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Set-Membership identification of linear systems with input backlash

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Abstract— **In this paper we present a two-stage procedure for deriving parameters bounds of linear systems with input backlash when the output measurement errors are bounded. First, using steady-state input-output data, parameters of the nonlinear dynamic block are tightly bounded. Then, given a suitable PRBS input sequence we evaluate tight bounds on the unmeasurable inner signal which, together with noisy output measurements are employed for bounding the parameters of the linear dynamic system.**

Index Terms— **Backlash, bounded uncertainty, output errors, errors-in-variable, parameter bounding, linear programming.**

I. Introduction

Control systems components, such as sensors and actuators, often exhibits backlash which, indeed, is a typical characteristic of mechanical connections (see, e.g. [1]). Backlash can be classified as a hard (i.e. non-differentiable) and dynamic nonlinearity. It is well known that this kind of nonlinearity may often cause delays, oscillations and inaccuracy which severely limit the performance of control systems (see, e.g. [2]). To cope with these limitations, either robust or adaptive control techniques can be successfully employed (see, e.g., [3], and [4] respectively) which, on the other hand, require the characterization of the nonlinear dynamic block. Amazingly, there are only few contributions in the literature on the identification of systems with backlash nonlinearity ([5],[6]). Therefore, the identification of such systems is an open theoretical problem of major relevance to applications.

The configuration we are dealing with in this paper, shown in Fig. 1, closely resembles that of a Hammerstein model which in turn consists of a static nonlinear part $\mathcal{N}$ followed by a linear dynamic system. The identification of such a model relies solely on input-output measurements, while the inner signal x_t is not assumed to be available. Identification of the Hammerstein structure has attracted the attention of many authors, as can be seen in [7], [8]. It must be stressed that existing identification procedures mostly require that the nonlinearity be static and differantiable, usually a polynomial (see e.g., [9], [10], [11] and the references therein). On the side of linear systems with hard input nonlinearities, Bai [5] considers the case of nonlinearities parameterized by one parameter. The proposed algorithm, based on the idea of separable least squares, can be applied to several common static and nonstatic input nonlinearities.

In identification, a common assumption is that the measurement error η_t is statistically described. A worthwhile alternative to the stochastic description of measurement errors is the bounded-errors characterization, where uncertainties are assumed to belong to a given set. In the bounding context, all parameter vectors belonging to the *Feasible Parameter Set* (FPS), i.e. parameters consistent with the measurements, the error bounds and the assumed model structure, are feasible solutions of the identification problem. The interested reader can find further details on this approach in a number of survey papers (see, e.g., [12], [13])and in the special issues edited by Norton [14], [15].

In this paper we present a scheme for the identification of linear systems with input backlash. More precisely, we address the problem of bounding the parameters of a stable single-input single-output SISO discrete time linear system with unknown backlash at the input (see Fig. 1) when the output error is considered to be bounded. Note that the inner signal $x(t)$ is not supposed to be measurable. Results obtained in this paper can be straightly employed in the procedure proposed by [3] where they assume that bounds on the parameters of uncertain backlash are available in order to derive a sliding mode technique for the stabilization of an intrinsically nonlinear plant with an uncertain backlash in the actuator. To the author's best knowledge, no contribution can be found in the literature which address the above described identification problem, except for the authors' work [16]. The results presented here, significantly improve paper [16], namely: (a) a more general model of the backlash is considered; (b) evaluation of the backlash parameters bounds and inner signal bounds does not rely any more on graphical inspection of the 2-dimensional parameter space, instead a couple of optimization results are given which provide tight bounds both on parameters and unmeasurable signal; (c) the simulated example has been revised accordingly. The paper is organized as follow. Section II is devoted to the formulation of the problem. In Section III, parameters of the nonlinear block are tightly bounded using input-output data collected from the steady-state response of the system to a square wave input. Then, in Section IV, through a dynamic experiment, for all u_t belonging to a suitable Pseudo Random Binary Signal (PRBS) sequence $\{u_t\}$, we compute tight bounds on the inner signal which, together with noisy output measurements are used for bounding the parameters of the linear part. A simulated example is reported in Section V.

