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Robust control from data via uncertainty model sets identification

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Abstract

In this paper, an integrated robust identification and control design procedure is proposed. It is supposed that the plant to be controlled is linear, time invariant, stable, possibly infinite dimensional and that input-output noisy measurements are available, together with some general information on the plant and on the noise characteristics. The emphasis is placed on the design of controllers guaranteeing robust stability and robust performances, and on the trade off between controller complexity and achievable robust performances. First, an uncertainty model is identified, consisting of a parametric model and a tight frequency bound on the magnitude of the modeling error, accounting for the dynamics not modeled by the parametric model. Second, an Internal Model Control, guaranteeing robust closed loop stability and best approximating the "perfect control" ideal target, is designed using H_∞ optimization techniques. This control structure is chosen because, if needed, it can be designed to be robust also in presence of input saturation. Then, the robust performances of the designed controller are computed, allowing to determine the level of model complexity needed to guarantee desired closed loop performances. A numerical example illustrates the effectiveness of the proposed design procedure.

1 Introduction

The typical problem a control designer has to face in most practical situations can be described as follows: given a physical plant, a control law has to be designed, able to drive the plant to reach, if possible, given performance specifications. The classical approach consists in building a mathematical model of the plant, on the basis of available information on it (physical laws, time invariance, linearity, etc.) and of input-output measurements, and then designing a control that meets the desired performance specifications for the identified model. However, this way it is not taken into account that any identified model is only an approximation of the actual plant. Indeed, the performances that can be actually achieved on the plant may be very poor, according to the size of the modeling error, and even the closed loop stability may be missed. These problems motivated the large interest devoted to robustness issues in the last decades. Robust control methodologies aim to design controllers guaranteeing to meet the specifications not for a single nominal model, but for all models obtained by given perturbations of the nominal model. However, the size of such perturbations has to account for the modeling error, which is not known and has to be estimated using available information and actual measurements on the plant. Moreover, the nominal model and the perturbation, indicated here as uncertainty model, have to be identified in a form suitable for the robust design, thus requiring strict interaction between identification and control design goal.

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In this paper, an integrated identification and control design procedure is proposed. It is supposed that the plant P^o to be controlled is linear, time invariant, stable, possibly infinite dimensional. Input-output noisy measurements in the time or frequency domain are available, together with some general information on the impulse response decay rate of the plant and on the characteristics of the noise, assumed to be pointwise (ℓ_∞) bounded and possibly with known deterministic-correlation properties. The emphasis is placed on the design of controllers guaranteeing robust stability and robust performances when used on the actual plant, and on the trade off between controller complexity and achievable robust performances.

The proposed design procedure is based on the following main steps.

First, an uncertainty model is identified, consisting of a model M and a frequency bound on the magnitude of the modeling error $\Delta = P^o - M$. The model is selected within a given class of parametric models $M(p)$, estimating the parameter vector $\hat{p}$ minimizing the H_∞ norm of the modeling error. The nonparametric part of the uncertainty model accounts for the unmodeled dynamics, by evaluating a tight frequency bound $W_\Delta(\omega)$ on their frequency response, assuring that, under the considered assumptions, the plant P^o is within the uncertainty model. In recent years, the uncertainty model identification problem has been widely studied, see e.g. the surveys [1]-[3] for an extensive list of references. However, most of the papers in literature use a nonparametric approach, leading to identified models of high order, greater or equal to the number of data. As a consequence, if the control is designed using H_∞ robust methods, the controller complexity is quite "high". More importantly, only rough bounds are usually derived on the magnitude of the identification error, whose conservativeness degree is unknown or quite high. Consequently, the guaranteed performances may result very poor and conservative. In order to overcome in a systematic way these drawbacks, here a mixed parametric and nonparametric approach is used, aimed to derive uncertainty models with low-order nominal models and tight bounds on the modeling errors, along the lines of [4]-[10].

Second, an Internal Model Control (IMC), guaranteeing robust closed loop stability and approximating frequency domain "perfect control" ideal target, is designed using H_∞ optimization techniques. Then, the robust performances of the designed controller, i.e. the performances that can be guaranteed for all systems belonging to the uncertainty model, are derived. The comparative evaluation of the robust performances of controllers designed on the basis of uncertainty models with parametric part of different complexity (order) allows also to determine the level of model complexity needed to guarantee the desired closed loop performances. This represents a systematic way of keeping low the controller complexity, since it depends on the model complexity. The derived controller can be further reduced by means of standard approximation techniques until the corresponding robust performances are considered acceptable.

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2 Notation

A glossary of main symbols and notations used or introduced throughout the paper is here reported.

