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A fractal interpretation of size-scale effects on strength, friction and fracture energy of faults

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Abstract

Experimental results indicate that large faults involved in earthquakes possess low strength, low friction coefficient and high fracture energy, in comparison with data obtained according to small scale laboratory tests on the same material. The reasons for such an unexpected anomalous behaviour have been the subject of several researches in the past and are still under debate in the Scientific Community. In this note, we propose a unifying interpretation of these size-scale effects according to fractal geometry, which represents the proper mathematical framework for the analysis of the multi-scale properties of rough surfaces in contact. This contribution sheds a new light on the non-linear properties of friction and on the understanding the fundamental physics governing the scaling of the mechanical properties in geophysics from the laboratory to a planetary scale.

1. Introduction

Over several tens of years, enormous elastic strains develop sometimes within the Earth's crust due to frictional sticking at the moving tectonic plate boundaries. When slip occurs between the crust and the tectonic plate, these stored elastic energies are suddenly released, causing damages during earthquakes [1].

While the motion of the tectonic plate is surely an observed fact, and the stick-slip process should be a major ingredient of any realistic model of earthquake, another established fact regarding the fault geometry is the multi-scale "fractal" nature of the roughness of the surfaces of the Earth's crust and the tectonic plate [2–4]. In fact, the surfaces involved in the process are the results of large scale fracture separating the crust from the moving tectonic plate. Hence, such rough surfaces obey the fundamental properties of self-affine fractals [5,6] and recent studies on the earthquake dynamics have already pointed out that the fracture mechanics of the stressed crust of the Earth forms self-affine fault patterns, with well-defined fractal dimensions of the contact areas of the major plates [7,8].

Since the times of Newton, an essential hypothesis which is put forward in the description of natural physics is that of differentiability. Smooth euclidean shapes have been adopted in almost all modellizations of the physical world. This hypothesis allowed physicists to write the equations of physics in terms of differential equations. The possibility of associating gradients and curvatures to euclidean surfaces implies the smoothness (or measurability) of the sets and there-

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fore their scale-independence. However, there is no *a priori* principle which imposes the laws of physics to be differentiable. Multi-scale phenomena are nowadays successfully interpreted by means of fractal models. As a consequence, the non-integer Hausdorff dimension of the domains on which the physical quantities are defined assumes a profound significance.

In this framework, the Renormalization Group Theory introduced by Wilson [9] has profitably been applied to determine synthetic scaling laws describing the mechanical behaviour of disordered materials with fractal boundaries. Pioneering studies in this research field have concerned the analysis of size-scale effects on the tensile strength and on the fracture energy of brittle materials [10,11], and the characterization of size-scale effects in contact problems between numerically generated fractal surfaces (see e.g. [12,13]).

In this study, we briefly review some of the theoretical results previously obtained by the Authors concerning the scaling laws for the nominal normal pressure, the nominal shear strength and the nominal friction coefficient. New applications of these concepts to the analysis of the mechanical behavior of faults are provided. Then, we extend these results to the analysis of size-scale dependence of the fracture energy of faults. None of the analyzed quantities is found to be scale-independent, as also experimentally evidenced in the literature. A rather good agreement between the proposed scaling laws and the experimental data ranging from the laboratory scale up to the scale of natural faults is achieved.

2. Size-scale dependences of the nominal normal pressure and of the nominal shear strength

A straightforward interpretation of size-scale effects in contact problems between bodies with fractal boundaries can be gained as a direct consequence of the fractality of the contact domain, C. Borri-Brunetto et al. [12] have shown that, considering rough interfaces with a fixed fractal dimension, Δ, but at different resolutions, δ, the concept of area of true contact [14] is no longer able to describe consistently (that is, in a scale-independent manner) the interface interactions. In fact, in correspondence to the same closure displacement of the two surfaces in contact, the real contact area A_{r} progressively decreases with increasing the resolution, ideally tending to zero in the theoretical limit of $\delta \to 0$. This behaviour implies the lacunarity of the contact domain, and therefore the necessity of abandoning its euclidean description and moving to a fractal model, characterized by the noninteger dimension Δ_σ ($\Delta_\sigma \leqslant 2$) of the domain C. This observation suggests that larger contact domains (i.e. larger apparent areas A_0) are less dense in the euclidean sense, that is, the probability of the occurrence of large zones without contact increases with the size of the interface.