II. Problem formulation

Consider the SISO discrete-time linear system with input backlash depicted in Fig. 1, where the nonlinear block transforms the input signal u_t into the unmeasurable inner

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variable x_t according to the following map (see, e.g., [2])

$$x_t = \begin{cases} m_l(u_t + c_l) & \text{for } u_t \leq z_l \\ m_r(u_t - c_r) & \text{for } u_t \geq z_r \\ x_{t-1} & \text{for } z_l < u_t < z_r \end{cases} \quad (1)$$

where $m_l > 0$, $m_r > 0$, $c_l > 0$, $c_r > 0$ are constant parameters characterizing the backlash and

$$z_l = \frac{x_{t-1}}{m_l} - c_l, \qquad z_r = \frac{x_{t-1}}{m_r} + c_r \quad (2)$$

are the u-axis values of intersections of the two m-slope parallel lines with the horizontal inner segment containing x_{t-1}. The linear dynamic part is modeled by a discrete-time system which transforms x_t into the noise-free output w_t according to the linear difference equation

$$A(q^{-1})w_t = B(q^{-1})x_t, \quad (3)$$

where $A(\cdot)$ and $B(\cdot)$ are polynomials in the backward shift operator q^{-1}, $(q^{-1}w_t = w_{t-1})$,

$$A(q^{-1}) = 1 + a_1 q^{-1} + \ldots + a_{na} q^{-na}, \quad (4)$$

$$B(q^{-1}) = b_0 + b_1 q^{-1} + \ldots + b_{nb} q^{-nb}. \quad (5)$$

In line with the work done by a number of authors, in the contest of identification of block oriented systems, we assume that (i) the linear system is asymptotically stable (see, e.g., [17], [18], [19], [20], [21]); (ii) $\sum_{j=0}^{nb} b_j \neq 0$, that is, the steady-state gain is not zero (see, e.g. [19], [20], [21]); (iii) the only *a priori* information needed is an estimate of the process settling-time (see, e.g., [22]). Let y_t be the noise-corrupted output

$$y_t = w_t + \eta_t. \quad (6)$$

Measurements uncertainty is known to range within given bounds $\Delta\eta_t$, i.e.,

$$\mid \eta_t \mid \leq \Delta\eta_t. \quad (7)$$

Unknown parameter vectors $\gamma \in R^4$ and $\theta \in R^p$ are defined, respectively, as

$$\gamma^T = [\ \gamma_1 \ \ \gamma_2 \ \ \gamma_3 \ \ \gamma_4\ ] = [\ m_l \ \ c_l \ \ m_r \ \ c_r\ ], \quad (8)$$

$$\theta^T = [\ a_1 \ \ \ldots \ \ a_{na} \ \ b_0 \ \ b_1 \ \ \ldots b_{nb}\ ], \quad (9)$$

where $na + nb + 1 = p$. It is easy to show that the parameterization of the structure of Fig. 1 is not unique. As a matter of fact, any parameters set $\tilde{b}_j = \alpha^{-1} b_j, j = 1, 2, \ldots, nb$, and $\tilde{\gamma}_k = \alpha\gamma_k, k = 1, 2$, for some nonzero and finite constant α, provides the same input-output behaviour. To get a unique parameterization, in this work we assume, without loss of generality, that the steady-state gain of the linear part be one, that is

$$g = \frac{\sum_{j=0}^{nb} b_j}{1 + \sum_{i=1}^{na} a_i} = 1 \quad (10)$$

In this paper we address the problem of deriving bounds on parameters γ and θ consistently with given measurements, error bounds and the assumed model structure.

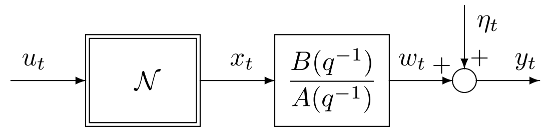

Fig. 1. Single-input single-output discrete-time linear system with input backlash $\mathcal{N}$.

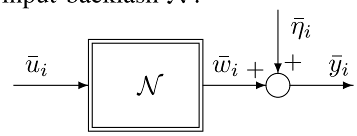

Fig. 2. Steady-state behaviour of the system under consideration when $g = 1$.