$\mathbf{Z}_m^n$	Set of integers ℓ such that $m \le \ell \le n$;
$\mathbf{R}^{m\times n}$	Set of real-valued $m \times n$ matrices;
$(\,\cdot\,)^T$	Transpose of a matrix;
$y^N,\ e^N$	Column vectors whose dimensions depend on the number N of experimental data;
P^o	Unknown plant to be controlled;
$\{h^{P^o}\}$	One-sided sequence $\{h_0^{P^o}, h_1^{P^o}, \cdots\}$ made by the impulse response of P^o;
$P^o(z)$	z-transform of h^{P^o} defined as $\sum_{k=0}^{\infty} h_k^{P^o} z^{-k}$;
$\|P^o(z)\|_\infty$	H_∞ norm of P^o defined as $\sup_{0\le\omega\le 2\pi} \left\|P^o(e^{j\omega})\right\|$;
p	Parameter vector $\in \mathbf{R}^q$ of the model $M_n(p)$.

3 From data to robust design

Consider a causal, linear, time-invariant, BIBO-stable, SISO-process P^o, unknown except for some noisy measurements, either in the time domain or in the frequency domain, and for some general prior information on plant memory and measurement noise.

The model based procedure for designing a robust control consists of the following three main steps:

1. *Uncertainty model set identification.*
   Evaluate, from the available prior information and measured data on the plant P^o:
   - a low-order parametric model $M_n(\hat{p})$ with transfer function of given order n depending on a parameter vector $p \in \mathbf{R}^q$;
   - a tight frequency domain bound $W_\Delta(\omega)$ on the transfer function magnitude of the modelling error $\Delta = P^o - M_n(\hat{p})$ guaranteeing that:
   $$\begin{aligned} P^o \in\ & \mathcal{M}(M_n(\hat{p}), W_\Delta) = \\ =\ & \{M_n(\hat{p}) + \Delta : |\Delta(\omega)| \le W_\Delta(\omega),\ \forall\omega\} \end{aligned} \tag{1}$$
   In the sequel, the subscript n in M_n is omitted when unnecessary.
2. *Robust IMC design.*
   Consider an Internal Model Control (IMC) structure and compute the parameter function $\hat{Q}_n$ taking into account the "perfect control" ideal target $1 - QM = 0$ and the robust stability constraint $\|Q\Delta\|_\infty < 1$. Reformulating the problem in terms of H_∞ optimization, a solution is obtained solving the following problem:
   $$\begin{aligned} &\min_{Q\in\mathcal{H}_\infty} \|1 - QM(\hat{p})\|_\infty \\ &\text{s.t. } \|Q\Delta(\omega)\|_\infty < 1 \end{aligned} \tag{2}$$
3. *Guaranteed closed loop performances computation.*
   Calculate the guaranteed closed loop performances, that is to say compute the frequency bounds for the considered performances:
   $$\begin{aligned} \underline{H_i}(\omega,\hat{Q},\mathcal{M}) \le |\tilde{H}_i(j\omega,\hat{Q},P)| \le \overline{H_i}(\omega,\hat{Q},\mathcal{M}), \\ \forall P \in \mathcal{M}(M(\hat{p}), W_\Delta) \end{aligned} \tag{3}$$
   where
   $$\overline{H_i}(\omega,\hat{Q},\mathcal{M}) = \sup_{P\in\mathcal{M}(M(\hat{p}),W_\Delta)} |\tilde{H}_i(j\omega,\hat{Q},P)|$$
   $$\underline{H_i}(\omega,\hat{Q},\mathcal{M}) = \inf_{P\in\mathcal{M}(M(\hat{p}),W_\Delta)} |\tilde{H}_i(j\omega,\hat{Q},P)|$$
   being $\tilde{H}_i(\cdot)$ the closed loop transfer functions of interest, like sensitivity, complementary sensitivity, etc. ■

In the next sections, the way the previous steps can be worked out is illustrated in some details.

Note that it could be possible to design robust control to directly minimize worst-case optimization criteria (using e.g. μ-synthesis techniques instead of the H_∞ ones), but such an approach usually leads to quite high order controllers and to overly conservative performances of the actual plant. In the proposed procedure, the control design is performed in order to guarantee robust stability, but the optimization criterion is minimized for the nominal model.

4 Uncertainty model set identification

In this section, it is shown how to perform the uncertainty model set identification using sampled data measurements of the plant. A discrete-time model is looked for to approximate the discrete-time system P^o, consisting of the plant and the input and output sampling devices.

Let $\mathcal{P}$ be the Banach space of causal, single-input single-output, linear time-invariant, discrete-time, BIBO-stable dynamical processes. Suppose that a plant $P^o \in \mathcal{P}$, possibly infinite-dimensional, has to be identified on the basis of noisy measurements and prior information on the system and the noise.

The measurements can be in the time, frequency or mixed time and frequency domain. Such an experimental information consists of a finite number N of samples and can be expressed in the form:

$$y^N = F_N h^{P^o} + e^N$$

where $y^N = [y_0 \ldots y_{N-1}]^T \in \Re^N$ is a known vector depending on the actual measurements, $F_N \in \Re^{N\times\infty}$ is a known matrix indicating how the measurements depend on P^o, $h^{P^o} = [h_0^{P^o}\ h_1^{P^o} \ \ldots]^T \in \Re^\infty$ is an unknown vector containing the impulse response $\{h^{P^o}\}$ of P^o, and $e^N \in \Re^N$ is an unknown vector representing the measurement noise. Explicit expressions of matrix F_N for time and/or frequency domain experiments can be found in [11, 12].

