Prof. Alberto Carpinteri

SCIENTIFIC PROFILE:

MAJOR RESEARCH TOPICS AND CUTTING-EDGE RESULTS

The academic activities by Alberto Carpinteri (AC) have been developed over the years towards different, although correlated, directions:

- (i) Teaching undergraduate, graduate, and postgraduate courses on Structural Mechanics Fundamentals, Advanced Structural Mechanics, Static and Dynamic Structural Instabilities, Theory of Plasticity, Fracture Mechanics;
- (ii) Organizing Workshops and Conferences at the national and international levels, in particular on topics regarding Fracture Mechanics and Structural Mechanics;
- (iii) Participating, initially as a member and later with higher responsibilities, to the life of different national and international Scientific Societies, in the fields of Theoretical and Applied Mechanics, Experimental Mechanics, Structural Mechanics, Fracture Mechanics, Civil Engineering, Metrology;
- (iv) Writing or editing volumes on different topics, reflecting various and ample scientific interests and teaching activities;
- (v) Publishing the major results of his studies in Refereed International Journals.

As regards the last two items, AC treated several specific topics, always giving them an original and personal contribution. In some cases, such a contribution resulted to be also innovative, anticipating even by years the trends in cutting-edge international research. Among these peculiar topics, it is significant to remind the following ones (references within round parentheses are related to the complete List of Publications reported in the website https://staff.polito.it/alberto.carpinteri/ - Capital letter B refers to the list of Books published by AC).

(1) Static-kinematic duality and its crucial role in Computational Mechanics

Scope and distinctive feature of the research topic developed by AC is that of proposing a new matrix-operator formulation of the *Finite Element Method*, which is introduced on the basis of the *Static-kinematic Duality* in a very general, direct, and unified way. Such demonstration is totally original and is reported in the text books on Structural Mechanics by AC (B8, B34, B38).

The mechanics of linear elastic bodies, in particular of bars, beams, plates loaded in their middle plane, plates in flexure, arches, shells, ropes, and membranes, is studied adopting an original matrix-operator formulation, which is suitable for Computational Mechanics applications. The static, constitutive, and kinematic equations, once composed, provide a matrix-operator equation presenting the generalized displacement vector as its principal unknown. Constant reference is made to *Duality*, i.e., to the strict correspondence between *Statics* and K*inematics* that emerges as soon as the two matrix-operators are recognised to be the *adjoint* of each other. In this context, it is easy to prove the mutual implication of *Duality* and *Principle of Virtual Work* (748, 830, 924). The Finite Element Method can be introduced as a discretization and interpolation procedure for the approximate solution of any elastic problem, in static or dynamic regimes, whatever are the dimensions of displacement vector and deformation vector, with or without intrinsic curvatures.

More in detail, after considering the case of 3-D body, for which static and kinematic matrices present only differential operators among their elements, resulting they the transpose of each other (3×6 and 6×3 are the respective dimensions), the sequence continues with 1-D and 2-D geometries, for which also algebraical (nondifferential) elements appear. In the case of rectilinear beam, the static matrix presents also the element –1, which is due to the well-known relationship between bending moment and shearing force (that are both internal actions), as well as the kinematic matrix presents also the element +1, which is due to the relationship between shearing strain and rotation (where the former is a deformation and the latter a generalised displacement). In the case of curved beam, the two matrices present also the intrinsic curvature 1/r of the beam among their elements, and appear to be the transpose of each other except for the algebraical elements, which result to be the opposite of each other. We can affirm that these matrices are the *adjoint* of each other. The logical sequence continues with the circular plate in flexure, the cylindrical shell, the thin dome (or membrane) of revolution, and the shell of revolution, the matrices of the last containing the elements of the previous three structural geometries. In all these key cases, the static and kinematic matrices are the adjoint of each other.