III. Assessment of tight bounds on the nonlinear static block parameters

Here we exploit steady-state operating conditions to bound the parameters of the backlash. The noisy output sequence is collected from the steady-state response of the system to a set of square wave inputs with different amplitudes. Due to the fact that the backlash deadzone is unknown (its evaluation is the main purpose of this section) we suggest to choose the input amplitude in such a way that the output shows any nonzero response. For each value of the input square wave amplitude, only one steady-state value of the noisy output is considered on the positive half-wave of the input and one steady-state value of the noisy output on the negative half-wave. Thus, given a set of square wave inputs with M different amplitudes, $2M$ steady-state values of the output are taken into account. We only assume to have a rough idea of the settling time of the system under consideration, in order to know when steady-state conditions are reached, so that steady-state data can be collected. Indeed, under conditions (i), (ii) and (iii) stated in Section II, combining equations (1), (3), (6) and (10) at steady-state, we get the following input-output description involving only the parameters of the backlash:

$$\bar{y}_i = m_r(\bar{u}_i - c_r) + \bar{\eta}_i, \quad \text{for } \bar{u}_i \geq \frac{\bar{x}_{i-1}}{m_r} + c_r \quad i = 1, \ldots, M; \quad (11)$$

$$\bar{y}_j = m_l(\bar{u}_j + c_l) + \bar{\eta}_j, \quad \text{for } \bar{u}_j \leq \frac{\bar{x}_{j-1}}{m_l} - c_l \quad j = 1, \ldots, M; \quad (12)$$

where the triplets $\{\bar{u}_i, \bar{y}_i, \bar{\eta}_i\}$ and $\{\bar{u}_j, \bar{y}_j, \bar{\eta}_j\}$ are collections of steady-state values of the known input signal, output observation and measurement error taken during the positive and the negative square wave respectively. A block diagram description of the steady-state response is depicted in Fig. 2 for equation (11) only; equation (12) leads to a similar block diagram representation. Since the pairs (m_l, c_l) and (m_r, c_r) affect the collected measurements, equations (11) and (12), separately, we can define the feasible parameter region of the backlash as

$$\mathcal{D}_\gamma = \mathcal{D}_\gamma^r \bigcup \mathcal{D}_\gamma^l \quad (13)$$

where

$$\mathcal{D}^r_\gamma = \{m_r, c_r \in R^+ : \bar{y}_i = m_r(\bar{u}_i - c_r) + \bar{\eta}_i, \quad | \bar{\eta}_i | \leq \Delta\bar{\eta}_i; \ \ i = 1, \ldots, M\} \quad (14)$$

$$\mathcal{D}^l_\gamma = \{m_l, c_l \in R^+ : \bar{y}_j = m_l(\bar{u}_j + c_l) + \bar{\eta}_j, \quad | \bar{\eta}_j | \leq \Delta\bar{\eta}_j; \ \ j = 1, \ldots, M\} \quad (15)$$

where $\{\Delta\bar{\eta}_i\}$ and $\{\Delta\bar{\eta}_j\}$ are the sequences of bounds on measurements uncertainty. From definition (13) it can be seen that $\mathcal{D}_\gamma$ is exactly described by the following constraints in the parameter space

$$\bar{y}_i - m_r(\bar{u}_i - c_r) \geq -\Delta\bar{\eta}_i, \quad \bar{y}_i - m_r(\bar{u}_i - c_r) \leq \Delta\bar{\eta}_i, \quad m_r > 0, c_r > 0, \qquad i = 1, \ldots, M \quad (16)$$

$$\bar{y}_j - m_l(\bar{u}_j + c_l) \geq -\Delta\bar{\eta}_j, \quad \bar{y}_j - m_l(\bar{u}_j + c_l) \leq \Delta\bar{\eta}_j, \quad m_l > 0, c_l > 0, \qquad j = 1, \ldots, M \quad (17)$$

Remark 1 — $\mathcal{D}^l_\gamma$ and $\mathcal{D}^r_\gamma$ are 2-dimensional sets lying on the $(m_l, c_l)-$plane and the $(m_r, c_r)-$plane respectively, i.e. they are disjoint sets, which means that they can be handled separately. We also note that they have the same mathematical structure, which means that they enjoy the same properties. Thus, from here on the results derived for one of the two sets, say $\mathcal{D}^r_\gamma$, will be also applicable to the other set ($\mathcal{D}^l_\gamma$). Throughout the paper it is assumed that $\mathcal{D}^r_\gamma$ ($\mathcal{D}^l_\gamma$) is a bounded set: to this end it suffices to collect at least two sets of measurements with different inputs u.

Below we present some possible descriptions of the feasible parameter set $\mathcal{D}^r_\gamma$. Introductory definitions and preliminary results are first given.