The prior information on plant P^o is a bound on its impulse response decay rate:

$$P^o \in K = \{P \in \mathcal{P} : \left|h_\ell^P\right| \le L\rho^\ell,\ \ \forall \ell \in \mathbf{Z}_0^\infty\}$$

where $L > 0$ and $0 < \rho < 1$ are known constants.

The prior information on noise e^N is given as:

$$e^N \in \mathcal{B}_e = \{\tilde{e}^N = [\tilde{e}_0 \ldots \tilde{e}_{N-1}]^T \in \Re^N : \|W_e^{-1} A\tilde{e}^N\|_\infty \le 1\}$$

where $A \in \Re^{l\times N}$ is a known matrix with $l \ge N$, $W_e = \operatorname{diag}(w_{e,1}, \ldots, w_{e,l}) \in \Re^{l\times l}$ is a known weighting matrix with $w_{e,k} > 0\ \forall k$, and $\|W_e^{-1} A\tilde{e}^N\|_\infty = \max_{1\le k\le l} w_{e,k}^{-1} \left|\left(A\tilde{e}^N\right)_k\right|$. This assumption can accommodate not only for information on maximal noise magnitude, i.e. $\|W_e^{-1}\tilde{e}^N\|_\infty = \max_{1\le k\le N} w_{e,k}^{-1} |\tilde{e}_k| \le 1$, as typically done in most of the literature, but can also account for possible information on deterministic cross-covariance or autocorrelation properties of the noise, see e.g. [13, 14, 15].

In the overall identification procedure, a key role is played by the Feasible Systems Set, often called "unfalsified systems set", i.e. the set of all processes consistent with prior information and measured data.

Definition 1. *Feasible Systems Set*

$$FSS = \{P \in K : \|W_e^{-1} A \cdot (y^N - F_N h^P)\|_\infty \le 1\}$$ ■

If prior assumptions on plant $P^o \in K$ and on noise $e^N \in \mathcal{B}_e$ are "true", this set includes P^o, an important property in view of subsequent use for robust control design. As required in any identification theory, the problem of checking the validity of priors arises. Indeed, the only thing that can be actually done is to check if measured data falsify the priors. This is equivalent to check if FSS is empty, being the FSS the set of unfalsified systems. Necessary and sufficient conditions for such a check are given in [16]. The FSS can be considered an uncertainty model set for P^o and, in the line with the robustness paradigm, control should be designed to be robust versus such an uncertainty model set. However, the FSS is not represented in a suitable form to be used by robust control design techniques, and model sets with such a property have to be looked for. Moreover, to be consistent with robust control design philosophy, uncertainty model sets including the set of unfalsified systems have to be looked for. This is formalized by the following definition.

Definition 2. *Model set for P^o*
A set of models $\mathcal{M} \subseteq \mathcal{P}$ is called a *model set for P^o* if:

$$\mathcal{M} \supseteq FSS \qquad \blacksquare$$

In this paper, additive frequency shaped model sets are considered, of the form:

$$\mathcal{M}(M(\hat{p}),W_\Delta) = \{M(\hat{p})+\Delta : |\Delta(\omega)| \le W_\Delta(\omega), \forall\omega \in [0,2\pi]\} \quad (4)$$

where, for given $M(\hat{p}) \in \mathcal{P}$, $W_\Delta(\omega)$ has to be chosen such that $\mathcal{M}(M(\hat{p}), W_\Delta)$ is a model set for P^o, i.e. the following condition has to be guaranteed $\forall\omega \in [0, 2\pi]$:

$$W_\Delta(\omega) \ge \sup_{P\in FSS} |P(\omega) - M(\omega,\hat{p})| = W_\Delta^*(\omega, M(\hat{p}))$$

For given frequency $\omega \in [0, 2\pi]$, the exact computation of $W_\Delta^*(\omega, M(\hat{p}))$ is not easy, but convergent upper and lower bounds are provided by the method presented in [16] and briefly summarized in the following.

For given plant $P \in \mathcal{P}$ with impulse response $\{h^P\}$, let P^ν be the FIR_ν system having the same first ν impulse response samples of P, i.e. $\{h^{P^\nu}\} = \{h_0^P, h_1^P, \ldots, h_{\nu-1}^P, 0, 0, \ldots\}$. For given $\nu \in \mathbf{Z}_1^\infty$ and $m \in \mathbf{Z}_3^\infty$, compute the points $\overline{t_k}(\omega) \in \Re^2$ and $\underline{t_k}(\omega) \in \Re^2$, $k \in \mathbf{Z}_1^m$, by solving the following linear programming problems:

$$\overline{t_k}(\omega) = \Omega(\omega)\cdot \underset{P^\nu\in\overline{FSS}^\nu}{\operatorname{argmin}} \left[-\Omega_1(\omega)\cdot\sin(s_k)+\Omega_2(\omega)\cdot\cos(s_k)\right]\cdot h^{P^\nu}$$

