A Kinetic Model of Crowd Evacuation from Bounded Domains

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Living Systems

Some common features of living systems

- **Ability to develop a strategy**: living entities are able to develop specific strategies and have organization abilities (e.g. for crowds: trend toward the exit, avoiding clusters, avoiding walls and obstacles, perception of signals, etc).

- **Heterogeneity**: the ability to express the said strategy is not the same for all entities

- **Interactions**: living entities interact with other entities and with the surrounding environment in a non-local and non-linear way
The choice of the scale for crowd dynamics

* **Microscopic**: Pedestrians identified singularly by $x = x(t)$ and $v = v(t) \rightarrow$ Large systems of ODE’s [Helbing et al.: “Social force model”, 1995]

* **Macroscopic**: The crowd is assimilated to a continuum, its state being described by average quantities (density, linear momentum, and energy) regarded as time and space-dependent variables $\rightarrow$ Systems of PDE’s [Hughes: a first order model, 2002]
The mean distance between pedestrians may be small or large, the ratio between the mean free path and the geometrical length scale (Knudsen number) spans a wide range of values in the same computational domain.

Using the macroscopic representation becomes a complex task because of the breakdown of continuum models in some regions of the physical domain.

The study of living complex systems always needs a **multiscale approach**, where the dynamics at the large scale need to be properly related to the dynamics at the low scales.
* **Mesoscopic** (kinetic): The microscopic state of pedestrians is still identified by position and velocity but the system is represented statistically through a distribution function over such a microscopic state → Integro-differential equations
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Living Systems


- **What**: Evacuation of pedestrians from a room with one or more exits.
- **How**: Development of the kinetic approach by [N. Bellomo, A. Bellouquid, D. Knopoff (2013)] to include
  - interactions with *walls*
  - flow through *exit doors*
Description of the system

- Bounded domain $\Sigma \in \mathbb{R}^2$, assumed convex (no obstacles)
- $E \subset \partial \Sigma$ outlet zone (exit), $E$ could be the union of disjoint sets
- $W = \partial \Sigma \setminus E$ wall
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Description of the system.

- Pedestrians are viewed as active particles
- Microscopic state (continuous-discrete hybrid features):
  * Position: $x = (x, y)$
  * Velocity: $v = (v, \theta)$

The velocity direction $\theta$ takes values in the discrete set

$$I_\theta = \left\{ \theta_i = \frac{i - 1}{n} 2\pi : i = 1, \ldots, n \right\}$$

The state of the system is defined by a distribution function over the micro-state:

$$\tilde{f}(t, x, \theta, v) = f(t, x, \theta) \delta(v - \langle v[f] \rangle)$$
Reduced distribution function

We neglect the heterogeneity of pedestrians in changing the velocity modulus
- changes of the velocity direction $\theta$: stochastic
- changes of the velocity modulus $v$: deterministic

From now on we consider the reduced distribution function

$$f(t, x, \theta) = \sum_{i=1}^{n} f_i(t, x) \delta(\theta - \theta_i), \quad f_i(t, x) = f(t, x, \theta_i)$$

$$f_i(t, x) dx = \text{number of pedestrians who, at time } t \text{ are in the infinitesimal rectangle } [x, x + dx] \times [y, y + dy] \text{ and move with direction } \theta_i$$

• Local density: $\rho(t, x) = \sum_{i=1}^{n} f_i(t, x)$
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Description of the system.

Velocity modulus and perceived density

Velocity modulus depends on

- (perceived) level of congestion
- quality of the environment, assessed by a parameter $\alpha \in [0, 1]$

In the free flow zone ($\rho \leq \rho_c(\alpha) = \alpha/5$) pedestrians move with the maximal speed $v_m(\alpha) = \alpha$ allowed by the environment. In the slowdown zone ($\rho > \rho_c(\alpha)$) pedestrians have a velocity modulus which is heuristically modeled by the 3rd order polynomial joining the points $(\rho_c(\alpha), v_m(\alpha))$ and $(1, 0)$ and having horizontal tangent in such points.
The mathematical structure

\[
\partial_t f_i(t, x) + \text{div}_x (v_i [\rho_i^p(t, x)] f_i(t, x)) = J_i[f](t, x)
\]

\(i = 1, \ldots, n\)

**transport term:**
net balance of particles in the elementary volume of the space of the microscopic states, moving with direction \(\theta_i\), due to transport

**interaction term:**
net balance of particles in the elementary volume of the space of the microscopic states, moving with direction \(\theta_i\), due to interactions
Interaction dynamics

We take into account the following effects:

