see the understeering curve of Fig. 3). The yaw rate, the sideslip angle and the vehicle velocity of the controlled and uncontrolled vehicle are compared in Figg. 8, 9 and 11 respectively. Such figures show the effectiveness of the proposed control system which improves the performance of the uncontrolled vehicle and satisfies all the specification (S1) - (S6). More specifically, the yaw rate of the controlled vehicle actually shows only one peak implicitly satisfying both specification (S1) and (S2). Besides, as clearly shown in Fig. 8, both the speed of response (specification (S4)) and the steady-state behaviour (specification (S5)) of the controlled vehicle are quite similar to those of the passive one. As far as specification (S3) is considered, the absolute value of the sideslip angle peak amplitude of the controlled vehicle is 4.2 degrees which is about 30% less than the maximum allowed one. Finally, as can be seen from Fig. 11, the maximum difference between controlled and passive vehicle velocities is about 4.5 km/h; thus, also specification (S6) is satisfied. Brakes pressures are depicted in Figg. 8, 9, 10 and 11.

VI. CONCLUSIONS

In this work a Vehicle Dynamics Control (VDC) system for tracking desired vehicle behavior is developed. A two degrees of freedom control structure is proposed to prevent vehicle skidding during critical maneuvers through the application of differential braking between right and left wheels in order to control yaw motion. The feed-forward filter is a reference generator which compute the desired yaw rate on the basis of the steering angle, while the feedback controller is designed to track the reference as close as possible and to satisfy suitable loop robustness requirements. Mixed-sensitivity minimization techniques are exploited in order to design the loop controller. The performance of the control system is evaluated through Hardware In-the-Loop Simulation (HILS) system both under emergency maneuvers and in non-critical driving conditions, i.e. when the VDC system is not supposed to intervene. The results show that the proposed system clearly improves the vehicle stability for active safety.

ACKNOWLEDGEMENTS

This research was supported in part by the italian *Ministero dell'Istruzione, dell'Università e della Ricerca (MIUR)*, under the plan "Control of advanced systems of transmission, suspension, steering and braking for the management of the vehicle dynamics", and by Fiat Spa under the contract "Metodologie di impostazione e sviluppo algoritmi e strategie di controllo dell'autotelaio — Logiche di controllo innovative basate sulla frenatura dinamica". The authors are grateful to Prof. M. Velardocchia, Dr. A. Morgando, Dr. A. Sorniotti, Ing. S. Campo and Ing. A. Fortina for the fruitful discussions during the development of the research.

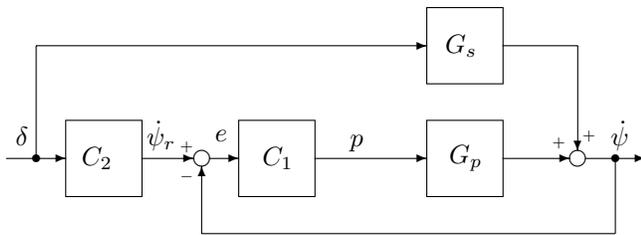


Fig. 1. Block diagram of the considered control system.

REFERENCES

[1] Y. Shibahata, K. Shimada, and T. Tomari, "The improvement of vehicle maneuverability by direct yaw moment control," in *Proceedings of AVEC 1992, Yokohama, Japan*, 1992.
 [2] S. Inagaki, I. Kshiro, and M. Yamamoto, "Analysis on vehicle stability in critical cornering using phase-plane method." in *Proceedings of AVEC 1994, Tsukuba, Japan*, 1994.
 [3] S. Matsumoto, H. Yamaguchi, H. Inoue, and Y. Yasuno, "Braking force distribution control for improved vehicle dynamics," in *Proceedings of AVEC 1992, Yokohama, Japan*, 1992.
 [4] A. V. Zanten, R. Erhardt, and G. Pfaff, "Vdc, the vehicle dynamics control system of bosch," in *SAE Technical Paper 950759*, 1995.
 [5] Y. Shibahata, A. Shimada, K. Shimada, and Y. Furukawa, "Improvement on limit performance of vehicle motion by chassis control," *Vehicle System Dynamics*, vol. 23, 1994.