$$\underline{t_k}(\omega) = \Omega(\omega)\cdot \underset{P^\nu\in\underline{FSS}^\nu}{\operatorname{argmin}} \left[-\Omega_1(\omega)\cdot\sin(s_k)+\Omega_2(\omega)\cdot\cos(s_k)\right]\cdot h^{P^\nu}$$

where:

$$\begin{aligned}
s_k &= 2\pi k/m \\
\Omega(\omega) &= \begin{bmatrix} \Omega_1(\omega) \\ \Omega_2(\omega) \end{bmatrix} = \begin{bmatrix} \Re e(\Psi(\omega)) \\ \Im m(\Psi(\omega)) \end{bmatrix} \in \Re^{2\times\infty} \qquad (5)\\
\Psi(\omega) &= \begin{bmatrix} 1 & e^{-j\omega} & e^{-j2\omega} & \cdots \end{bmatrix} \in C^{1\times\infty} \\
\overline{FSS}^\nu &= \{P^\nu \in K : \|(W_e+\widetilde{W})^{-1}\cdot A\cdot(y^N - F_N h^{P^\nu})\|_\infty \le 1\} \\
\underline{FSS}^\nu &= \{P^\nu \in K : \|W_e^{-1} A\cdot(y^N - F_N h^{P^\nu})\|_\infty \le 1\} \\
\widetilde{W} &= \varepsilon_\nu \operatorname{diag}(\|a_1\|_1, \ldots, \|a_l\|_1) \in \Re^{l\times l},\ a_\ell : \ell\text{-th row of } A \\
\varepsilon_\nu &= L\rho^\nu/(1-\rho)
\end{aligned}$$

and compute also the intersection points $\overline{v_k}(\omega) \in \Re^2$ of lines l_k and $l_{\mathrm{mod}(k,m)+1}$, $k \in \mathbf{Z}_1^m$, being l_k the line in $\Re^2$ with slope s_k passing through $\overline{t_k}(\omega)$. Convex hulls $VI_m^\nu(\omega)$ and $VO_m^\nu(\omega)$ of points $\underline{t_k}(\omega)$ and $\overline{v_k}(\omega)$, $k \in \mathbf{Z}_1^m$, provide convergent inner and outer approximations respectively of the value set $V(\omega)$, i.e. the set of transfer function values at frequency ω of all feasible systems in FSS, [16].

For given model $M(\hat{p}) \in \mathcal{P}$, $\omega \in [0, 2\pi]$, $\nu \in \mathbf{Z}_1^\infty$ and $m \in \mathbf{Z}_3^\infty$, $W_\Delta^*(\omega, M(\hat{p}))$ is then bounded as:

$$\underline{W}_m^\nu(\omega) \le W_\Delta^*(\omega, M(\hat{p})) \le \overline{W}_m^\nu(\omega)$$

where:

$$\begin{aligned}
\underline{W}_m^\nu(\omega) &= \max_{k=1,\ldots,m} \|M(\omega,\hat{p}) - \underline{t_k}(\omega)\|_2 \\
\overline{W}_m^\nu(\omega) &= \max_{k=1,\ldots,m} \|M(\omega,\hat{p}) - \overline{v_k}(\omega)\|_2 + \varepsilon_\nu
\end{aligned}$$

with the convergence property:

$$\lim_{\nu,m\to\infty} \overline{W}_m^\nu(\omega) = \lim_{\nu,m\to\infty} \underline{W}_m^\nu(\omega) = W_\Delta^*(\omega, M(\hat{p}))$$

Obviously, $W_\Delta^*(\omega, M(\hat{p}))$ can be actually evaluated only on a finite set of frequencies. If $P \in K$, the variation rate of $|\Delta(\omega)|$ is bounded by $\max_{0\le\omega\le2\pi} \frac{d}{d\omega}|\Delta(\omega)| \le \frac{L\rho}{(1-\rho)^2}$, see e.g. [17]. This information can be used to choose how coarse the gridding has to be for the intersample variation to be negligible. The error between P^o and a given nominal model $M(\hat{p})$ is measured by a weighted H_∞ norm:

$$\|P^o - M(\hat{p})\|_\infty^{W_C} = \sup_{0\le\omega\le2\pi} \left|W_C^{-1}(\omega)\cdot[P^o(\omega) - M(\omega,\hat{p})]\right|$$

where $W_C(\omega)$ is a known positive weighting function, suitably selected according to the intended use of the identified model. For example, if identification is aimed to derive an uncertainty model set for H_∞ robust control design, $W_C(\omega)$ can be computed so that the identified model optimally approximates the closed loop performance underlying the control design, see e.g. [18]. Since P^o is only known to belong to FSS, the following identification error may be defined.