Historical remark: The kinematic expressions for the *shearing strain* of beam (Timoshenko), plate (Mindlin),shell of revolution along the meridian (Fluegge), and generic shell along the two principal directions of curvature (Novozhilov) are easily defined according to the Static-kinematic Duality, whereas they were not in the classical treatments.

Main conclusions:

(a) The change in algebraical sign of the nondifferential elements is due to the rule of integration by parts, which presents the sign minus before the integral just for the differential-operator elements and not for the algebraical elements;

(b) The complex structure of the composite matrix-operator provided by the product of static, constitutive, and kinematic matrix-operators permits to immediately recognise the identity between FEM global stiffness matrix and Ritz-Galerkin matrix. In other terms, FEM strictly is a particular case of Ritz-Galerkin Method.

(2) Applications of Dimensional Analysis to scaling transitions in solids and fluids

The application of Dimensional Analysis (Buckingham's Theorem for physical similitude and scale modelling) leads to the definition of the dimensionless *Brittleness Number* in the scaling competition between plastic collapse and brittle fracture, which are failure mechanisms governed by generalised forces with different physical dimensions.

The stress-singularity-based *Brittleness Number* is a function of strength (FL^{-2}), toughness ($FL^{-3/2}$), and size-scale (L) of the solid body. In this context, it is possible to confirm, and even to generalize, the *small-scale yielding* condition, which is just local (small plastic zone around the crack tip) and related to an infinite plate. The innovative idea of directly comparing the two different failure modes in finite-sized plates was firstly proposed in the papers (13, 17, 23, 31, 44). Additional studies on microcracking coalescence were proposed later (859, 860), as well as a generalization to critical phenomena besides fracture, like resonance and turbulence (923).

By using complex potentials, some light was recently shed on the analogy between the singularity problems arising in fluid dynamics and in fracture mechanics –in particular, those concerning planar fluid flows around sharp obstacles (957) and plane elasticity in cracked bodies. Applications to two equivalent geometries were shown: a thin plate transversally immersed in a uniform fluid flow and a crack subjected to uniform out-of-plane shearing stress at infinity (Mode III). The matching between the fluid velocity field and the shearing stress field is consistent with the hydrodynamic analogy. A *velocity-intensity factor* was defined by AC to predict the *vortex shedding* phenomenon generated by a transversal sharp obstacle. It is important to remark that the *velocity-intensity factor* presents physical dimensions that are intermediate between those of fluid velocity and those of kinematic viscosity. Therefore, it was demonstrated that the size-scale affects the occurrence of *natural-to-forced turbulence* transitions, i.e., from Reynolds' to von Karman's critical phenomena in fluids, in a strict analogy to the ductile-to-brittle failure transitions in solids. By this relevant analogy, a new dimensionless number emerges governing the *turbulence-to-vortex shedding* transition. As the dimensional mismatch between strength and toughness implies a scale-dependent ductile-to-brittle failure transition in solids, so a scale-dependent turbulence-to-vortex shedding transition is implied in fluids by the difference in physical dimensions between kinematic viscosity and v*ortex shedding toughness* (i.e., the critical value of velocity-intensity factor).

(3) Ductile-to-brittle size-scale transition in structural behaviour: Definition of Cohesive Crack Model in the Catastrophe Theory context

Decreasing the fracture energy of the material, increasing its tensile strength, and/or increasing the size-scale of the structural element, the structural response in terms of load versus deflection diagram becomes more and more brittle, and finally a cusp-catastrophe emerges, with a positive slope in the post-peak softening branch (*snap-back instability*). It was the first time that such a general and rational interpretation was provided to brittle mechanical behaviour, with the possibility of capturing also the post-peak branch. From a numerical point of view, the crack length was selected as the monotonically increasing input parameter (*Crack Length Control Scheme*), whereas the input parameter in the experiments was the crack mouth opening displacement. The theoretical model and the related numerical algorithm were called the *Cohesive Crack Model* by AC and proposed in the papers (41, 52, 56, 59, 71, 73, 74, 75, 76, 77, 78, 99, 108, 109). On the other hand, the studies on this nonlinear model confirmed the importance of the energy-based *Brittleness Number* in the description of the ductile-to-brittle transition by increasing the structural size. Large bodies are much more brittle than small bodies with the same shape and made up of the same material. More recent applications to *superplasticity in polycrystalline ceramic materials* (369) appear to capture the experimental trends.

