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A GLOBAL APPROACH FOR THREE-DIMENSIONAL ANALYSIS OF TALL BUILDINGS

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SUMMARY

In the literature, several approximate approaches have been proposed to analyse the lateral loading distribution of external loads in high-rise buildings; in this paper, a general method is proposed for the analysis of the lateral loading distribution of three-dimensional structures composed of any kind of bracings (frames, framed walls, shear walls, closed and/or open thin-walled cores and tubes) under the customary assumption of floor slabs being undeformable in their planes. This general formulation allows analyses of high-rise structures by taking into account the torsional rigidity of the elements composing the building without gross simplifications, even in the case of very complex shapes and with the contemporary presence of different kinds of bracing. The method is aimed at gaining an insight into the force flow in the structure, in order to understand how the building response is governed by decisive structural parameters and to compare preliminary calculations with other approaches such as the structural finite element analysis. Copyright © 2009 John Wiley & Sons, Ltd.

1. INTRODUCTION

The design of very tall structures poses a fascinating problem involving some of the most significant aspects of architecture and civil engineering. The symbolic value of a tall building in an urban landscape is very powerful and, in dimensioning such a building, its overall geometry should be viewed as one of the primary elements affecting structural behaviour. From the structural viewpoint, a tall building means a multi-storey construction in which the effects of horizontal actions and the need to limit the relative displacements take on primary importance (Taranath, 1988, 2005). Historically, the development of high-rise buildings hinged on the economic primacy and technological progress of the countries that engaged in their construction. Nowadays, this competition is spreading throughout the world, even to countries regarded as less advanced, which, however, are experiencing a fast industrial growth. The latest skyscrapers rise to over 400 m, the tallest of all being the Taipei Financial Center at 508 m high, while in Dubai they are currently constructing the 'Burj Dubai' tower to exceed 800 m.

A profound understanding of the force flow in these complex structural systems is often very difficult, and a huge commitment in terms of design, technology and economic resources is required. While in the design of low-rise structures the strength requirement is the dominant factor, with increasing height, the importance of the rigidity and stability requirements to be met to counter wind and earthquake actions grows until they become the prevailing design factors. The difficulties in the design and construction of a tall building are not linearly correlated with total height and number of storeys.

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Furthermore, the vibration problems of high-rise buildings subject to certain dynamic loads, such as earthquake or wind loads, have kindled interest in these studies. With increasing height, in fact, the traditional solutions providing for load-bearing main and secondary parts tend to be forsaken in favour of a global approach, whereby the structure is conceived in a unitary fashion, i.e. as a single cantilever beam projecting out from the foundations. At any rate, the key issue in structural design continues to be the choice of an appropriate design model that is able to faithfully reproduce the actual conditions of a structure.

Concerning the model choice, the structural analysis of this kind of structures can be carried out on two levels. On one hand, because of the widespread availability of powerful computers and the variety of finite element (FE) software, structural designers could resort to complex, three-dimensional, multi-bay, multi-storey models of structures with a high degree of accuracy. These models, which have become commonplace over the last 20 years, can take into account any level of detail as they model the structure in its entirety. However, as remarked by several authors (Howson, 2006; Steenbergen and Blaauwendraad, 2007), these models have their drawbacks. Even using powerful computers, the modelling is usually very demanding, and the lengthy and time-consuming procedure of handling all the data can always be a source for errors. In addition and most importantly, because of the complexity of the results, they are difficult to be interpreted and it is not easy to understand from FE simulations which structural parameters govern the response of the structure and the interaction among its elements, or clearly to identify the paths of force flow in the structure. In a sense, the importance of the key structural elements is somehow concealed behind the great number of input and output data. As remarked by Howson (2006), the use of FE models is not in question for the detailed design and analysis, but the development of such a model at an early stage in a design process can be not only time-consuming but also unproductive, if used during a period of rapid evolution of the concept.

Simplified procedures relying on carefully chosen approximations represent the alternative possibility for the analysis. A simplified global model can offer a number of potential advantages. Data preparation and analysis is definitely faster; the modelling procedures are likely to be simpler and more transparent, thus, less prone to be a potential source of error; the accuracy, although not as high as in FE simulations, is sufficient for the preliminary design stage. Most importantly, the use of this kind of procedures offers a clear picture of the structural behaviour, allowing us to gain insight into the key structural parameters governing the tall building behaviour. Within these approaches that rely directly on the engineer's experience and judgement, the engineer is again at the heart of the design and analysis procedure, in a way that can sometimes appear to be lost when using fully automated, general kinds of software (Howson, 2006). In any case, both approaches have their importance, as they are reciprocally complementary. In the early stage of conceptual design, the main structural characteristics can be profitably established using an approximate method. On the other side, a detailed FE static model of the structure can follow, eventually providing the final structural computation.

Over the last three decades, it seems that a large part of the engineering community has followed a path towards the use of FE models also in the early stage of design; however, in the last few years, the discussion and debate on the advantages and disadvantages of abandoning the use of simplified approximate models seems to set a reverse trend. These models cannot be renounced to fully understand the complex behaviour of high-rise structures; in this paper, we propose a general method for the static analysis of tall building structures designed by using any combination of frames, framed walls, shear walls, internal cores, external tubular elements and bracings, with open or closed thin-walled cross-section. The capabilities of the method to provide deep insight into the complex features of the interaction and collaboration of resisting elements in high-rise structures will also be shown.

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2. APPROXIMATE MODELS: STATE OF THE ART

In the past, several approximate approaches have been proposed to determine the lateral loading distribution of external loads (Taranath, 1988; Jayachandran, 2003); according to Zalka (2006), they share some simplifying assumptions:

(a) the material of the structures is homogeneous, isotropic and obeys Hooke's law;
(b) the floor slabs are stiff in their plane and flexible perpendicularly to their plane; the loads are applied statically and maintain their direction (they are conservative forces);
(c) the location of the shear centre only depends on geometrical characteristics;
(d) concerning stability and buckling, the structures have no geometrical imperfections, they develop small deformations, and the incremental effects of the deformations due to the axial forces are negligible.

Very often, they also imply some additional hypotheses, depending on the structure type that they are intended to model. Most of the models, in fact, refer to a single structural type.