Definition 3. *Model set identification error*

$$E(\mathcal{M}(M(\hat{p}), W_\Delta^*)) = \sup_{P\in FSS} \|P - M(\hat{p})\|_\infty^{W_C} \qquad \blacksquare$$

This error gives a measure of the model uncertainty, being the radius of the minimal ball in the $\|\cdot\|_\infty^{W_C}$ norm with center $M(\hat{p})$ and containing the set of unfalsified systems FSS. Exact computation of $E(\mathcal{M}(M(\hat{p}), W_\Delta^*))$ is hard, but the following convergent upper and lower bounds are provided by the procedure described above:

$$\underline{E}_m^\nu(\mathcal{M}(M(\hat{p}),W_\Delta^*)) \le E(\mathcal{M}(M(\hat{p}),W_\Delta^*)) \le \overline{E}_m^\nu(\mathcal{M}(M(\hat{p}),W_\Delta^*))$$

where:

$$\begin{aligned}
\underline{E}_m^\nu(\mathcal{M}(M(\hat{p}), W_\Delta^*)) &= \sup_{0\le\omega\le2\pi} \underline{W}_m^\nu(\omega)\, W_C^{-1}(\omega) \\
\overline{E}_m^\nu(\mathcal{M}(M(\hat{p}), W_\Delta^*)) &= \sup_{0\le\omega\le2\pi} \overline{W}_m^\nu(\omega)\, W_C^{-1}(\omega)
\end{aligned} \qquad (6)$$

The model set uncertainty can be minimized by appropriately choosing the nominal model $M(\hat{p})$. The best model set corresponds to $M(p^*)$, where

$$p^* = \arg\min_p E(\mathcal{M}(M(p), W_\Delta^*)) \qquad (7)$$

can be found by means of iterative nonlinear optimization algorithms. The model $M(p^*)$, where p^* is solution of (7), is the so called "conditional center" of FSS, [19]. Finding conditional centers is hard, see e.g. [19, 20] for some results in the case that the class of models $M(p)$ is linear in the parameters, e.g. FIR, Laguerre, Kautz or orthonormal basis functions. These models are essentially with fixed poles and the problem arises of choosing "good" basis, i.e. "good" model poles, since this choice affects, for given model order, the identification accuracy. On the other hand, models nonlinear in the parameters, able to arrange also their poles, may improve the identification accuracy, but make problem (7) non-convex, and trapping in local minima may take place. In such a case, the choice of a good starting point to be used by iterative nonlinear optimization algorithms for solving (7) is very important. The following procedure has been proposed in [16]:

- compute the "nearly optimal" FIR_ν model $M_\nu^{no} \in FSS$, whose impulse response is obtained as solution of the following linear programming problem:

$$h^{M_\nu^{no}} = \arg\min_{P^\nu\in FSS} \|s^* - \Omega^*\cdot h^{P^\nu}\|_\infty^{W_C} \qquad (8)$$

where:

$$s^* = \begin{bmatrix} s^*(\omega_1) \\ \vdots \\ s^*(\omega_q) \end{bmatrix} \in \Re^{4q},\quad s^*(\omega) = \begin{bmatrix} \max_{s=\left[\begin{smallmatrix}s_1\\s_2\end{smallmatrix}\right]\in VO_m^\nu(\omega)} s_1 \\ \max_{s=\left[\begin{smallmatrix}s_1\\s_2\end{smallmatrix}\right]\in VO_m^\nu(\omega)} s_2 \\ \min_{s=\left[\begin{smallmatrix}s_1\\s_2\end{smallmatrix}\right]\in VO_m^\nu(\omega)} s_1 \\ \min_{s=\left[\begin{smallmatrix}s_1\\s_2\end{smallmatrix}\right]\in VO_m^\nu(\omega)} s_2 \end{bmatrix} \in \Re^4$$

$$\Omega^* = \begin{bmatrix} \Omega^*(\omega_1) \\ \vdots \\ \Omega^*(\omega_q) \end{bmatrix} \in \Re^{4q \times \nu}, \ \Omega^*(\omega) = \begin{bmatrix} \Omega^\nu(\omega) \\ \Omega^\nu(\omega) \end{bmatrix} \in \Re^{4 \times \nu}$$

with $q \in \mathbf{Z}_1^\infty$ and $\Omega^\nu(\omega)$ given by the first ν columns of the matrix $\Omega(\omega)$ defined in (5);

- compute reduced order models $M_n^r(\hat{p})$ of M_ν^{no} by means of Hankel norm approximation.

The interest of this procedure is due to the fact that M_n^r is a "nearly central" projection model in the Hankel norm, [16]. Indeed, central projection models are known to provide good approximations of conditionally central models, see e.g. [20]. Thus, recalling the well-known relationship between Hankel and H_∞ norms, M_n^r results to be a good approximation of the conditional center.

The above method allows to derive "hard" uncertainty models, making use of the knowledge of deterministic constraints on measurement noise. If probabilistic information on the noise is available, "soft" uncertainty models, guaranteeing the inclusion property (1) with assigned probability, can be estimated using the method of [21].

5 Robust IMC design and guaranteed closed loop performances

The robust Internal Model Control (IMC) design is carried out in the continuous-time setting, using the continuous-time uncertainty model derived by applying the bilinear transformation to the discrete-time uncertainty model identified with the method presented in the previous section.

