INdAM-GNFM Young Researchers Project 2011

Title

Multiscale methods and models for collective behaviours in living complex systems

Scientific description

Contextualization of the research

Recently living complex systems have attracted interest in various fields, from Sociology to Ethology and Biology. The main reason is that they pose new stimulating scientific challenges with respect to more traditional systems. Indeed they are particle systems (such as e.g., crowds, animal groups, cell colonies) whose individuals, unlike inter matter (such as e.g., fluid or gas molecules), possess decisional abilities. This makes the collective behaviour more complex than the simple superposition of individual actions: interactions arise among subjects, which produce spontaneously different dynamics with respect to the case of isolated particles. One of the great challenges of modern Applied Mathematics is therefore the design of innovative analytical and computational tools to study these systems. A further motivation is provided by an increasing number of applications, which have started to face the necessity to understand, predict, and control the aforesaid behaviour.

Existing approaches and open problems

In the literature numerous contributions, also from the experimental side, are already available to this research line. Mathematical models are formulated at either the microscopic scale, by means of systems of ordinary differential equations, or the macroscopic scale, by means of partial differential equations and conservation laws. Usually the former do not allow one to easily aggregate the individuals in order to simplify the system and get an overview of it. The latter are more suited to this goal, but those currently available often exhibit technical difficulties, which limit their use mostly to one-dimensional scenarios. Moreover, some multiscale aspects are certainly crucial for understanding the spontaneous emergence of self-organized collective dynamics. In fact overall dynamics stem from an intrinsic coupling between discrete and continuous dynamics at the levels of single agents and distributed group.

Proposed approach

Mathematical framework

Our research aims at the development of modelling, analytical, and computational methods for living complex systems in two-dimensional and three-dimensional domains. The starting point are a few recent works, where we proposed discrete and continuous techniques for crowd dynamics based on a common approach relying on measure theory. The fundamental idea is to represent the spatial distribution of the agents by means of the probability distribution of their positions. This leads to a deterministic Eulerian description of the macroscopic behaviour from a stochastic Lagrangian modelling of the microscopic interactions.

Multiscale aspects

The description of the system via abstract measures does not force one to choose *a priori* the representation scale, which can then be selected *a posteriori* without changing the modelling structure. In particular, at the microscopic scale the description is discrete, i.e., each agent is represented individually. From our viewpoint this corresponds to atomic probability measures. Conversely, at the macroscopic scale the description is continuous, i.e., agents are represented by means of a density in space. From our viewpoint this corresponds to an absolutely continuous probability measure with respect to the volume. It is also possible to design models in which the probability contains both an atomic and an absolutely continuous component. This gives rise to a description which is neither fully discrete nor fully continuous, thus able to incorporate the microscopic granularity in the macroscopic flow.

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