The cusp catastrophe was demonstrated by AC to coincide with the classical Griffith-Irwin instability in the framework of Linear Elastic Fracture Mechanics (LEFM), when the size of the process zone tends to zero. How can a relatively simple nonlinear constitutive law, which is scale-independent, generate a size-scale dependent ductile-to-brittle transition? Consistently with the different dimensionalities of the two abscissa axes (strain and crack opening displacement), constant reference has to be made to Dimensional Analysis and to the definition of suitable nondimensional *Brittleness Numbers* that govern the transition. These numbers can be defined in different ways, according to the selected theoretical model, although they always contain information on material strength and toughness, as well as on structural size-scale (B40). The simplest way is that of directly comparing critical LEFM conditions and plastic limit analysis results. This is an equivalent way −although more effective for finite-sized cracked plates− if compared to the traditional evaluation of the crack tip plastic-zone extension in an infinite plate. In extremely brittle cases, the plastic zone or process zone tends to disappear and the cusp catastrophe conditions prevail over the strain-softening ones, tending to coincide with the LEFM critical conditions in the case of initially cracked plates. In the case of initially uncracked plates, they tend to coincide with the ultimate strength critical conditions.

Besides the *Cusp-catastrophe* interpretation of brittle crack propagation, also *frictional stick-slip* (550) and *buckling in thin cylindrical and spherical shells* (B38) can analogously be interpreted in the context of *Catastrophe Theory* (see the early contributions by Theodore von Karman and his School).

(4) Stability of cracking and crushing in bar- and/or fibre-reinforced concrete elements: Definition of Bridged Crack Model in the discontinuous function context

The original solution proposed by AC to the problem of propagation stability for cracks bridged by reinforcements and/or fibres was based on rigorous conditions of static equilibrium and kinematic compatibility on the beam cross-section, besides on a condition of LEFM criticality at the brittle-matrix crack tip (*Bridged Crack Model*). The first two papers on the subject (21, 37) deal with a single reinforcement layer or fibre in a bent beam traversed by an edge crack. The stability condition is governed by a reinforcement-based *Brittleness Number* where also the reinforcement percentage or the fibre volume fraction appears. Based on this analytical model, numerous applications to cementitious materials followed over the years. Repeated or cyclic loading and hysteretic behaviour were considered in (38, 444, 899, 943), as well as the problem of *minimum reinforcement* in reinforced concrete members (64, 97, 98, 103, 116, 117, 754).

The model was then extended to beams with multiple longitudinal reinforcements and/or fibres (150, 168, 189, 190, 878, 911), as well as to reinforced concrete beams with a nonlinear (*quasi-brittle*) matrix in tension (309, 335). Later, also the nonlinear behaviour in compression of the concrete matrix was considered through the definition of the *Overlapping Crack Model* (459, 538, 543, 544, 545, 546, 582, 584, 587, 590, 633, 634, 643, 644, 645, 661, 668, 705, 706, 871).