IMC methods have been investigated in past years for designing robust control in the face of unmodeled dynamics, [22, 23, 24]. It is well know [22] that the stable transfer function Q in the IMC structure depicted in figure 1 parameterizes, in the case of a stable plant, all the stabilizing controllers C (figure 2): $C = Q \cdot (1 - QM)^{-1}$. The design of the parameter Q is then equivalent to the design of the controller C, but some remarkable advantages, such as stability preservation, are obtained by using the IMC implementation of figure 1 in presence of saturating actuators, see [22, 25].

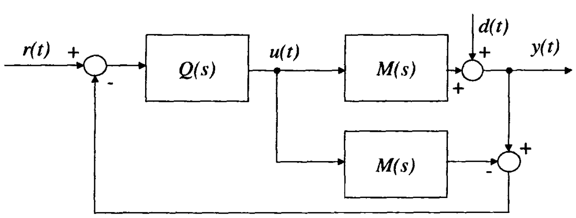

Figure 1: IMC structure

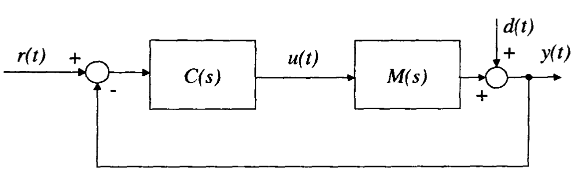

Figure 2: Classical feedback structure

Now consider the IMC structure of figure 3, where model uncertainty Δ is explicitly taken into account, and define as usual the sensitivity and the complementary sensitivity functions as: $S(s) = Y(s)/D(s)$, $T(s) = Y(s)/R(s)$. It is straightforward to compute that, if

$$1 - QM = 0 \qquad (9)$$

and the closed loop is robustly stable in the face of model uncertainty $\Delta(s)$, then $S(s) = 0$ and $T(s) = 1$ ("perfect

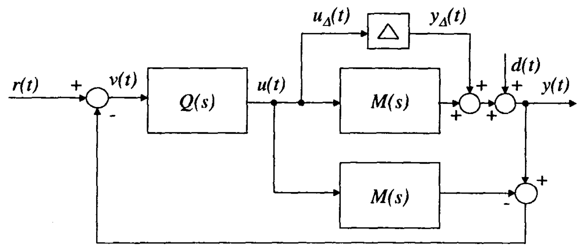

Figure 3: IMC structure with unmodeled dynamics

control") for all stable $\Delta(s)$. However, it is also usually required that Q is proper and stable and these requirements cannot be met by the choice $Q = M^{-1}$ if M is strictly proper or nonminimum-phase.

In standard IMC approach (see [22, 23, 24]), the parameter Q is obtained as $Q = \tilde{Q}F$ and a two stage design procedure is adopted. First, a nominal design (i.e. $\Delta(s) = 0$) is performed computing $\tilde{Q}$ by minimizing $\|(1 - M\tilde{Q})v\|_2$ for some canonical signal $v = r - d$ (typically step signal). Then, the filter F is chosen as a rational function such that Q is proper and nominal closed loop is internally stable. The filter F is designed depending on a parameter λ, whose value is related to the nominal closed loop bandwidth. Then, on the basis of model uncertainty, the parameter λ is chosen to give the maximal bandwidth compatible with stability and performance of the actual ($\Delta(s) \neq 0$) closed loop, looking at the quality of the actual response to the considered canonical reference signals.

In the approach proposed in this paper, the IMC design problem is reformulated in terms of H_∞-optimization. Considering the "perfect control" ideal target (9), a cost function is defined in terms of the H_∞ norm of $(1 - QM_n(\hat{p}))$. Moreover the robust stability constraint imposed by the Small Gain Theorem requires $\|Q\Delta\|_\infty < 1$, being $-Q$ the transfer function from y_Δ to u_Δ (see figure 3), for which a sufficient condition is $\|QW_\Delta^*(\omega)\|_\infty < 1$. Then Q is obtained as solution of the following constrained optimization problem:

$$\hat{Q}_n = \arg\min_{Q \in \mathcal{H}_\infty} \|[1 - QM_n(\hat{p})]W\|_\infty \quad \text{s.t. } \|QW_\Delta^*(\omega)\|_\infty < 1$$

where W is a performance weighting function defined on the basis of spectral features of signals $r(t)$ and $d(t)$.

The solution of this problem can be computed using standard H_∞-optimization algorithms (see for example [26]), but it is required to have model sets of the form:

$$\mathcal{M}(M(\hat{p}), W_r) = \{M(\hat{p}) + \Delta : |\Delta(\omega)| \leq W_r(\omega), \forall \omega \in [0, 2\pi]\}$$

where $W_r(\omega)$ is the magnitude of a stable rational function, possibly of low order, since this order affects the complexity of the designed controller. Thus, $W_r(\omega)$ has to be computed, tightly overbounding $W_\Delta^*(\omega)$ for a suitable set of frequency values. A systematic approach for such overbounding can be found in [27].

The H_∞-optimization algorithm also requires the approximation of the performance weighting function W (not necessarily rational) with a rational function $W_s(\omega)$.