The earlier models, developed in the 1960s and 1970s, addressed the case of shear wall versus frame interaction; among them, we could cite the approaches by Khan and Sbarounis (1964), Coull and Irwin (1972), Heidebrecht and Stafford Smith (1973), Rutenberg and Heidebrecht (1975), and Mortelmans *et al*. (1981). In all these models, however, only one degree of freedom (DOF) per storey is considered, and the torsional and flexural problems are treated separately.

Most of the models are based on the continuum medium technique, which was proposed in the pioneering papers by Beck *et al*. (1968) and Beck and Schäfer (1969); within this framework, the whole structure is idealized as a single shear-flexural cantilever. According to the development of the tall building typologies, these models were devoted to framed wall structures, as in the papers by Stamato and Mancini (1973) and Gluck and Krauss (1973), as well as to framed-tube structures, as in the papers by Khan (1974) and Coull and Bose (1977), among others. The continuum medium technique has then been extended to other structural typologies (Stafford *et al*., 1991; Hoenderkamp and Snijder, 2000; Lee *et al*., 2007).

A very peculiar model, specifically developed for framed-tube tall buildings, although the underlying theory is derived from aeronautics, is the stringer-shear panel developed by Connor and Poungare (1991), in which the tall framed-tube is viewed as a stringer-shear panel assembly. The stringers carry only axial loads and have no bending stiffness. The shear panels have shear rigidity, but no axial or bending rigidity, and carry only shear. The tall building is replaced by stringers on any side and shear panels in between. Another peculiar model is that proposed by Takabatake *et al*. (1995), in which a core tube is replaced by an equivalent rod, including the effect of the bending, transverse shear deformation, shear lag and torsion. The discontinuous variation of sectional stiffness of the tubular element is taken into account as a continuous function by means of the extended Dirac function. The closed-form solutions obtained for the deflection, shear lag, and torsional angle of the variable tube structures with braces are based on the elastic theory by Takabatake and Matsuoka (1983).

A different research direction, again derived from aerospace engineering, is that based on subdivision of the structure into substructures; it is somehow in between the simplified models and the FE approach, as the substructures are used to formulate super-elements, either by condensation or by analytical procedures with exact solutions. Among others, we could cite the papers by Leung (1985), Leung and Wong (1988), Wong and Lau (1989), as well as the so-called finite story method (FSM) by Pekau, Lin and Zielinski (1995, 1996). More recent works include, among others, the approaches presented by Kim and Lee (2003) and Steenbergen and Blaauwendraad (2007).

The main problem with all these simplified models is the lack of generality, as the same formulation often cannot be used to analyse buildings with different underlying structural typologies. In addition, several of them consider two-dimensional plane structures. However, not all buildings can be modelled as plane structures, especially in the case of very complex shapes, e.g. the buttressed core shear wall structure, adopted for the record-breaking 'Burj Dubai' tower. With the aim at acquiring insight into the effects of the stability element typologies and arrangements in tall buildings within a unified framework and the capability of modelling complex structures and different typologies, in this paper, we propose a three-dimensional formulation based on the work by Carpinteri and Carpinteri (1985). In this paper, the formulation is extended to encompass any combination of bracings, including bracings with open thin-walled cross-sections, which are analysed in the framework of Timoshenko-Vlasov's theory (Timoshenko, 1936; Vlasov, 1961) of sectorial areas and according to the approach by Capurso (1981). More in detail, section 3 summarizes the original approach to the case with closed thin-walled cross-sections and frames by Carpinteri and Carpinteri (1985), whereas section 4 recalls that by Capurso (1981), with open thin-walled cross-sections only. In section 5, the two approaches are merged within the framework of the formulation by Carpinteri. Section 6 presents a numerical example showing the effectiveness and flexibility of the combined approach.

3. CARPINTERI'S FORMULATION

The general formulation of the problem of the external lateral loading distribution between the bracings of a three-dimensional civil structure, originally presented by Carpinteri and Carpinteri (1985), will be revisited in this section. The structure is idealized as consisting of M bracings interconnected through floors undeformable in their planes, and the axial deformations of bracings are not considered. With these hypotheses, the floor movement can be expressed by three generalized coordinates: the two translations in X- and Y-direction of the global coordinate system origin (Figure 1) and the floor rotation.

If N is the number of stories, the external load will be represented by a $3N$-vector F, whose elements are three elementary loads for each floor and, more exactly, two shear forces and the torsion moment. In the same way, the internal loading transmitted to the i-th element will be represented by a $3N$-vector S_i and obtained from the preceding F through a pre-multiplication by a distribution matrix.

Let p_i be the $2N$-vector representing the shear loadings, on the i-th element in the global coordinate system XY (Figures 1 and 2), and m_i is the N-vector representing the torsion moments, so that

$$S_i = \begin{Bmatrix} p_i \\ m_i \end{Bmatrix} \tag{1}$$

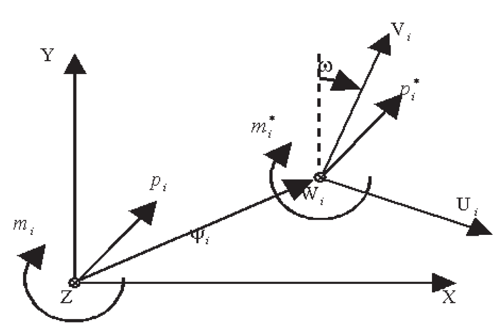

Figure 1. Global and local coordinate systems. The Z-axis completes the right-handed global system XYZ and W_i completes the right-handed local system U_i V_i W_i

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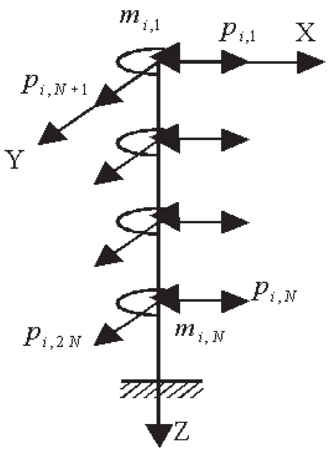