[6] M. Nagai, S. Yamanaka, and Y. Hirano, "Integrated control of active rear wheel steering and yaw moment control using braking forces," *JSME International Journal*, vol. 42, no. 2, pp. 301–308, 1999.
 [7] J. Park and W. Ahn, "H-infinity yaw moment control with brakes for improving driving performance and stability," in *Proceedings of the IEEE/ASME International Conference on Advanced Intelligent Mechatronics*, September 19-23, 1999, Atlanta, GA, USA, pp. 747–752.
 [8] B. Güvenç, T. Bünte, D. Odenthal, and L. Güvenç, "Robust two degree-of-freedom vehicle steering controller design," *Ieee Transactions On Control Systems Technology*, vol. 12, no. 4, pp. 627–636, 2004.
 [9] S. Anwar, "Yaw stability control of an automotive vehicle via generalized predictive algorithm," in *Proc. American Control Conference*, June 8-10, 2005. Portland, OR, USA, pp. 435–440.
 [10] S. Zheng, H. Tang, Z. Han, and Y. Zhang, "Controller design for vehicle stability enhancement," *Control Engineering Practice*, vol. 14, p. 14131421, 2006.
 [11] A. Sorniotti, "Hardware in the loop for braking systems with anti-lock braking system and electronic stability program," *SAE Technical paper 2004-01-2062*, 2004.
 [12] M. Velardocchia and A. Sorniotti, "Hardware-in-the-loop to evaluate active braking systems performance," *SAE Technical paper 2005-01-1580*, 2005.
 [13] R. Rajamani, *Vehicle Dynamics and Control*. Springer, 2006.
 [14] H. B. Pacejka, *Tyre and vehicle dynamics*. SAE International, 2001.
 [15] R. Schrama, "Accurate identification for control: the necessity of an iterative scheme," *IEEE Trans. Automatic Control*, vol. 37, no. 7, pp. 991–994, 1992.
 [16] P. V. den Hof and R. Schrama, "Identification and control: Closed-loop issues," *Automatica*, vol. 31, no. 12, pp. 1751–1770, 1995.
 [17] M. Milanese, M. Taragna, and P. V. den Hof, "Closed-loop identification of uncertainty model for robust control design: a set membership approach," in *Proc. of the 36th IEEE Conference on Decision and Control*, 1997, pp. 2447–2452.

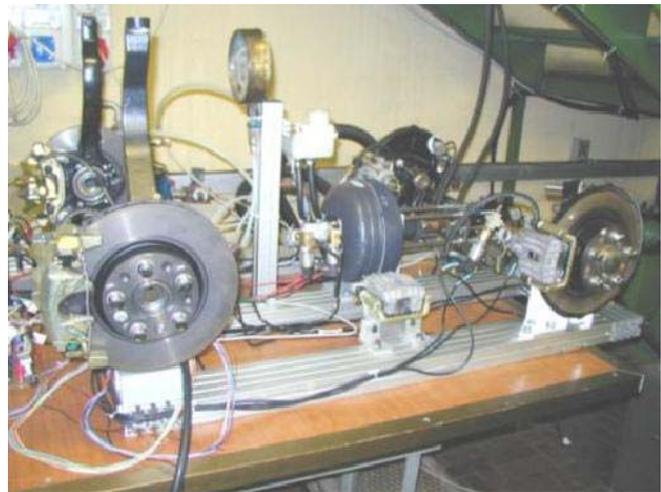


Fig. 2. Hardware in the loop test-bench.

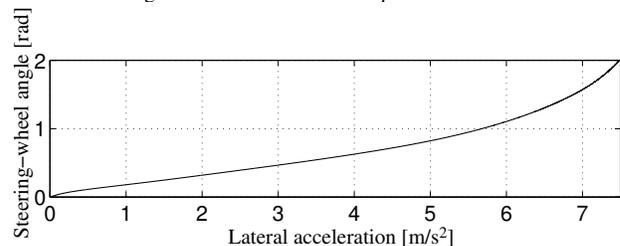


Fig. 3. Vehicle understeering curve considered in this work.

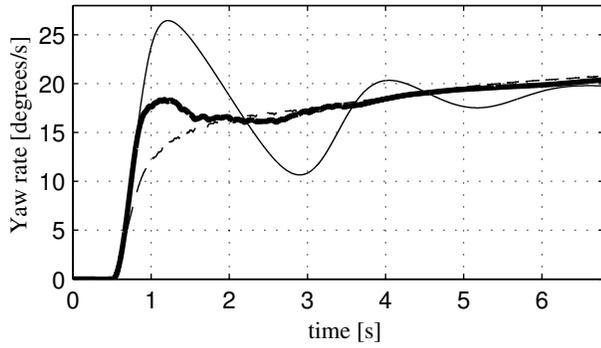


Fig. 4. Yaw rate response to a Step steer test (test M1) of 110 degrees: controlled vehicle(thick), uncontrolled vehicle (thin) and reference (dashed).

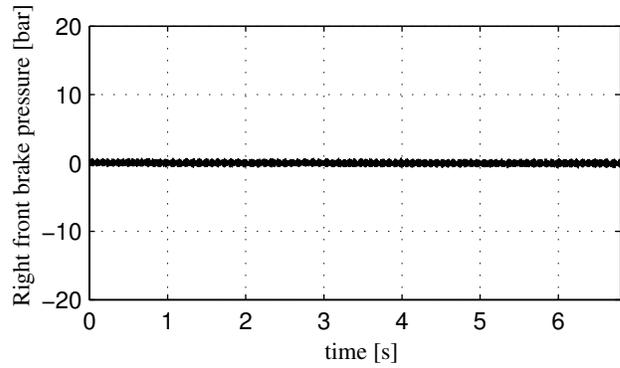


Fig. 8. Right front brake pressure.