Remark: The robust IMC design approach proposed above can be applied also in presence of saturating actuators. In order to guarantee the robust stability, the constraint $\|QW_r(\omega)\|_\infty < 1$ is replaced by $\|Q\|_\infty \cdot \|W_r(\omega)\|_\infty < 1$, as shown in [25]. ■

Once completed the design procedure, the uncertainty model can be used to calculate the guaranteed closed loop performances. Let $\hat{H}_i(\cdot)$ be the closed loop transfer functions of interest, like sensitivity, complementary sensitivity, etc. The frequency bounds on the guaranteed performance are then computed according to (3) for given $\omega \in \Re^+$, $\hat{Q} \in \mathcal{H}_\infty$ and $\mathcal{M}\left(M(\hat{p}), W_\Delta\right) \subseteq \mathcal{P}$. Note that for some functions (e.g. sensitivity) such bounds can be computed exactly, while for other functions (e.g. complementary sensitivity) only approximated (though tight) values can be obtained, see e.g. [28].

The frequency behavior of the gap width $\overline{H_i}(\omega,\hat{Q},\mathcal{M}) - \underline{H_i}(\omega,\hat{Q},\mathcal{M})$ gives interesting information on how sensitive the performance is when all plants belonging to the uncertainty model are considered.

It has to be noted that this *a posteriori* robustness analysis is independent of the employed controller-design procedure and can be applied to any given stabilizing controller.

6 Example

Robust control from data of the following nonminimum-phase continuous-time system is considered:

$$P(s) = \frac{-0.0609s^4 - 0.4871s^3 - 0.4871s^2 + 1.9482s + 2.9224}{s^4 + 0.4311s^3 + 2.6764s^2 + 0.4384s + 0.1412}$$

The available experimental information is made up by $N = 1100$ samples of the system output to a PRBS input of unitary amplitude, using $T_s = 1s$ as sampling time. These samples are corrupted by a pointwise bounded noise e^N with $|e_i| \leq 4$, $i = 0, 1, \ldots, 1099$. This data set is used to identify a discrete-time uncertainty model guaranteeing to contain the discrete-time system P^o, consisting of the plant and the input and output sampling devices.

The prior information assumed on P^o is $L = 6$ and $\rho = 0.93$, which has been validated using Theorem 2 in [16]. This is a quite "safe" prior assumption, since stronger assumptions are still not invalidated by data, e.g. $L = 6$ and $\rho = 0.91$, or $L = 5$ and $\rho = 0.93$.

Different nominal models have been identified, with their corresponding model sets:

- a 150-th order FIR model M_{150}^{no} provided by the "nearly optimal" algorithm (8), with $\nu = 150, m = 16, q = 500$ and $W_C(\omega) = 1\ \forall\omega \in [0, 2\pi]$;
- approximations of M_{150}^{no} of order $2 \div 6$, denoted by $M_2 \div M_6$, provided by the procedure described in section 4.

The $\sup_{0 \leq \omega \leq 2\pi}$ computations required in (6) have been carried on by gridding ω on a sufficiently coarse set of frequencies. Indeed, the value sets $V(\omega)$ have been computed on a set T_ω of 500 equispaced frequencies in the range $[0,\pi]$.

The obtained values of the lower and upper bounds on the identification error $E(\mathcal{M})$ for the identified model sets $\mathcal{M}_{150}^{no}$ and $\mathcal{M}_n$ (with nominal model M_n), for $n = 2, \cdots, 6$. are reported in the following table:

$\mathcal{M}$	$\mathcal{M}_{150}^{no}$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$
$\underline{E}_{16}^{150}(\mathcal{M})$	3.57	6.49	5.31	3.48	3.48	3.48
$\overline{E}_{16}^{150}(\mathcal{M})$	3.61	6.52	5.34	3.49	3.49	3.49

The magnitude frequency response of the identified model sets $\mathcal{M}_2$ and $\mathcal{M}_4$ are reported in figure 4 and compared with the actual system P^o. From these results it is seen that model sets of order higher than 4 do not allow to improve the identification error. Then the control design is performed on model sets $\mathcal{M}_2 \div \mathcal{M}_4$.

The nominal reduced order models $M_2 \div M_4$, mapped in s-domain using inverse bilinear transformation with a sampling time $T_s = 1s$, and the corresponding information about identification error are used for computing $\hat{Q}_2 \div \hat{Q}_4$ as described in section 5.

Attenuation of $50\,dB$ of disturbance $d(t)$ in the frequency band $[0, 10^{-4}]\ rad/s$ and maximum sensitivity peak of $2\,dB$