More recent developments deal with the brittle behaviours of *high-performance reinforced concrete* beams occurring for particularly low or high reinforcement percentages. In the former case, the loading drop is due to *tensile cracking*, whereas in the latter it is due to *compression crushing* of concrete at the opposite beam edge. For the former case, the *Bridged Crack Model* is able, through an original rotational compatibility condition, to deduce the redundant closing forces applied by the longitudinal reinforcement to the crack faces. This elementary analytical model is conceptually relevant, since it permits to define the *minimum reinforcement* condition. The linear elasticity of the matrix and the LEFM stress-singularity at the crack tip provide a power-law for the minimum reinforcement percentage, which results to be proportional to the beam depth raised to the exponent –1/2. On the other hand, introducing a numerical model where concrete is considered as a cohesive softening material both in tension and in compression (*Cohesive/Overlapping Crack Model*), we can obtain a double size-scale brittle-to-ductile-to-brittle transition. When the steel percentage is too low (or the beam depth too small), then the peak load is higher than the ultimate perfectly-plastic plateau, and a condition of vertical loading-drop prevails (*hyper-strength*). On the other hand, when the steel percentage is too high (or the beam depth too large), then the ultimate perfectly-plastic plateau reduces its extension up to zero and a condition of vertical loading-drop prevails again, in this case due to crushing at the beam extrados (*over-reinforcement*) (119, 486, 948). On these geometrical bases, by equating peak load and ultimate plastic load in the former case, and tending the plastic plateau extension to zero in the latter, it is possible to establish very robust criteria to determine *minimum* and *maximum* *reinforcement percentages* (925). By applying Dimensional Analysis and a best-fitting procedure, both in tension and in compression, it is possible to find the scaling laws for minimum and maximum reinforcement percentages, respectively. The two exponents become equal to –0.15 and –0.25, respectively. The absolute values of the exponents are both lower than the absolute value of the reference LEFM exponent 0.50 (scaling of extreme severity) and agree with the available experimental results very satisfactorily. The former has recently been assumed as the reference value in the AASHTO Guidelines for the minimum flexural reinforcement. Unfortunately, we can not affirm the same yet for the most well-known National and International Standard Codes.

Very recently, the scaling laws of minimum and maximum reinforcement percentages have been extended to *next-generation reinforced concrete* materials: Prestressed concrete, FRP-bar reinforced concrete, Hybrid (steel bars and fibres) reinforced concrete (921, 922, 941, 942, 944, 949, 954, 955, 964, 965).

(5) Fractals in deformation, damage, fracture, and fatigue

Based on some experimental evidence, fractal patterns to describe rough fracture surfaces, damaged cross-sections at peak load, and uniaxial crack-band deformations were assumed by AC in (136, 137, 223, 284, 313, 360). This innovative conjecture led him to define the *renormalized* (or *fractal*) counterparts of fracture energy, tensile strength, and ultimate strain, which present anomalous and noninteger physical dimensions and represent the really scale-invariant material properties. On the other hand, the usual nominal quantities consequently become scale-dependent and vary with the specimen size according to peculiar power-laws where the scaling exponent is connected to the fractal dimension of the same fractal set over which the quantity is defined.

More recent developments by AC deal with the occurrence of self-similar and fractal patterns in a wider class of mechanical phenomena: *deformation, damage, microcracking, fracture, crushing, fragmentation, comminution, wear, creep* (908, 912, 913), and *fatigue* (182, 542, 585, 625, 627, 749, 811, 866, 883, 926, 938, 939 , 959), in cementitious, ceramic, polymeric, and metallic materials. Analogously, he considered the consequent apparent scaling in the related nominal mechanical properties. Such a scaling is negative (*lacunar fractality*) for tensile strength and fatigue limit, whereas it is positive (*invasive fractality*) for fracture energy, fracture toughness, and fatigue threshold. At the same time, corresponding fractal (or renormalized) quantities emerge, which are the true scale-invariant properties of the material. They take the role of constant factor in the power-law relating the nominal canonical quantity to the size-scale of observation. In this fractal context, it is then possible to define a scale-invariant constitutive law: the so-called *Fractal Cohesive Crack Law*, in which stress and strain are defined over two orthogonal lacunar fractal sets, as well as the fracture energy in an invasive fractal set that is the Cartesian product of the two previous lacunar sets (202, 284, 313, 864).