The Renormalization Group Theory introduced by Wilson [9] can be then profitably applied to determine synthetic scaling laws describing the mechanical behaviour of disordered materials with fractal boundaries [10–12,15,16]. As regards the normal contact problem, considering the applied normal load as a scale-invariant quantity, it is possible to obtain a scaling law which yields the dependence of the nominal pressure, σ_0, on the characteristic linear size of the specimen, b [12]:

$$\log_{10}\sigma_0 = \log_{10}\sigma^* - (2 - \Delta_\sigma)\log_{10}b \tag{1}$$

where σ^* is the fractal mean pressure defined by the anomalous physical dimensions $[F][L]^{-\Delta_\sigma}$, which results to be scale-independent.

Another fundamental aspect to be highlighted is the dimensional evolution of the contact domain C, which is initially very rarefied and progressively increases its density at larger loads. The total saturation of the contact domain C (or, at least, of some islands) would imply $\Delta_\sigma = 2$. This value, in real materials, can be attained only under very high normal loads. In this limit case, the size-scale effect would disappear (see Eq. (1)) and the Euclidean description would be consistent and the physical quantities would retain their usual integer dimensions.

A multiscale analysis of the domains where the shear resistance is activated was also proposed by Borri-Brunetto et al. [17] and Chiaia [18]. Repeating the same reasoning as that for the normal contact problem, they found the following scaling law yielding the dependence of the nominal tangential stress, τ_0, on the characteristic linear size of the specimen, b, (see Fig. 1a for a graphic representation of this scaling law):

$$\log_{10}\tau_0 = \log_{10}\tau^* - (2 - \Delta_\tau)\log_{10}b \tag{2}$$

In the field of rock mechanics, size-scale effects on shear strength were experimentally detected by Bandis et al. [19], who observed that the peak shear stress before sliding increases by reducing the size of the tested specimens. They casted 360–400 mm long replicas of eleven natural joint surfaces with a wide range of different roughnesses, using artificial rock material. For each of the natural joint surfaces considered, several specimens were prepared which, for practical purposes, could be considered identical. For each natural joint, a full size replica was subjected to direct shear testing under constant compressive stress. Then, another replica of the same joint was sawn into four parts with each part being