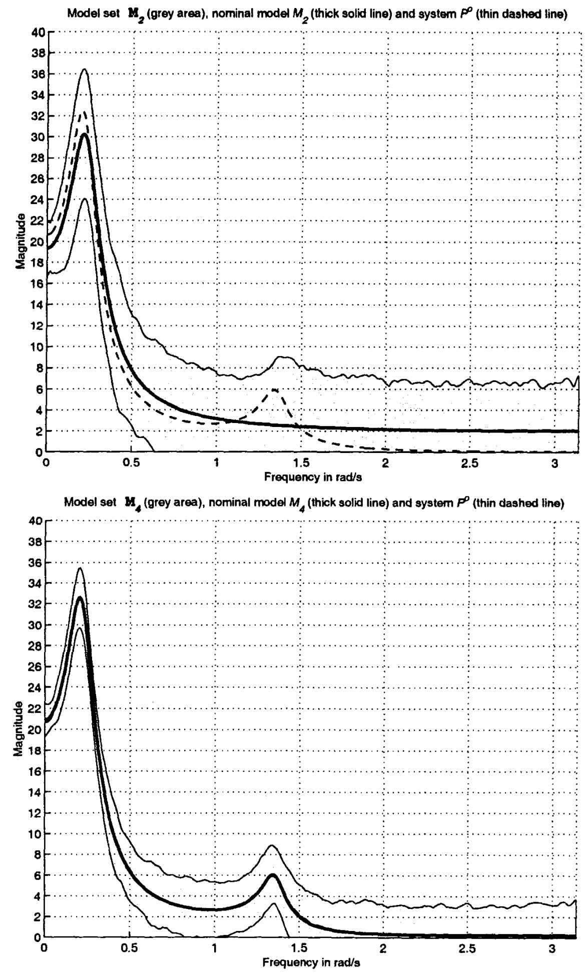

Figure 4: Magnitude frequency responses of $\mathcal{M}_2$ and $\mathcal{M}_4$

are the considered performance specifications. This means that the closed loop transfer function of interest $\tilde{H}(\cdot)$ is the sensitivity, and the following performance weighting function is assumed:

$$W = \begin{cases} 312.5 & \text{for } \omega \in \left[0, 10^{-4}\right]\ rad/s \\ 0.5 & \text{for } \omega \in \left]10^{-4}, \infty\right]\ rad/s \end{cases}$$

In order to use the H_∞-optimization algorithm available under MATLAB, the two rational functions $W_r(s)$ and $W_s(s)$ mentioned in section 5 must be used respectively in place of W_Δ^* and W. Considering that W_Δ^* is approximately constant, a suitable choice for W_r is $W_r(s) = \overline{E}_{16}^{150}(\mathcal{M}_n)$ for $n = 2, 3, 4$. A reasonable low-order rational approximation of W is:

$$W_s(s) = \frac{0.50119\,(s + 0.3786)}{s + 0.0006}$$

The obtained guaranteed sensitivity upper and lower bounds for model sets $\mathcal{M}_2$ and $\mathcal{M}_4$ are shown in figure 5. The analysis of these results may give interesting information to the designer. For example, it can be noted that the sensitivity $\tilde{H}(j\omega,\hat{Q}_4,\mathcal{M}_4)$, guaranteed by the controller $\hat{Q}_4$ designed on the basis of $\mathcal{M}_4$, robustly satisfies the performance requirement of $50\,dB$ disturbance attenuation over the band $[0, 3 \cdot 10^{-4}]\ rad/s$.

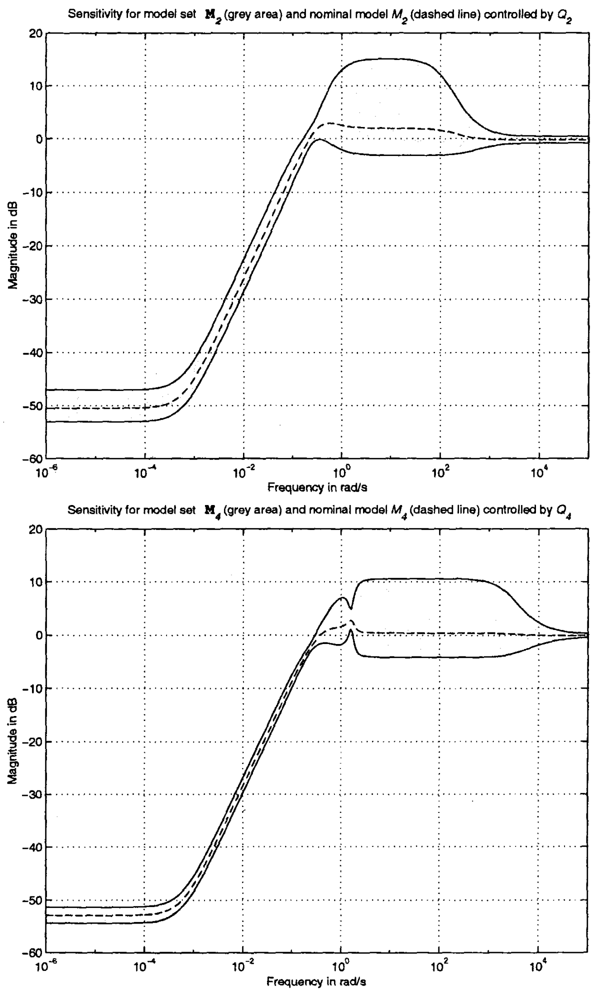

Figure 5: Upper and lower bounds on sensitivity transfer functions $\tilde{H}(j\omega, \hat{Q}_2, \mathcal{M}_2)$ and $\tilde{H}(j\omega, \hat{Q}_4, \mathcal{M}_4)$

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