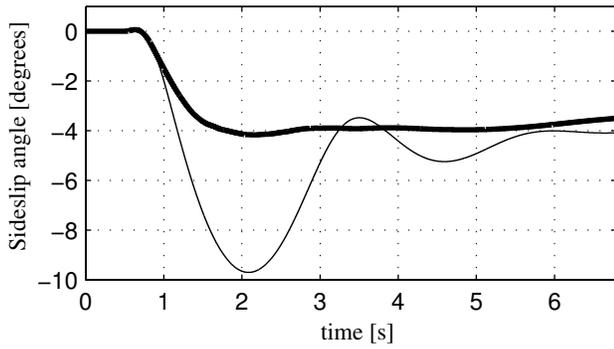


Fig. 5. Sideslip angle response to a Step steer test (test M1) of 110 degrees: controlled vehicle (thick), uncontrolled vehicle(thin).

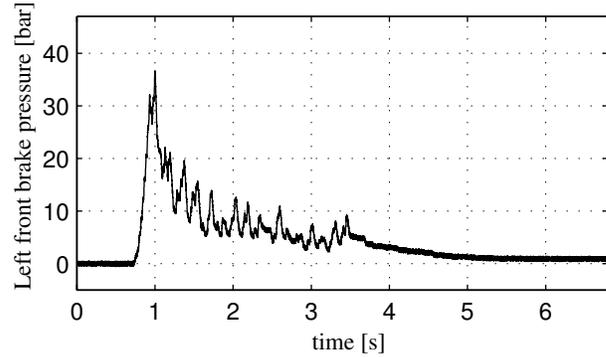


Fig. 9. Left front brake pressure.

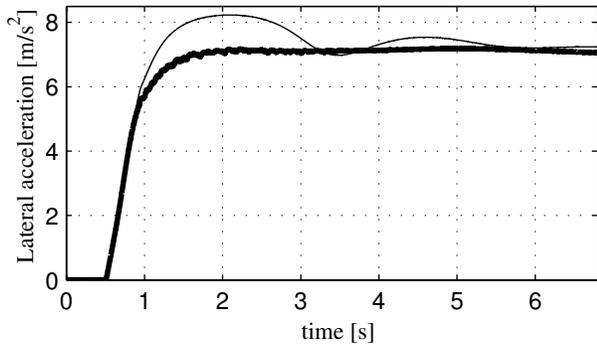


Fig. 6. Lateral acceleration response to a Step steer test (test M1) of 110 degrees: controlled vehicle (thick), uncontrolled vehicle (thin)

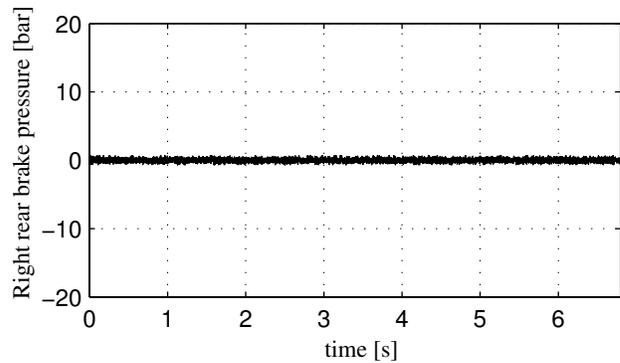


Fig. 10. Righth rear brake pressure.

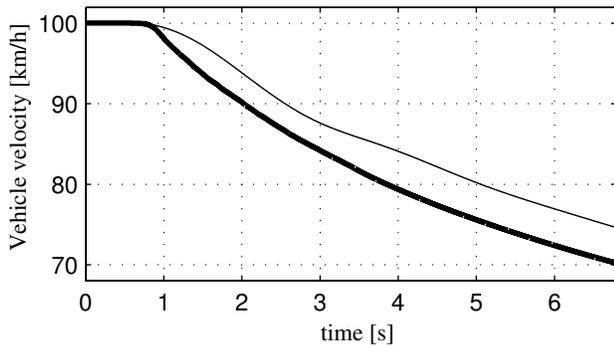


Fig. 7. Vehicle velocity response to a Step steer test (test M1) of 110 degrees: controlled vehicle (thick), uncontrolled vehicle (thin).

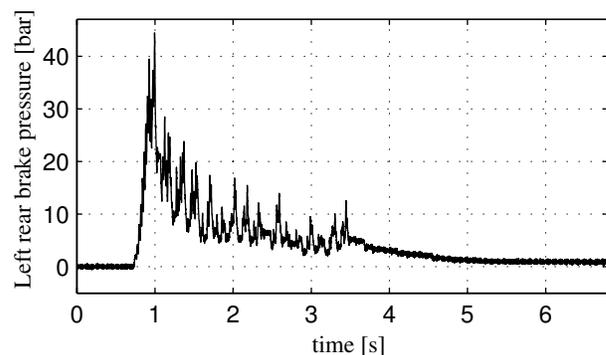


Fig. 11. Left rear brake pressure.