Figure 2. Carpinteri's approach (Carpinteri and Carpinteri, 1985): internal loadings S_i (transmitted to the i-th bracing) in the global coordinate system. Note that the highest floor is indicated with 1 and the lowest with N

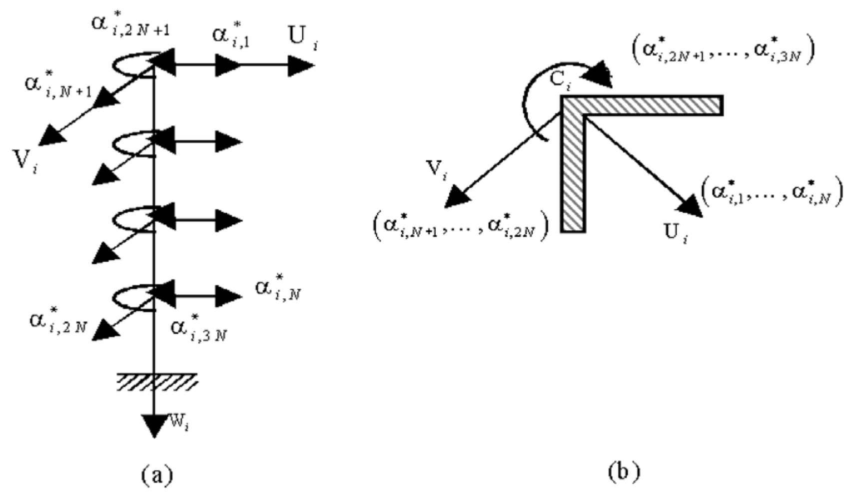

Figure 3. Carpinteri's approach (Carpinteri and Carpinteri, 1985): DOFs of the i-th bracing in the local coordinate system U_i V_i W_i. Axonometry (a) and top view (b)

The internal loadings S_i transmitted to the i-th bracing and related to the global coordinate system XY are connected with the same loadings S_i^* related to the local coordinate system U_iV_i (the origin of this system is in the centre of twist C_i and the U_iV_i axes are parallel to the central ones, see Figures 1 and 3):

$$p_i^* = N_i p_i \tag{2}$$

$$m_i^* = m_i - \psi_i \times p_i \cdot u_z \tag{3}$$

where the superscript * is used to indicate the loadings in the local coordinate system U_iV_i, N_i is the orthogonal matrix of transformation from the system XY to the system U_iV_i, ψ_i is the co-ordinate vector of the origin of the local system U_iV_i in the global one XY, and u_z is the unit vector in the Z-direction (note that $\psi_i \times p_i \cdot u_z$ is a scalar triple product). The orthogonal matrix N_i is represented as

$$\mathrm{N}_i = \begin{bmatrix} \cos\omega & \sin\omega \\ -\sin\omega & \cos\omega \end{bmatrix} \tag{4}$$

where each element represents a diagonal $N \times N$-matrix, and ω is the angle between the X-axis and the U_i-axis (Figure 1).

Equations (2) and (3) may be represented in the matrix form

$$S_i^* = A_i S_i \tag{5}$$

where

$$\mathrm{A}_i = \begin{bmatrix} \mathrm{N}_i & 0 \\ \mathrm{C}_i^T & \mathrm{I} \end{bmatrix} \tag{6}$$

I is the identity matrix, 0 is the null matrix, and the $N \times 2N$-matrix C_i^T is defined as

$$C_i^T = -u_z \times \psi_i, \tag{7}$$

or

$$C_i^T = [-y_i x_i] \tag{8}$$

where each element is a diagonal $N \times N$-matrix, and (x_i, y_i) are the components of vector ψ_i.

The displacements α_i in the global coordinate system XY are then connected with the displacements α_i^* in the local system U_iV_i:

$$\delta_i^* = N_i \delta_i, \tag{9}$$

$$\varphi_i^* = \varphi_i, \tag{10}$$

where δ_i represents the translations and φ_i the rotations. Equations (9) and (10) may be represented in the matrix form

$$\alpha_i^* = B_i \alpha_i, \tag{11}$$

where

$$B_i = \begin{bmatrix} N_i & 0 \\ 0 & I \end{bmatrix} \tag{12}$$

The internal loadings S_i^* are connected with the displacements α_i^* through the relation:

$$S_i^* = K_i^* \alpha_i^*, \tag{13}$$

where K_i^* is the stiffness matrix in the local system. Recalling Equations (5) and (11), we get

$$A_i S_i = K_i^* B_i \alpha_i. \tag{14}$$

Pre-multiplying both the members by the inverse A_i^{-1}:

$$S_i = \left(A_i^{-1} K_i^* B_i \right) \alpha_i \tag{15}$$

It follows that the stiffness matrix in the global system for the i-th bracing is

$$K_i = A_i^{-1} K_i^* B_i \tag{16}$$

where

$$K_i^* = \begin{bmatrix} K_{pi}^* & 0 \\ 0 & K_{mi}^* \end{bmatrix} \tag{17}$$

The displacement $3N$-vector α_i of the i-th element is connected with the displacement $3N$-vector α of the rigid floors by the relation

$$\alpha_i = T_i \alpha \tag{18}$$

where the transformation $3N \times 3N$-matrix T_i is

$$T_i = \begin{bmatrix} I & C_i \\ 0 & I \end{bmatrix} \tag{19}$$

The $2N \times N$-matrix C_i is

$$C_i = \begin{bmatrix} \psi_i \times u_x \\ \psi_i \times u_y \end{bmatrix} \tag{20}$$

or

$$C_i = \begin{bmatrix} -y_i \\ x_i \end{bmatrix} \tag{21}$$

where each element is a diagonal $N \times N$-matrix, and (x_i, y_i) are the components of vector ψ_i. Equation (1) can be rewritten:

$$S_i = K_i T_i \alpha \tag{22}$$

or

$$S_i = \bar{K}_i \alpha \tag{23}$$

where $\bar{K}_i = K_i T_i$ is the stiffness of the i-th element with respect to the floor displacements. For the global equilibrium we have

$$\sum_{i=1}^{M} S_i = \sum_{i=1}^{M} \bar{K}_i \alpha, \tag{24}$$

$$F = \bar{K}\alpha, \tag{25}$$

where $\bar{K} = \sum_{i=1}^{M} \bar{K}_i$ is the global stiffness matrix of the rigid floors. Recalling Equations (23) and (25), we get