In addition, stress-intensity attenuation or mitigation with respect to the LEFM stress-singularity power ½ was considered, when the material is power-law strain-hardening (31), the sharp crack is replaced by a re-entrant corner or V-notch (57, 358, 548, 549, 555, 592, 626, 628, 631, 632, 676, 695, 696, 742, 743, 756, 769, 770, 823), and/or the crack faces are invasive (extremely rough) fractal sets (137). In all these three cases, the stress-intensity factor assumes anomalous physical dimensions, which rule the scale-effect bi-logarithmic diagrams with slopes smaller than that of the most severe case, i.e., a sharp and smooth-faced crack in a linear elastic material (B40).

Particularly relevant applications were proposed by AC to size effects on fracture energy of concrete and fatigue threshold of metals. As the former is demonstrated to be a power-law that is increasing with the specimen size, the latter is shown to be a power-law decreasing by decreasing the initial crack length. In this context, a consistent explanation was provided to the *short crack problem* (900). As for rough cracks invasive fractal sets are applied (dimension greater than 2), so for damaged cross-sections at peak load lacunar fractal sets (dimension lower than 2) are considered. The tensile strength of concrete is proved to be a power-law with negative exponent, so that it decreases with the size of the specimen. Analogously, the fatigue limit of metals decreases by increasing the specimen size (901). All these trends are faithfully confirmed by extensive experimental results reported in the current literature (526).

Multi-fractal Scaling Laws (MFSL) for tensile strength and fracture energy of concrete-like materials best-fit the experimental data when the size-scale ranges are beyond one order of magnitude. If the concept of *self-affinity* (in addition to that of *self-similarity*) is applied, the absolute value of slope in the bi-logarithmic scaling laws is the highest and equal to 1/2 (1/4 for the fatigue threshold) at the smallest scales, whereas it tends to vanish at the largest scales (147, 148, 149, 171, 172, 184, 185, 204). This homogenization effect for tensile strength is captured by the Multi-Fractal Scaling Law (MFSL) very satisfactorily for initially uncracked structural elements (147, 148). The MFSL has been adopted by the European Standards for concrete and reinforced concrete structures. It also works well for ceramic materials at the micro- and nano-scales (400). On the other hand, the Size Effect Law (SEL) works well only for initially cracked specimens, when the initial crack length is proportional to the structural size-scale, and its concavity is downwards. On the contrary, the experimental relative crack depth at the peak load is large in small specimens and small in large specimens, when the specimens are initially uncracked. In addition, the concavity of the experimental data results to be upwards, like in the MFSL.

AC provided original and innovative contributions on *fragmentation, comminution, wear,* and *drilling* (289, 291, 307, 314, 336, 337, 338, 340, 342, 357, 367, 693). Very interesting and new scaling laws for these phenomena were established on the basis of fractal geometry and related power-laws. Additional papers by AC deal with a fundamental aspect like the size-scale effects on *friction coefficient* and *criticality of rock slopes* (222, 334, 370, 476, 517). The *Gutemberg-Richter statistics* for seismic analysis was reconsidered, providing the *b-value* with a fractal meaning in a very original way (521, 534, 554, 669, 771, 773, 814, 872). Also the problems of dynamics and stability of fractal-shaped structures (trees, antennas, etc.) were considered and solved with their peculiar implications in Nature (520, 593, 629).

Fractional Calculus (B9) was applied by AC to the field and boundary equations of an elastic body that is deformable only over a fractal subset of its domain. The duality of the static and kinematic fractional operators was demonstrated, so that a fractal version of the Virtual Work Principle emerges (258, 285, 326, 328, 329, 330, 331, 332, 630, 697, 750, 817, 842).

(6) Fracto-emissions as seismic precursors

TeraHertz phonons are produced in solids and fluids by *mechanical instabilities at the nano-scale* (e.g., fracture and cavitation) (B36). Their frequency is close to the resonance frequency of atomic lattices (*Debye frequency*) as well as their energy is close to that of thermal neutrons. A series of fracture experiments on natural rocks and the systematic monitoring of seismic events have demonstrated that TeraHertz phonons are able to induce fission reactions in medium-weight chemical elements (in particular, Iron and Calcium) with neutron and/or alpha particle emissions (529, 530, 579, 636, 639, 670, 673, 702, 776, 778, 821, 956).