1. Geometrical effects
   - Exit
   - Walls

2. “Congestion” effects
   - Stream
   - Vacuum
Optimal geometrical direction

\[ \tilde{\omega}_G(x, \theta_h) = (1 - d_E(x)) \tilde{v}_E(x) + (1 - d_W(x, \theta_h)) \tilde{\nu}_W(x, \theta_h) \]

whose direction \( \theta_G \) is the optimal geometrical direction

Geometrical transition probability or “table of games”

\[
A_h(i) = \beta_h(\alpha) \delta_{s,i} + (1 - \beta_h(\alpha)) \delta_{h,i}, \quad i = 1, \ldots, n
\]

where \( s := \arg \min_{j \in \{h-1, h+1\}} \{d(\theta_G, \theta_j)\} \)

\[
d(\theta_*, \theta^*) = \begin{cases} 
|\theta_* - \theta^*| & \text{if } |\theta_* - \theta^*| \leq \pi \\
2\pi - |\theta_* - \theta^*| & \text{if } |\theta_* - \theta^*| > \pi 
\end{cases}
\]

\[
\beta_h(\alpha) = \begin{cases} 
\alpha & \text{if } d(\theta_h, \theta_G) \geq \Delta \theta \\
\frac{\alpha d(\theta_h, \theta_G)}{\Delta \theta} & \text{if } d(\theta_h, \theta_G) < \Delta \theta
\end{cases}
\]
Optimal interaction-based direction

\[ \vec{\omega}_P(x, \theta_h, \theta_k) = \varepsilon \vec{\sigma}_k + (1 - \varepsilon) \vec{\gamma}(x, \theta_h) \]

\( \theta_P \) direction of \( \vec{\omega}_P \)

* \( \varepsilon \) close to 0 \( \rightarrow \) normal conditions
* \( \varepsilon \) close to 1 \( \rightarrow \) panic conditions

“Congestion” transition probability

\[ B_{hk}(i)[\rho] = \beta_{hk}(\alpha) \rho \delta_{r,i} + (1 - \beta_{hk}(\alpha) \rho) \delta_{h,i}, \quad i = 1, \ldots, n \]

where \( r := \arg\min_{j \in \{h-1, h+1\}} \{d(\theta_P, \theta_j)\} \)

\[ \beta_{hk}(\alpha) = \begin{cases} \alpha & \text{if } d(\theta_h, \theta_P) \geq \Delta \theta \\ \alpha \frac{d(\theta_h, \theta_P)}{\Delta \theta} & \text{if } d(\theta_h, \theta_P) < \Delta \theta \end{cases} \]
Some words about panic


**Panic**: Breakdown of ordered, cooperative behavior of individuals due to anxious reactions to a certain event.

Typical features of panic conditions

- people develop blind actionism
- move considerably faster
- moving and passing of a bottleneck becomes incoordinated
- herding behavior, i.e. people do what other people do
Interaction terms

\[ J_i[f](t, x) = J^G_i[f](t, x) + J^P_i[f](t, x) \]

\[ J^G_i[f](t, x) = \mu[\rho(t, x)] \left( \sum_{h=1}^{n} A_h(i) f_h(t, x) - f_i(t, x) \right) \]

\[ J^P_i[f](t, x) = \eta[\rho(t, x)] \left( \sum_{h,k=1}^{n} B_{hk}(i) [\rho] f_h(t, x) f_k(t, x) - f_i(t, x) \rho(t, x) \right) \]

\( J^G_i \): difference between the gain and the loss of particles moving with direction \( \theta_i \) due to geometrical effects

\( J^P_i \): difference between the gain and the loss of particles moving with direction \( \theta_i \) due to interactions among particles
Case-studies

The specific case-studies are selected to analyze the influence on evacuation time of

1. the initial distribution
2. the parameter $\varepsilon$
Case-studies

The specific case-studies are selected to analyze the influence on evacuation time of

1. the initial distribution
2. the parameter $\varepsilon$

In our simulations:

- $\Sigma$ square domain of side length 10 m
- quality of the environment $\alpha = 1$
- 8 different velocity directions in $I_\theta = \left\{ \frac{i-1}{8}2\pi : i = 1, \ldots, 8 \right\}$
The influence of initial distribution

* density referred to $\rho_M = 7 \text{ ped/m}^2$,
* velocity modulus referred to $v_M = 2 \text{ m/s}$,
* time referred to the minimal evacuation time $T_{ev,0} = 13 \text{ s.}$
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Simulations and numerical results.

The role of the parameter $\varepsilon$
Further ideas

- More tests on the model (e.g. which is the initial distribution that minimizes the evacuation time?)
- Moving towards more complex geometries, including obstacles, interconnected areas and inlet zones (finite volume methods)