$$\alpha = \bar{K}_i^{-1} S_i = \bar{K}^{-1} F \tag{26}$$

and then

$$S_i = \bar{K}_i \bar{K}^{-1} F \tag{27}$$

Equation (27) solves the problem of the external loading distribution between the resistant elements of a building. It is formally analogous to the equation for the distribution of a force between different in-parallel resistant elements in a plane problem. In fact, the distribution matrix $\bar{K}_i \bar{K}^{-1}$ is the product of the partial stiffness matrix by the inverse of the total stiffness matrix, as well as in the plane problem, the distribution factor is the product of the partial stiffness by the inverse of the total stiffness. The sum of the distribution matrices is equal to the unit matrix. Details on the condensation procedure for the computation of the stiffness matrices can be found in Humar and Kandhoker (1980).

It can be observed that the above presented formulation is general and allows one to treat any kind of structural elements, such as frames, shear walls and thin-walled sections either open or closed. In the paper by Carpinteri and Carpinteri (1985), however, the case of open thin-walled cross-section was not included. In this paper, we will extend this formulation also to the case of bracing elements of this kind. To this aim, in the next section we will revisit the formulation by Capurso (1981), which, on the contrary, takes into account bracings with open thin-walled cross-sections only.

4. CAPURSO'S APPROACH

To introduce the numerical approach by Capurso, we should briefly recall some fundamentals from the Timoshenko–Vlasov (Timoshenko, 1936; Vlasov, 1961) torsion theory of beams with thin, open cross-section, characterized by the presence of the warping stiffness. With reference to Figure 4, let us consider a generic beam; if the beam sections are undeformable in their planes, the section movements can be expressed by three displacements—the two translations ξ and η (in X- and Y-direction of the global coordinate system origin; see Figure 4) and the floor rotation ϑ. Under the customary assumption that the external loads are concentrated transverse flexural loads and only torsional moments so that the longitudinal force N at any value of z should be zero, it can be shown that the following relations hold:

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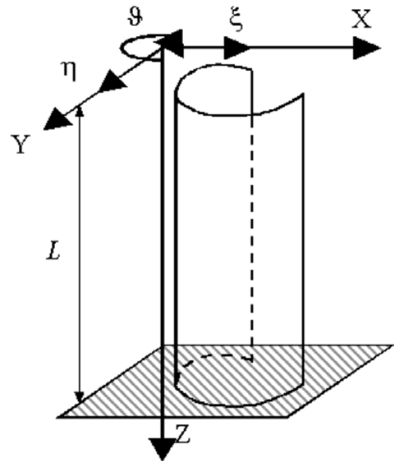

Figure 4. Capurso's approach (Capurso, 1981): displacements of the *i*-th bracing in the global coordinate system XYZ

$$\begin{aligned} M_y &= -E(J_{yy}\xi'' + J_{xy}\eta'' + J_{y\omega}\vartheta''), \\ M_x &= -E(J_{xy}\xi'' + J_{xx}\eta'' + J_{x\omega}\vartheta''), \\ B &= -E(J_{y\omega}\xi'' + J_{x\omega}\eta'' + J_{\omega\omega}\vartheta''). \end{aligned} \tag{28}$$

where M_x and M_y are the bending moments, B is the bimoment (Timoshenko, 1936; Vlasov, 1961), E is the Young's modulus, and the apex $'$ corresponds to derivations with respect to the axial coordinate z. J_{xx}, J_{xy} and J_{yy} are the (second-order) moments of inertia, $J_{\omega\omega}$ is the sectorial moment of inertia, and $J_{x\omega}$ and $J_{y\omega}$ are the sectorial products of inertia. Analogously, the shear forces F_x and F_y and the torsional moment M_z are given by the following expressions:

$$\begin{aligned} T_x &= -E(J_{yy}\xi''' + J_{xy}\eta''' + J_{y\omega}\vartheta'''), \\ T_y &= -E(J_{xy}\xi''' + J_{xx}\eta''' + J_{x\omega}\vartheta'''), \\ M_z &= -E(J_{y\omega}\xi''' + J_{x\omega}\eta''' + J_{\omega\omega}\vartheta'''). \end{aligned} \tag{29}$$

If we now introduce the following vectors

$$\delta \equiv \begin{Bmatrix} \xi \\ \eta \\ \vartheta \end{Bmatrix}, \quad M = \begin{Bmatrix} M_y \\ M_x \\ B \end{Bmatrix}, \quad T = \begin{Bmatrix} T_x \\ T_y \\ M_z \end{Bmatrix} \tag{30}$$

and the matrix of inertia related to the central axes:

$$J = \begin{bmatrix} J_{yy} & J_{xy} & J_{y\omega} \\ J_{xy} & J_{xx} & J_{x\omega} \\ J_{y\omega} & J_{x\omega} & J_{\omega\omega} \end{bmatrix}, \tag{31}$$

Equations (28) and (29) can be rewritten in the synthetic matrix form:

$$M = -EJ\delta'' \tag{32}$$

$$T = -EJ\delta''' \tag{33}$$

If we denote with F the vector of external loadings

$$F = \begin{Bmatrix} p_x \\ p_y \\ m_z \end{Bmatrix} \tag{34}$$

by further deriving Equation (33), we get

$$F = EJ\delta^{IV} \tag{35}$$

Because the matrix of inertia J is symmetric and positive definite, aside from some anomalous cases (detailed by Capurso, 1981), Equation (35) can be inverted and the components of vector δ^{IV} can be computed:

$$\delta^{IV} = \frac{1}{E} J^{-1} F \tag{36}$$

Then, the displacements δ can be obtained by integration, with the appropriate boundary conditions

$$\delta = 0, \quad \delta' = 0, \tag{37}$$

at the clamped base of the beam ($z = L$), and

$$\delta'' = 0, \quad \delta''' = 0, \tag{38}$$

at the free-end ($z = 0$). Given the δ, all the displacements and stresses in the beam can be computed (Capurso, 1981).