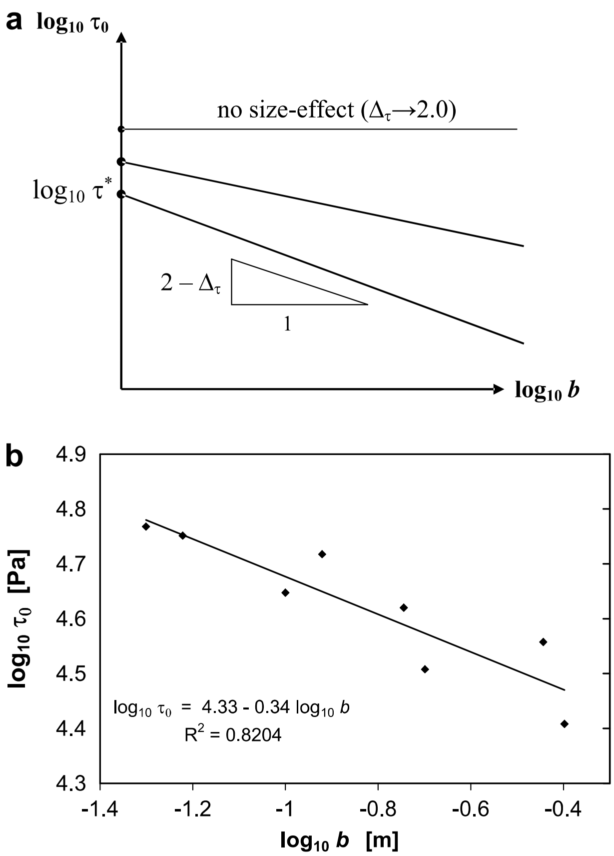

Fig. 1. Size-scale effects on the nominal shear strength: (a) Schematic representation of Eq. (2). (b) Experimental assessment of Eq. (2) (data by Bandis et al. [19]).

subjected to shear testing under the same nominal compressive stress. On the remaining samples, further subdivisions were created and tested. Plotting the experimentally determined values of the peak shear stress vs. the characteristic specimen size, the fractal dimension of the contact domain where the shear strength is activated can be computed and it turns out to be equal to $\Delta_\tau \cong 1.66$ (see the regression curve in Fig. 1b and Eq. (2)).

3. Size-scale dependence of the nominal friction coefficient

A satisfactory understanding of how large is the friction resistance of faults during earthquakes is one of the major research topics in physics and has enormous implications for the dynamics of seismic rupture. For about 20 years, engineers and geophysicists have been very divided about the fundamental question on the magnitude of the shear stress resisting slip along the major faults, like the San Andreas fault in southern California. In fact, although recent in situ experimental results indicate that these faults support a low frictional strength [20], these observations are in contrast with the values of the friction coefficient determined at the laboratory scale [21]. Currently, a possible explanation of this phenomenon has been attributed to either the slip-weakening effect [22,23], or to a rate- and state-dependent friction law [24,25]. More recently, melt lubrication has been indicated as a possible cause of low frictional strength [26], although increases in heat flow supporting this hypothesis have not been found near active faults [27].

According to the fractal analysis of the contact domain previously summarized, a fractal friction coefficient, f^*, which takes into account the dimensional disparity between normal and tangential stresses and represents the scale-invariant property of the interface, can be introduced to explain the size-scale effects on the nominal friction coefficient [13]. Postulating a fractal Coulomb law to link the fractal normal and tangential stresses, the following scaling law can be deduced (see Fig. 2a for a graphic representation of this scaling law):

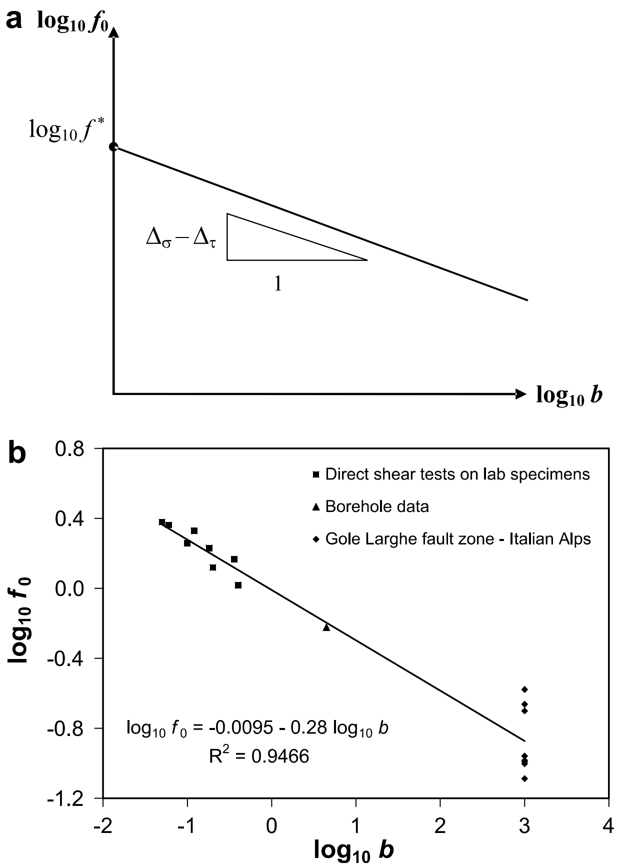

Fig. 2. Size-scale effects on the nominal friction coefficient: (a) Schematic representation of Eq. (3). (b) Experimental assessment of Eq. (3).