The same phenomenon appears to occur in several different situations and to explain puzzles related to the history of our planet (641, 642, 862, 874), like the primordial carbon pollution (and correlated Iron depletion) or the ocean formation (and correlated Calcium depletion), as well as scientific mysteries, like the so-called *cold fusion* (777, 961, 962), or the correct radio-carbon dating of organic materials (675). Very important implications to and applications in earthquake precursors, early-stage fatigue diagnostics, climate change, and energy production are likely to develop in the next future (916).

Three different forms of emitted energy can be used as *earthquake precursors*. At the tectonic scale, *Acoustic Emission* (AE) prevails, as well as *Electro-Magnetic Emission* (EME) at the meso-scale, and *Neutron Emission* (NE) at the nano-scale. The three fracto-emissions tend to anticipate the next seismic event with an evident and chronologically ordered shifting: small cracks, high frequencies, and neutron emission about one week before, whereas longer cracks, lower frequencies as well as electromagnetic and acoustic waves later and temporally closer to the seismic event (3-4 days and one day, respectively). The very repeatable experimental observations over a period of six years reveal a strong correlation between the three fracto-emission peaks and the major earthquakes occurring in the areas close to the seismic station (839, 868).

(7) Dynamics and stability: From nano- to mega-structures

Dynamics and stability were studied by AC for nano-structures (*proteins* and *macromolecular structures*) and mega-structures (*long-span bridges* and *high-rise buildings*).

Proteins and macromolecular structures represent one of the most important building blocks for a variety of biological processes. Their biological activity is performed in a dynamic fashion, so that their mechanical instabilities and vibrations can help to explain how proteins function (B41). Great attention was given by AC to computational approaches and modern experimental techniques (837, 838, 865, 877, 906, 914, 920, 931, 932).

The Euler-Prandtl coupled problem was investigated in the case of *thin-walled open-section Vlasov beams* (960). The same kind of vertical elements were considered also in the complex problem of *high-rise buildings*. An analytical algorithm was originally implemented (45), where just three degrees-of-freedom per floor are contemplated. This model, once enriched by the Vlasov theory, results to be in a good agreement with the most time-consuming FEM codes, both in statics and dynamics (581, 667, 708, 744, 746, 813, 818, 875, 876, 897, 904, 909, 910, 929 , 940). Also for *long-span suspension bridges,* a new analytical model was developed to evaluate wind velocity and frequency of *Flutter Instability* (753, 928 , 958). As in the previous case, the comparison with FEM calculations is very positive, the computing times being much shorter and the results absolutely comparable (700, 835, 836). For *shallow space and shell roofing structures*, the interaction between *Buckling* and *Snap-through* was studied, evidencing the related transitions by varying *shallowness* and/or *thinness* of continuous or trussed shells (812, 833, 834, 915).

Nonlinear and chaotic dynamics in cracked solids was investigated in (362, 363). A transverse crack can change its state, from open to closed and vice-versa (*breathing crack*), when the beam vibrates. As a consequence, a nonlinear dynamic behaviour emerges. Considering a continuous variation in the crack state (also partially open or closed), it was possible to simulate *period doubling* and the consequent *transition to chaos*.

The mechanics of *nanostructured* and *hierarchical materials* was studied by AC and co-workers in (361, 373, 403, 409, 483, 484, 487, 488, 489, 490, 491, 492, 494, 518, 525, 527, 541, 598), as well as the mechanics of *layered* and *functionally graded materials* in (288, 308, 371, 390, 397, 398, 399, 404, 405, 438, 439, 477, 478, 479, 528, 586, 589, 677, 768). In all these cases, original and innovative results were achieved.

May 2025