As in the previous case of Carpinteri's approach, the structure is idealized as consisting of M bracings interconnected through floors undeformable in their planes but with infinite out-of-plane flexibility; as shown by Taranath (1988, chapter 10), however, the effect of the out-of-plane stiffness of the slabs does not affect much the behaviour of the building. Eventually, the axial deformations of bracings are not considered. With these hypotheses, the floor movement can be expressed by three generalized coordinates: the two translations in X- and Y-direction of the global coordinate system origin (see Figure 4) and the floor rotation. Concerning the loads, they are given by two distributed shear forces and a distributed torsion moment. Notice the different ordering of the DOFs in Capurso's approach (compare Figure 5 with Figure 3). Because of this difference, in the following equations, we will denote the vectors ordered according to Carpinteri's convention with the subscript (1), whereas the vectors ordered according to the Capurso's one will be denoted by the subscript (2).

If we denote $S_{i(2)}$ the 3N-vector that collects the shear forces and the torsional moment acting on the generic i-th shear wall at the levels of the floors, and with $\alpha_{(2)}$ as the 3N-vector collecting the floor displacements, according to Equation (35), we can write

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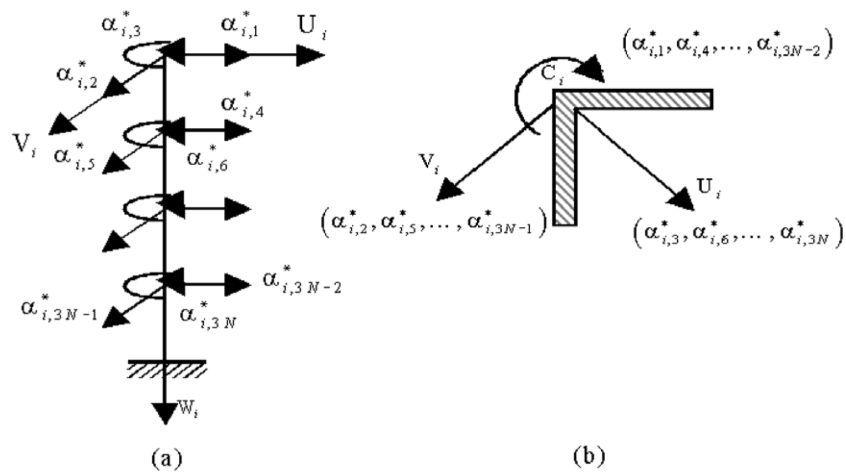

Figure 5. Capurso's approach (Capurso, 1981): DOFs of the i-th bracing in the local coordinate system U_i V_i W_i. Axonometry (a) and top view (b)

$$S_{i(2)} = EJ_i\alpha_{(2)}^{\mathrm{IV}}, \tag{39}$$

where J_i is the $3N \times 3N$-matrix collecting all the moments of inertia of the transverse section of the i-th bracing. If we denote with $F_{(2)}$ the $3N$-vector containing the shear and torsional components of the external loads applied to the whole building, necessarily we get

$$F_{(2)} = \sum_{i=1}^{M} S_{i(2)} \tag{40}$$

and therefore we obtain

$$F_{(2)} = E\sum_{i=1}^{M} J_i\alpha_{(2)}^{\mathrm{IV}} \tag{41}$$

The whole building behaves as if it were a single cantilever having the following inertia matrix

$$J = \sum_{i=1}^{M} J_i \tag{42}$$

If the matrix J is non-singular, Equation (41) can be inverted

$$\alpha_{(2)}^{\mathrm{IV}} = \frac{1}{E} J^{-1} F_{(2)} \tag{43}$$

which is formally analogous to Equation (36), valid for a single bracing. By substituting Equation (43) into Equation (39) now, we get

$$S_{i(2)} = J_i J^{-1} F_{(2)} \tag{44}$$

Equation (44) solves the problem of the external loading distribution between the resistant elements of a building in the case of open thin-walled, cross-sections only. It is formally analogous, as Equation (27) is, to the equation for the distribution of a force between different in-parallel resistant elements in a plane problem.

5. A GENERAL FORMULATION MERGING CARPINTERI'S AND CAPURSO'S APPROACHES

In this section, Carpinteri's approach will be extended in order to encompass as well bracings with open thin-walled cross-section. To this aim, the stiffness matrix of this kind of elements should be computed. In Capurso's approach, the inertia matrix J_f (of size 3×3; the subscript f indicates that the matrix refers to a single floor) of the i-th bracing at the generic floor is given by Equation (31); therefore, the total $3N \times 3N$-matrix of inertia of the whole i-th bracing J_i is given by

$$J_i = \begin{bmatrix} J_f & 0 & 0 & 0 & 0 \\ 0 & J_f & 0 & 0 & 0 \\ 0 & 0 & \ddots & 0 & 0 \\ 0 & 0 & 0 & J_f & 0 \\ 0 & 0 & 0 & 0 & J_f \end{bmatrix}. \tag{45}$$

Before proceeding, we must recall that the order of the DOFs is different in the two approaches. In Carpinteri's algorithm, the DOFs are collected in the $3N$-vector $\alpha_{(1)}$ by considering first the displacements in X-direction, then the displacements in the Y-direction and eventually the rotations (see Figure 3). In Capurso's algorithm, the order in the vector $\alpha_{(2)}$ is different, as the DOFs at each floor are grouped together in triplets (see Figure 5). Therefore, to pass from one ordering convention to the other, a transformation matrix Z is needed:

$$\alpha_{(1)} = \begin{Bmatrix} \xi_1 \\ \xi_2 \\ \cdots \\ \xi_N \\ \eta_1 \\ \eta_2 \\ \cdots \\ \eta_N \\ \vartheta_1 \\ \vartheta_2 \\ \cdots \\ \vartheta_N \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & \cdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & \cdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \xi_1 \\ \eta_1 \\ \vartheta_1 \\ \xi_2 \\ \eta_2 \\ \vartheta_2 \\ \cdots \\ \eta_{N-1} \\ \vartheta_{N-1} \\ \xi_N \\ \eta_N \\ \vartheta_N \end{Bmatrix} = Z\alpha_{(2)}. \tag{46}$$