$$\log_{10} f_0 = \log_{10} f^* - (\Delta_\sigma - \Delta_\tau)\log_{10} b \tag{3}$$

where the difference $(\Delta_\sigma - \Delta_\tau)$ is generally a positive quantity.

This scaling law can be profitably applied to interpret the phenomenon of size-scale effect on the friction coefficient. More specifically, shear test data by Bandis et al. [19] can be considered for the laboratory scale (b ranging from 50 to 400 mm), whereas the natural data by Di Toro et al. [26,28] concerning the Gole–Larghe fault zone in the Italian Alps are assumed to be representative of large faults ($b \sim 1$ km). Borehole measures performed by Brudy et al. [29] and Zoback et al. [30] can also be included in the analysis to characterize the intermediate scale range (b ranging from 2 to 10 m). The log-log plot of the friction coefficient vs. the characteristic linear size, b, is reported in Fig. 2b, clearly showing the aforementioned decrease in the friction coefficient revealed in large faults. Di Toro et al. [26] suggested that melt produced by friction during earthquakes may act as a coseismic fault lubrication as evidenced in the high-velocity frictional experiments, thus reducing the friction coefficient. It has to be argued that this interpretation which completely refuses the existence of size-scale effects is highly questionable at least for two reasons: (i) increases in heat flow proving melting have not been found near active faults, and (ii) borehole data in Fig. 2b cannot be explained according to this theory. On the contrary, our proposed fractal interpretation seems to be fully consistent with the whole scale range. The difference $(\Delta_\sigma - \Delta_\tau)$ in Eq. (3) can be computed according to a best-fitting procedure on the frictional data and it turns out to be equal to 0.28. The noninteger dimension Δ_σ of the normal contact domain S should be then equal to $\Delta_\sigma = 1.94$.

4. Size-scale dependence of the nominal fracture energy

Size-scale effects on the fracture energy can also be interpreted in the framework of the slip-weakening model incorporating the size-scale effects on the peak shear stress. Slip along the fault associated to the stick-slip event is in fact usually followed by a large and rapid drop in shear stress from the peak value τ_0. Sliding continues at a lower, relatively constant residual shear stress level, τ_r, until the end of sliding [31]. This observed fault slip weakening has been recently interpreted in terms of an apparent fracture energy required for continued fault growth, or energy release rate, G. An idealized model of fault slip-weakening behaviour proposed by Ida [22] and Andrews [23], in which the decrease in shear stress proceeds linearly with increasing fault slip, is shown in Fig. 3a, where the fracture energy is given by the area of the shaded region.

The problem of scaling of G has been addressed in the past and it has been concluded that the fracture energy measured for earthquakes is several orders of magnitude larger than laboratory scale measurements [32,33]. Moreover, field data have shown that the critical fault displacement w_c in Fig. 3a is positively correlated with the fault length: $w_c \propto b^{1.5}$ according to Marrett and Allmendinger [34] and Gillespie et al. [35], and $w_c \propto b^2$ according to Watterson [36]. More recently, Scholz et al. [37] and Scholz [38] have found that the critical fault displacement increases linearly with b over seven orders of magnitude. If the peak shear stress were a scale-invariant quantity, this result would imply that the fracture energy is also linearly correlated with b, as recently suggested by Scholz [38]. However, if the power-law scaling law for the shear strength is considered according to Eq. (2), and if we assume $w_c \propto b$ according to the correlation by Scholz, then we expect the following power-law scaling for the fracture energy:

$$G \cong \frac{1}{2}(\tau_0 - \tau_r) w_c \propto b^{\Delta_\tau - 1} \tag{4}$$

Since $\Delta_\tau \cong 1.66$, we expect a positive scaling with an exponent equal to 0.66. An experimental assessment of this relationship is provided in Fig. 3b using the data collected in [33], including both laboratory and large scale measurements of fracture energy. A best-fitting analysis on these data provide an exponent equal to 0.67, in close agreement with our predicted value. It has to be remarked that this value is also in agreement with that proposed by Chambon et al. [39], who determined an exponent equal to 0.60 for rotary gouge friction experiments. A higher value equal to 1.28 was instead determined by Abercrombie and Rice [40], although their finding was based on data in the slip range from 0.2 mm to 0.2 m.

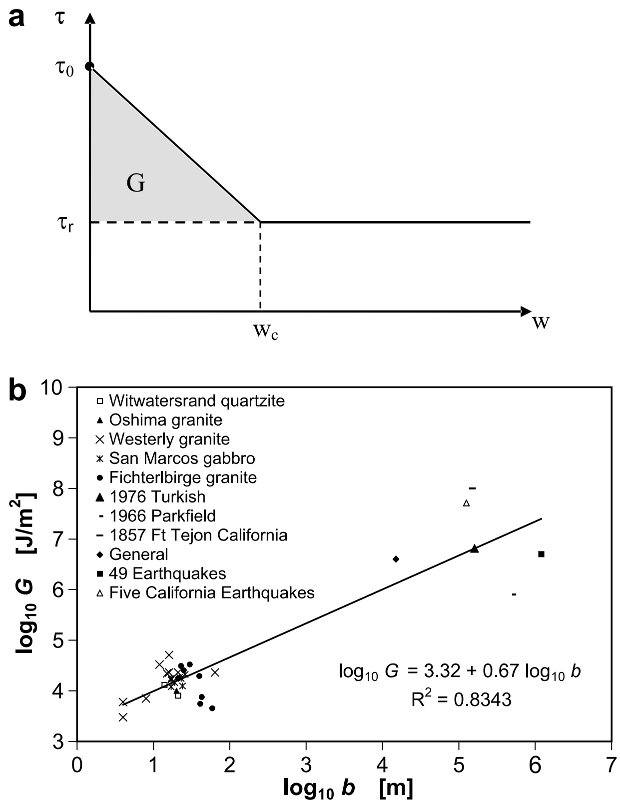

Fig. 3. Size-scale effects on the nominal fracture energy: (a) Schematic representation of the slip-weakening model. (b) Experimental assessment of Eq. (4) using the data in [33].

5. Conclusion

It is well-known that the conventional statistical methods present some limits to describe the multi-scale features of real surface geometries. For these reasons, the ability of fractal geometry to give a scale-independent description of reality has exerted a strong appeal to many researchers, who applied the fractal concepts to various branches of mechanics. Many forms of scaling invariance appear in seismic phenomena. The most impressive feature is the well-known Gutenberg–Richter law for the magnitude distribution of earthquakes. In this case, there is experimental evidence suggesting that the epicentre distribution is self-similar both in space and in time. The concept of self-organized criticality and fractal geometry are then profitably applied to capture the basic features of magnitude distribution of earthquakes [41,42].

As far as the shear strength, the friction coefficient and the fracture energy of faults are concerned, experimental observations reveal that they cannot be considered as universal, scale-independent, mechanical properties. At present, these anomalous features have not yet been described satisfactorily in a systematic manner.

Our proposed interpretation of size-scale effects on the shear strength, on the friction coefficient and on the fracture energy of faults aims at fostering the use of fractal geometry also for the analysis of the aforementioned size-scale effects. This contribution sheds a new light on the non-linear properties of friction and on the understanding the fundamental physics governing the scaling of the mechanical properties in geophysics from the laboratory to a planetary scale.

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