A. Definitions and preliminary results

Definition 1 — $h^+_r(\overline{u}_s)$ and $h^-_r(\overline{u}_s)$ are the constraints boundaries defining the FPS $\mathcal{D}^r_\gamma$ corresponding to the s-th sets of data:

$h^+_r(\overline{u}_s) \doteq \{m_r \in R^+, c_r \in R^+ : \overline{y}_s + \Delta\eta_s = m_r(\overline{u}_s - c_r)\}$

$h^-_r(\overline{u}_s) \doteq \{m_r \in R^+, c_r \in R^+ : \overline{y}_s - \Delta\eta_s = m_r(\overline{u}_s - c_r)\}$

Definition 2 — Boundary of $\mathcal{D}^r_\gamma \doteq \mathcal{H}(\mathcal{D}^r_\gamma)$

Definition 3 — The constraints boundaries $h^+_r(\overline{u}_s)$ and $h^-_r(\overline{u}_s)$ are said to be active if their intersections with $\mathcal{H}(\mathcal{D}^r_\gamma)$ is not the empty set:

$$h^+_r(\overline{u}_s) \bigcap \mathcal{H}(\mathcal{D}^r_\gamma) \neq \emptyset \Longleftrightarrow h^+_r(\overline{u}_s) \text{ is active.}$$

$$h^-_r(\overline{u}_s) \bigcap \mathcal{H}(\mathcal{D}^r_\gamma) \neq \emptyset \Longleftrightarrow h^-_r(\overline{u}_s) \text{ is active.}$$

Remark 2 — It is trivial to see that the constraints boundaries $h^+_r(\overline{u}_s)$ and $h^-_r(\overline{u}_s)$ may either (a) intersect $\mathcal{H}(\mathcal{D}^r_\gamma)$ or (b) be external to $\mathcal{H}(\mathcal{D}^r_\gamma)$, hence be external to $\mathcal{D}^r_\gamma$.

Definition 4 — Edges of $\mathcal{D}^r_\gamma$.

$$\tilde{h}^+_r(\overline{u}_s) \doteq h^+_r(\overline{u}_s) \bigcap \mathcal{D}^r_\gamma = \{\ m_r, c_r \in \mathcal{D}^r_\gamma : \overline{y}_s + \Delta\eta_s = m_r(\overline{u}_s - c_r)\}$$

$$\tilde{h}^-_r(\overline{u}_s) \doteq h^-_r(\overline{u}_s) \bigcap \mathcal{D}^r_\gamma = \{\ m_r, c_r \in \mathcal{D}^r_\gamma : \overline{y}_s - \Delta\eta_s = m_r(\overline{u}_s - c_r)\}$$

Definition 5 — Constraints intersections.
The set of all the pairs $(m_r, c_r) \in R^2$ where intersections among the constraints occur is defined as

$$\mathcal{I}^r_\gamma = \{(m_r, c_r) \in R^2 : \{h^+_r(\overline{u}_i), h^-_r(\overline{u}_i)\} \bigcap \{h^+_r(\overline{u}_j), h^-_r(\overline{u}_j)\} \neq \emptyset; i, j = 1, \ldots, M; i \neq j\} \quad (18)$$

Definition 6 — Vertices of $\mathcal{D}^r_\gamma$.
The set of all the vertices of $\mathcal{D}^r_\gamma$ is defined as the set of all the intersection couples belonging to the feasible parameter set $\mathcal{D}^r_\gamma$:

$$\mathcal{V}(\mathcal{D}^r_\gamma) = \mathcal{I}^r_\gamma \bigcap \mathcal{D}^r_\gamma \quad (19)$$

B. Exact description of $\mathcal{D}^r_\gamma$

An exact description of $\mathcal{D}^r_\gamma$ can be given in terms of edges, each one being described, from a practical point of view, as a subset of an active constraint lying between two vertices. An effective procedure for deriving active constraints, vertices and edges of $\mathcal{D}^r_\gamma$ is reported in [23].