An analogous relation can be written for the vector of static actions, as in Carpinteri's approach the loads on the i-th bracing in the local coordinate system are collected in the $3N$-vector $S_{i(1)}$ by considering first the forces in X-direction, then the forces in the Y-direction and eventually the torsions; in Capurso's algorithm, the order in the vector $S_{i(2)}$ is different, as the DOFs at each floor are grouped together in triplets:

$$S_{i(1)} = ZS_{i(2)} \tag{47}$$

Equation (39) can be rewritten with the order adopted by Carpinteri; to this aim, Equations (46) and (47) have to be substituted into Equation (39), and taking into account Equation (18) we obtain

$$Z^{-1}S_{i(1)} = EJ_iZ^{-1}T_i\alpha_{(1)}^{\mathrm{IV}} \tag{48}$$

Pre-multiplying the preceding expression by the transformation matrix Z we get

$$S_{i(1)} = E\left(ZJ_iZ^{-1}\right)T_i\alpha_{(1)}^{\mathrm{IV}} \tag{49}$$

and it can be easily shown that the reordered inertia matrix $\overline{J}_i = ZJ_iZ^{-1}$ is given by nine diagonal sub-matrices of dimension $N \times N$:

$$\overline{J_i} = ZJ_iZ^{-1} = \begin{bmatrix} J_{\mathrm{yy}} & J_{\mathrm{xy}} & J_{\mathrm{y}\omega} \\ J_{\mathrm{xy}} & J_{\mathrm{xx}} & J_{\mathrm{x}\omega} \\ J_{\mathrm{y}\omega} & J_{\mathrm{x}\omega} & J_{\omega\omega} \end{bmatrix}. \tag{50}$$

Considering the inverse of Equation (49) and integrating it with the boundary conditions already detailed (Equations (37) and (38)), we obtain the relation between the displacements $\alpha_{i(1)}$ and the loads $S_{i(1)}$; therefore, the compliance matrix C_i of the i-th bracing

$$\alpha_{(1)} = \frac{1}{E}T_i^{-1}\overline{J}_i^{-1}L^3\overline{Q}S_{i(1)} \tag{51}$$

where the matrix $\overline{Q}$ is a $3N \times 3N$-matrix of non-dimensional influence coefficients determined through the integration, and L is the storey height. Its structure is block-diagonal, with three equal (full) sub-matrices, Q:

$$\overline{Q} = \begin{bmatrix} Q & 0 & 0 \\ 0 & Q & 0 \\ 0 & 0 & Q \end{bmatrix} \tag{52}$$

In case of a bracing with constant cross-section, the computation of the terms of the $N \times N$ sub-matrix Q provides the generic term q_{ij}

$$q_{ij} = \frac{1}{6}(N - j + 1)^2\,(2N + j - 3i + 2). \tag{53}$$

In the case of bracings with variable cross-section, the computation can be performed by means of the Mohr's theorem.

By inverting the compliance matrix C_i we finally get the expression of the stiffness matrix K_i of the i-th bracing with open thin-walled cross-section:

$$K_i = C_i^{-1} = \frac{E}{L^3}\overline{Q}^{-1}\overline{J}_iT_i. \tag{54}$$

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6. NUMERICAL EXAMPLE

The numerical example chosen to show the flexibility and effectiveness of the proposed approach is presented in this section. The structure is an asymmetric 20-storey tube-in-tube system with square plane layout. The asymmetry is chosen in order to investigate both the flexural and the torsional behaviour of the building when subjected to horizontal wind loads. As shown in Figure 6, the internal core is closed, whereas the external tube is made by two bracings with 'C'-shaped open cross-sections. The Young's modulus is $E = 2{\cdot}4 \times 10^4$ MPa, the Poisson ratio $v = 0{\cdot}1$. The story height is $H = 3{\cdot}5$ m, corresponding to a total height of $L = 70$ m. To evaluate the effect of the building height on the structural behaviour, we also consider smaller buildings, with 10 and 5 storeys, corresponding to $L = 35$ m and $L = 17{\cdot}5$ m, respectively. The member cross-section properties are given in Table 1.

Concerning the loads, in this example, we consider the actions of wind only. For the sake of simplicity, we assume constant wind pressure over the height of the structure. Because of the hypothesis of infinite rigidity of the floors in their plane, the wind actions can be applied as a system of concentrated horizontal loads passing through the barycentre of the pressure distribution. Because the shape of the building plane is squared, the actions in the X and Y directions are equal. The intensity of wind actions is computed in a simplified way by considering the reference kinetic pressure provided by the Italian Technical Regulations (Ministero dei Lavori Pubblici, 1996), that adopt the same models of

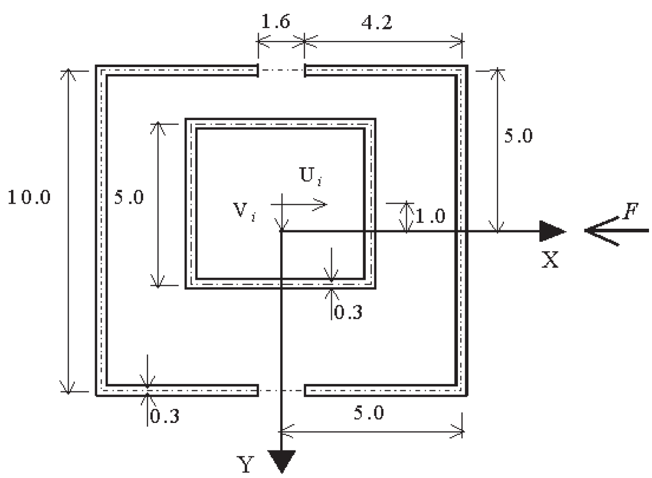

Figure 6. Floor plane of the example building. Tube-in-tube structure with asymmetric core

Table 1. Cross-section properties of the open thin-walled core and of the external tube

	Core	'C'-shaped bracings
Second moment J_{xx} (m^4)	25·09	88·00
Second moment J_{yy} (m^4)	25·09	43·21
Warping moment $J_{\omega\omega}$ (m^6)	0·00	171·54
Torsional rigidity (à la de Saint Venant (m^4))	37·50	0·166
Global coordinate x_c of the shear centre (m)	0·00	13·27
Global coordinate y_c of the shear centre (m)	−1·00	±6·50
Angle ω (rad)	0·00	0·00