C. Tight orthotope description of $\mathcal{D}^r_\gamma$

Edges provide exact description of $\mathcal{D}^r_\gamma$ which, on the downside, could be not so easy to handle. A somewhat more practical description, although approximate, can be obtained by the computation of the following orthotope outer-bounding set $\mathcal{B}^r_\gamma$ tightly containing $\mathcal{D}^r_\gamma$:

$$\mathcal{B}^r_\gamma = \{\gamma \in R^2 : \gamma_j = \gamma^c_j + \delta\gamma_j, | \delta\gamma_j | \leq \Delta\gamma_j, j = 1, 2\} \quad (20)$$

where

$${\gamma_j}^c = \frac{\gamma^{min}_j + \gamma^{max}_j}{2}, \qquad \Delta\gamma_j = \frac{| \gamma^{max}_j - \gamma^{min}_j |}{2} \quad (21)$$

$$\gamma^{min}_j = \min_{\gamma \in \mathcal{D}^r_\gamma} \gamma_j, \quad \gamma^{max}_j = \max_{\gamma \in \mathcal{D}^r_\gamma} \gamma_j. \quad (22)$$

Since constraints (16) defining $\mathcal{D}^r_\gamma$ are nonlinear in m_r and c_r, at least in principle the solution of the above optimization problems (22) requires the use of nonconvex optimization techniques which, however, do not guarantee the finding of the global optimal solution. Problems (22) can be solved thanks to the result reported below.

Proposition 1 — The global optimal solutions of problems (22) occur on the vertices of $\mathcal{D}^r_\gamma$.

Proof — First (i) we notice that each level curve of functionals (22) — parallel lines to m_r-axis and c_r-axis respectively — intersect the boundary of each constraint in (16) only once. Next, (ii) objective functions in (22) are monotone which implies that the optimal solution lies on the boundary of $\mathcal{D}^r_\gamma$. Thanks to (i) the optimal value cannot lie on one edge between two vertices: if that was true, it would mean that there is a suboptimal value where the functional intersect the edge twice: that would contradict (i). Then the global optimal solutions of problems (22) can only occur on the vertices of $\mathcal{D}_\gamma^r$.

Remark 3 — Given the set of vertices $\mathcal{V}(\mathcal{D}_\gamma^r)$ computed via the procedure reported in [23], the evaluation of (22) is an easy task since it only requires the simple calculation of (a) the objective functions on a set of points $\leq 4M$ and (b) the maximum over a set of real values.

IV. Bounding the parameters of the linear dynamic model

In the second stage of our procedure we evaluate bounds on the parameters of the linear dynamic block. In this stage, we excite the system to be identified with a pseudo random binary signal (PRBS) which takes the values $\pm u^\star$. We recall that, thanks to its nice properties, a PRBS input is successfully employed in linear system identification ([24], [25]). Although PRBS inputs are not suitable for nonlinear system identification in general ([5], [26]) since it may not adequately excite the unknown nonlinearity, in [27] it is shown that such a signal can be effectively used to decouple the linear and nonlinear parts in the identification of Hammerstein model with static nonlinearity. In this paper we show that the use of a PRBS sequence is profitable for the identification of linear system with input backlash. The key idea underlying our contribution is based on the following result:

Result 1 — Under a PRBS input whose levels are $\pm u^\star$, $u^\star > c_r$ and $-u^\star < -c_l$, the output of a backlash described by (1) is still a PRBS with levels $\bar{x}^\star = m_r(u^\star - c_r)$, $\underline{x}^\star = m_l(u^\star - c_l)$.
The proof of Result 1 is not reported since it is a trivial one.
Given the exact description of the feasible parameter set (FPS) $\mathcal{D}_\gamma^r$, tight bounds on the magnitude $\bar{x}^\star$ of the unmeasurable pseudo random inner signal x_t can be computed $\forall t$ through the following expressions

$$\begin{aligned}\bar{x}^{\star min} &= \min_{m_r, c_r \in \mathcal{D}_\gamma^r} m_r(u^\star - c_r), \quad \text{for } u^\star \geq c_r \\ \bar{x}^{\star max} &= \max_{m_r, c_r \in \mathcal{D}_\gamma^r} m_r(u^\star - c_r), \quad \text{for } u^\star \geq c_r\end{aligned} \tag{23}$$

Computation of bounds in equation (23) requires, at least in principle, the solution of 2 nonconvex optimization problems with 2 variables and $4M$ constraints. However, the computational efforts can be dramatically reduced thanks to the results reported below, where we exploit the following definition:

Definition 7 — x-level curve of the objective function to be optimized:

$$g_r(u^\star, x) \doteq \{m_r \in R^+, c_r \in R^+ : x = m_r(u^\star - c_r)\} \tag{24}$$

Proposition 2 — The global optimal solutions of problems (23) occur on the vertices of $\mathcal{D}_\gamma^r$.
Proof — The proof of Proposition 2 follows the same lines as the proof of Proposition 1. First (i) we notice that the each x-level curve $g_r(u^\star, x)$ of functional (24) intersect each constraint boundary in (16) only once. Next, (ii) the objective function (24) is a monotone function which implies that the optimal solution lies on the boundary of $\mathcal{D}_\gamma^r$. Thanks to (i) the optimal value cannot lie on an edge between two vertices: if that was true, it would mean that there is a suboptimal value where the functional intersect the edge twice: that would contradict (i). Then the global optimal solutions of problems (23) can only occur on the vertices of $\mathcal{D}_\gamma^r$.
■

Here, same comments reported in *Remark 3* apply.
Now, if we define the quantities

$$x_t^c = \frac{\bar{x}^{\star min} + \bar{x}^{\star max}}{2}, \qquad \Delta x_t = \frac{\bar{x}^{\star max} - \bar{x}^{\star min}}{2} \tag{25}$$

the following relation can be established between the unknown inner signal x_t and the central value x_t^c:

$$x_t^c = x_t + \delta x_t \tag{26}$$

$$|\ \delta x_t \ | \leq \Delta x_t. \tag{27}$$

We can now formulate the identification of the linear model in terms of the noisy output sequence $\{y_t\}$ and the uncertain inner sequence $\{x_t^c\}$ as shown in Fig. 3. Such a formulation is commonly referred to as an errors-in-variables problem (EIV), i.e. a parameter estimation problem in a linear-in-parameter model where the output and some or all the explanatory variables are uncertain. As a matter of fact,

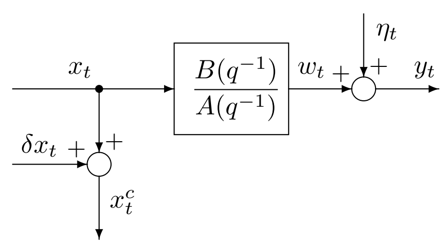

Fig. 3. Errors-in-variables set-up for bounding the parameters of the linear system.

combining equations (3), (4), (5), (6), (26) we get

$$y_t = -\sum_{i=1}^{na}(y_{t-i} - \eta_{t-i})a_i + \sum_{j=0}^{nb}(x_{t-j}^c - \delta x_{t-j})b_j + \eta_t. \tag{28}$$

The definition of the feasible parameter region for the linear system is:

$$\begin{aligned}\mathcal{D}_\theta = \{\theta \in R^p\colon & A(q^{-1})[y_t - \eta_t] = B(q^{-1})[x_t^c - \delta x_t]; \\ & g = 1; |\ \eta_t \ | \leq \Delta\eta_t; |\ \delta x_t \ | \leq \Delta x_t; t = 1, \ldots, N\}.\end{aligned} \tag{29}$$

where $g = 1$ takes account of condition (10) on the steady-state gain. From equation (28) it can be seen that consecutive regressions are related deterministically by uncertain output samples and uncertain input samples; that occurrence qualifies the problem as a dynamic EIV. It is referred to as a static EIV problem when the uncertain variables appearing in successive regressions are supposed to vary independently. The relations between successive regressions in the dynamic EIV case give rise to possibly nonlinear exact parameter bounds, which could be not easily and exactly computed [28]. On the other end, in the static EIV case exact parameter bounds are piecewise linear and, although generally non convex, the feasible parameter region is the union of at most 2^p convex sets: each being the intersection of the FPS with a single orthant of the p-dimensional parameter space (a detailed discussion on the geometrical and topological structure of the feasible parameter region for static EIV problems can be found in [29]). Thus, as shown in [28], the FPS of static EIV can be more conveniently handled than the FPS of dynamic EIV. That motivates the use, in this paper, of results from the static EIV [29]; since in model (28) the uncertain variables appearing in successive regressions are deterministically related, only outer approximations of the exact feasible parameter region will be obtained. Thus, in this work, a polytopic outer approximation $\mathcal{D}'_\theta$ of the exact FPS $\mathcal{D}_\theta$, i.e. $\mathcal{D}'_\theta \supset \mathcal{D}_\theta$, will be presented, together with an orthotope-outer bounding set $\mathcal{B}_\theta$ of $\mathcal{D}'_\theta$, which provides parameter uncertainties intervals. When we apply results from [29] to our problem we get the following description of the feasible parameter set $\mathcal{D}'_\theta$ at the single time t

$$(\phi_t - \Delta\phi_t)^{\mathrm{T}}\theta \leq y_t + \Delta\eta_t, \quad (\phi_t + \Delta\phi_t)^{\mathrm{T}}\theta \geq y_t - \Delta\eta_t \quad (30)$$