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wind actions contained in the Eurocode 1 (European Committee for Standardization, 2002), in which the wind actions are supposed to be static and directed according to the principal axes of the structure. According to the Eurocode 1, the reference kinetic pressure is a function of the wind velocity v_{ref}:

$$q_{ref} = \frac{v_{ref}^2}{1.6} 10^{-3} \left[kN/m^2\right]. \tag{55}$$

The reference velocity of wind is, in turn, a function of the region and of the altitude; in this example, we suppose that the building is located in Turin (Piedmont, Italy). From the regulations, we get v_{ref} = 25 m/s and therefore q_{ref} = 390·62 N/m^2. For the sake of simplicity, we do not consider the exposition-, shape- and dynamic-coefficients (European Committee for Standardization, 2002) that vary depending on several parameters (roughness, shape of the building, topography . . .). The obtained concentrated loads have the following values: F = 13·67 kN, except at the top floors of the three buildings where the loads are halved. In the computations, the wind force is applied in the X-direction only.

Results are summarized in Figures 7–11. In Figure 7(a) and (b), displacements in the X-direction and rotations at the floor levels are reported, respectively; as can be seen, the flexural deformed shape does not display change of sign of the curvature, whereas rotations do. An inflection point is clearly visible at the level of floor 8 in Figure 7(b). This fact is in tight connection with the diagrams of the torsion moment M_z, reported in Figure 8(a) and (b) for the internal core and the external 'C'-shaped bracings, respectively. In both plots, it can be clearly seen that the primary part M_t of the torsional moment displays a maximum at the same height. Regarding the internal closed core, it obviously supports a larger part of the total moment and presents a null warping moment M_ω. On the other hand, the 'C'-shaped cross-sections display smaller primary moment and larger warping moment M_ω. The

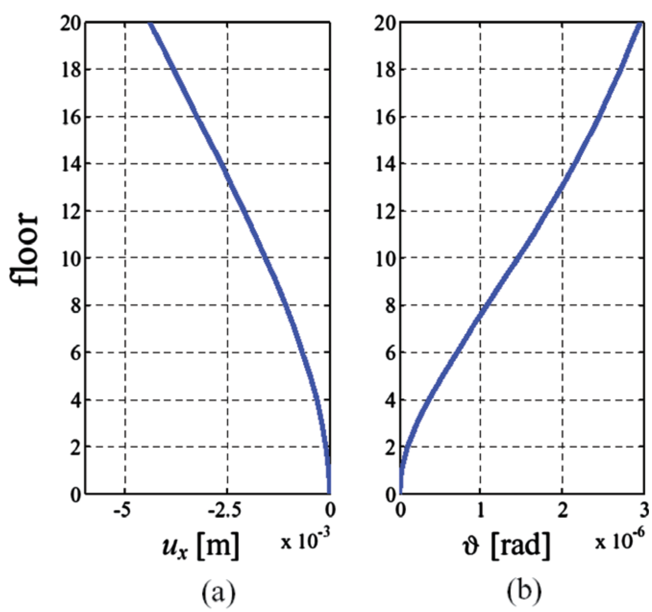

Figure 7. Building with N = 20 storeys: displacements of the floors in the global coordinate system. Translation in the X-direction (a) and rotation (b)

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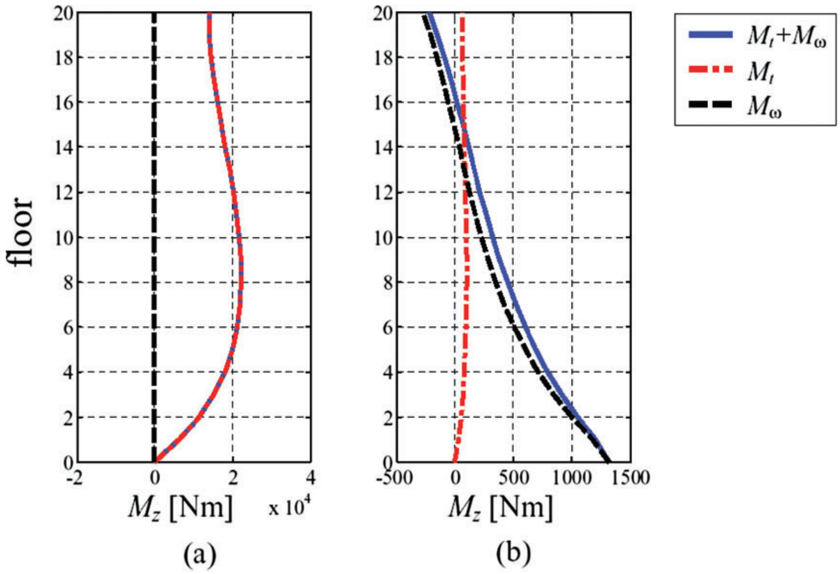

Figure 8. Building with N = 20 storeys: torsion of the internal core (a) and of the external ‘C’-shaped bracings (b) along the building height. M_t and M_ω represent the primary and warping moment, respectively