$$[1 \quad \dots \quad 1 \quad -1 \quad -1 \quad \dots \quad -1]\,\theta = -1 \quad (31)$$

where

$$\phi_t^{\mathrm{T}} = \left[-y_{t-1} \dots - y_{t-na} \; x_t^c \; x_{t-1}^c \dots x_{t-nb}^c\right] \quad (32)$$

$$\Delta\phi_t^{\mathrm{T}} = [\Delta\eta_{t-1}\mathrm{sgn}(a_1) \dots \; \Delta\eta_{t-na}\mathrm{sgn}(a_{na}) \;\; \Delta x_t \mathrm{sgn}(b_0) \\ \Delta x_{t-1}\mathrm{sgn}(b_1) \dots \; \Delta x_{t-nb}\mathrm{sgn}(b_{nb})] \quad (33)$$

Equation (31) takes account of condition (10) on the steady-state gain. The orthotope-outer bounding set $\mathcal{B}_\theta$ is defined as

$$\mathcal{B}_\theta = \{\theta \in R^p : \theta_j = \theta_j^c + \delta\theta_j, \mid \delta\theta_j \mid \leq \Delta\theta_j, j = 1, \dots, p\}, \quad (34)$$

where

$$\theta_j^c = \frac{\theta_j^{min} + \theta_j^{max}}{2}, \qquad \Delta\theta_j = \frac{\mid \theta_j^{max} - \theta_j^{min} \mid}{2} \quad (35)$$

$$\theta_j^{min} = \min_{\theta \in \mathcal{D}'_\theta} \theta_j, \quad \theta_j^{max} = \max_{\theta \in \mathcal{D}'_\theta} \theta_j. \quad (36)$$

Parameter vectors γ^c and θ^c are Chebishev centers in the ℓ_∞ norm of $\mathcal{D}_\gamma$ and $\mathcal{D}'_\theta$ respectively and are commonly referred to as central estimates.

V. A SIMULATED EXAMPLE

In this section we illustrate the proposed parameter bounding procedure through a numerical example. The system considered here is characterized by a linear block with $A(q^{-1}) = (1 + 0.5q^{-1} - 0.1q^{-2})$ and $B(q^{-1}) = (0.2q^{-1} + 1.2q^{-2})$ and a nonsymmetric backlash with $m_l = 0.24$, $m_r = 0.26$, $c_l = 0.035$, $c_r = 0.070$. Thus, the true parameter vectors are $\gamma = [m_l \; c_l \; m_r \; c_r]^{\mathrm{T}} = [0.24 \;\; 0.035 \;\; 0.26 \;\; 0.070]^{\mathrm{T}}$ and $\theta = [a_1 \; a_2 \; b_1 \; b_2]^{\mathrm{T}} = [0.5 \;\; -0.1 \;\; 0.2 \;\; 1.2]^{\mathrm{T}}$. We emphasize that the backlash parameters have been realistically chosen: as a matter of fact we considered the parameters of a real world precision gearbox which features a gear ratio equal to 0.25 and a deadzone as low as 0.0524 rad ($\approx 3^o$) and simulated a possible fictitious nonsymmetric backlash with gear ratio $m_l = 0.24$, $m_r = 0.26$ and deadzone $c_l = 0.035$ ($\approx 2^o$), $c_r = 0.070$ ($\approx 4^o$). Bounded absolute output errors have been considered when simulating the collection of both steady state data, $\{\bar{u}_s, \bar{y}_s\}$, and transient sequence $\{u_t, y_t\}$. Here we assumed $\mid \eta_t \mid \leq \Delta\eta_t$ and $\mid \bar{\eta}_s \mid \leq \Delta\bar{\eta}_s$ where η_t and $\bar{\eta}_s$, are random sequences belonging to the uniform distributions $U[-\Delta\eta_t, +\Delta\eta_t]$ and $U[-\Delta\bar{\eta}_s, +\Delta\bar{\eta}_s]$ respectively. Bounds on steady-state and transient output measurement errors were supposed to have the same value, i.e., $\Delta\eta_t = \Delta\bar{\eta}_s \doteq \Delta\eta$. Eight different values of $\Delta\eta$ were chosen in such a way as to simulate the measurement errors of eight commercial absolute binary encoder with a number of bits n_{bit} varying from 8 to 15. For a given $\Delta\eta$, the length of steady-state and the transient data are $M = 50$ and $N = [100, 1000]$ respectively. The steady-state input samples $\bar{u}_s$ are equally spaced values from 0.6 and 3, while the transient input sequence $\{u_t\}$ is a PRBS which takes the values ± 1. Results about the nonlinear and the linear block are reported in Figures 4, 5 and 6 respectively. For low noise level ($n_{bit} > 10$ bits) and for all N, the central estimates of both the nonlinear static block and the linear model are consistent with the true parameters. For higher noise levels ($n_{bit} \leq 10$ bits), both $\gamma^c = [m^c, c^c]$ and θ^c give satisfactory estimates of the true parameters. As the number of observations increases (from $N = 100$ to $N = 1000$), parameter uncertainty bounds $\Delta\theta_j$ decreases, as expected.

VI. CONCLUSION

A two-stage parameter bounding procedure for linear systems with input backlash in presence of bounded output errors has been outlined. First, using steady-state input-output data the two parameters of the backlash have been tightly bounded. Then, for a given input transient sequence we have computed bounds on the unmeasurable inner signal which, together with output noisy measurements have been used to overbound the parameters of the linear part. The numerical example showed the effectiveness of the proposed procedure.

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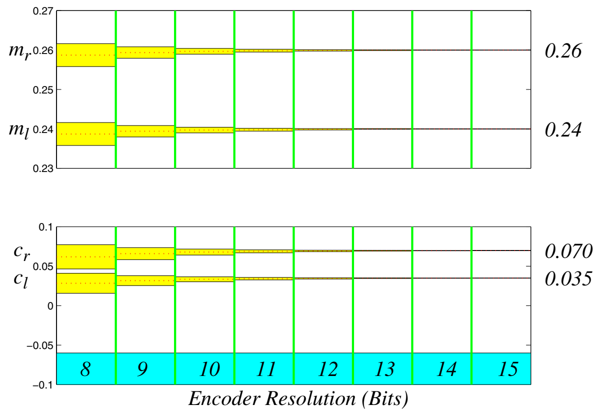

Fig. 4. Backlash parameter identification: Central estimates (red-dotted) and parameters uncertainty intervals (yellow-shaded) versus Encoder Resolution ($M = 50$).

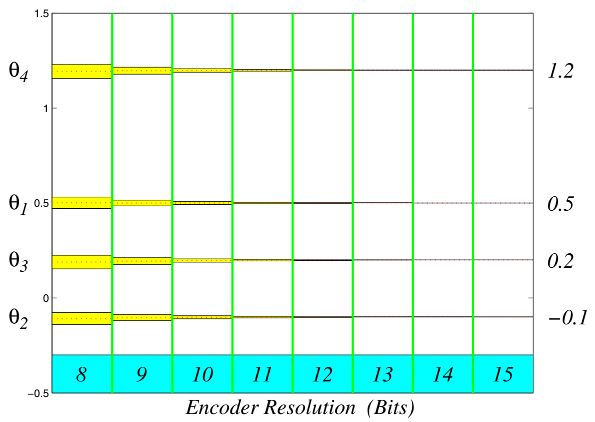

Fig. 5. Linear system parameter identification: Central estimates (red-dotted) and parameters uncertainty intervals (yellow-shaded) versus Encoder Resolution ($N = 100$).

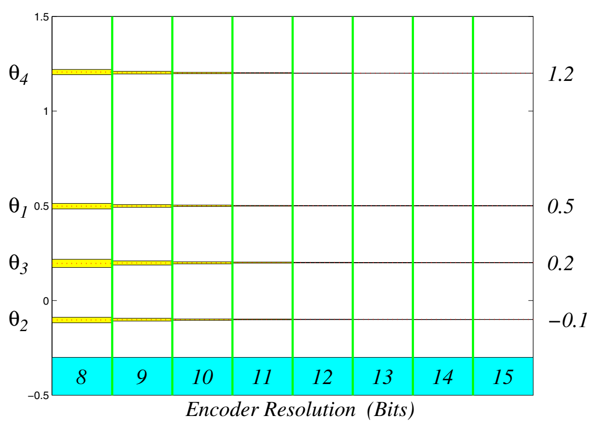

Fig. 6. Linear system parameter identification: Central estimates (red-dotted) and parameters uncertainty intervals (yellow-shaded) versus Encoder Resolution ($N = 1000$).

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