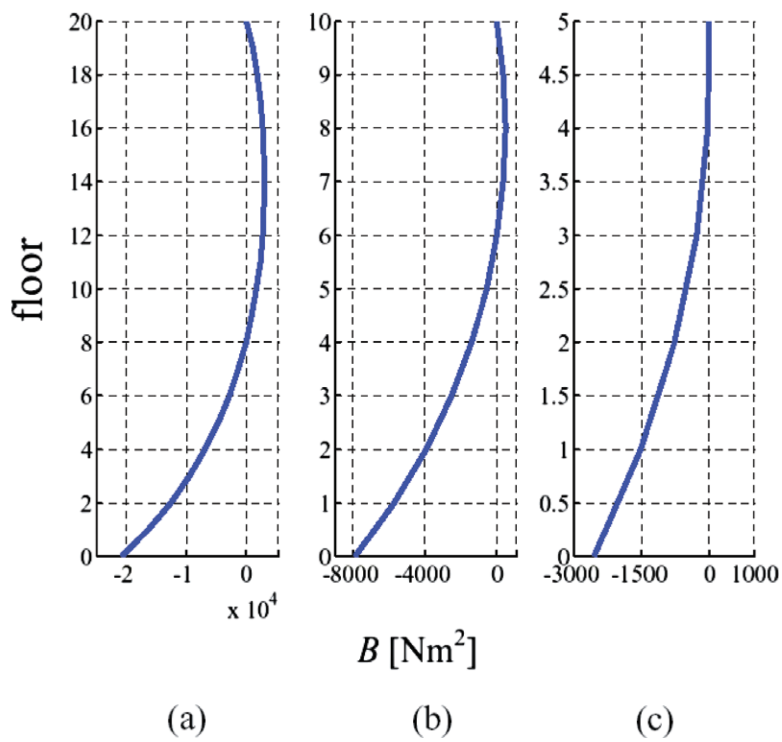

Figure 9. Bimoments in the external ‘C’-shaped bracings along the building height: N = 20 (a) N = 10 (b) N = 5 (c)

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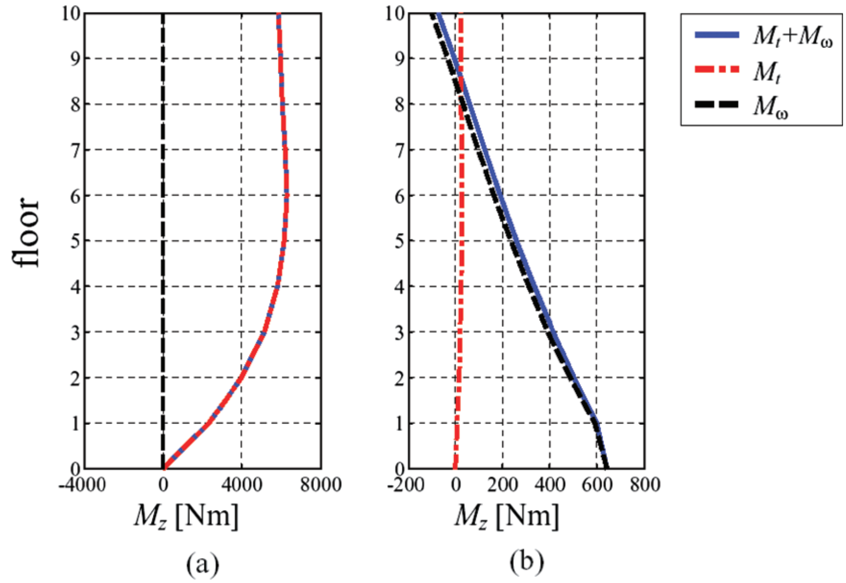

Figure 10. Building with $N = 10$ storeys: Torsion of the internal core (a) and of the external 'C'-shaped bracings (b) along the building height. M_t and M_ω represent the primary and warping moment, respectively

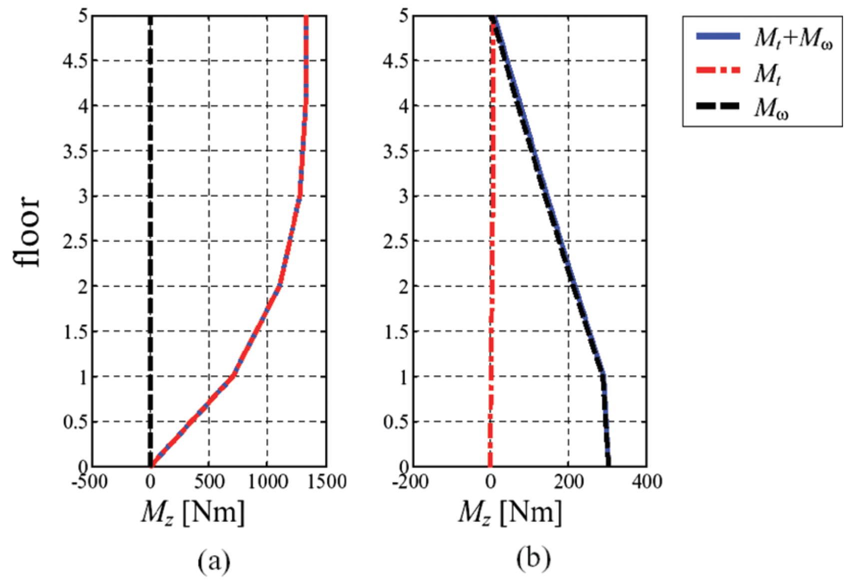

Figure 11. Building with $N = 5$ storeys: Torsion of the internal core (a) and of the external 'C'-shaped bracings (b) along the building height. M_t and M_ω represent the primary and warping moment, respectively

latter changes its sign from the top of the building (where it is negative) to the ground, and shows a faster rate of increase below the floor 8. The primary moment is negligible with respect to the warping one in the external bracings. This is a consequence of the fact that the internal core is closed and can sustain large torsional loads, thus influencing the behaviour of the other bracings. All this can be confirmed if we look at the bimoment *B* in the external ‘C’-shaped bracings (plotted in Figure 9(a)): we can see that the sign changes at the level of floor 8, being positive above it. As expected, the maximum value is at the ground floor, and the bimoment is null at the top of the building.

Concerning the effect of the building height, looking at Figures 10 and 11, which refer to the buildings with 10 and 5 storeys, respectively, we can see that, by reducing the height, the maximum of the primary moment shifts towards the top of the building and eventually reaches it in the smallest building (see Figure 11).

7. CONCLUSIONS

The numerical algorithm for the lateral loading distribution between the elements of a three-dimensional civil structure (Carpinteri and Carpinteri, 1985), extended in this paper by introducing thin-walled bracing elements with open cross-section, can be employed to predict the gross structural deformations of tall buildings with different structural typologies, i.e. composed of any kind of bracings (frames, framed walls, shear walls, closed and/or open thin-walled cores and tubes). The general formulation presented in this paper offers, compared with a detailed FE simulation, ease of use and reduced effort in preparing the model, as well as in the result interpretation, with sufficient accuracy in the preliminary and conceptual design stage. In addition, such a global approach provides the structural engineer with a clear picture of the structural behaviour and a deeper insight into the key structural parameters governing the tall building behaviour.

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