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Size-Scale Effects on Strength, Friction and Fracture Energy of Faults: A Unified Interpretation According to Fractal Geometry

By

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Summary

Experimental results clearly indicate that large faults involved in earthquakes possess low strength, low friction coefficient and high fracture energy, in comparison with data obtained from small scale laboratory tests on rock samples. The reasons for such an unexpected anomalous behaviour have been the subject of several studies in the past and are still under debate in the Scientific Community. In this paper we propose a unifying interpretation of these size-scale effects on the basis of fractal geometry, which represents the proper mathematical framework for the analysis of multi-scale properties of rough surfaces in contact. A rather good agreement between the proposed scaling laws and the experimental data ranging from the laboratory scale up to the planetary scale typical of natural faults is achieved.

Keywords: Faults, friction, size-scale effects, fractals, geomechanics, contact mechanics.

1. Introduction

Over several tens of years, enormous elastic strains develop sometimes within the Earth's crust during frictional sticking at moving tectonic plate boundaries. When slip occurs between the crust and the tectonic plate, this stored elastic energy is suddenly released, causing damages during earthquakes (Scholz, 2002).

While the motion of the tectonic plate is surely an observed fact, and the stick-slip process should be a major ingredient of any realistic model of earthquake, another established fact regarding the fault geometry is the multi-scale "fractal" nature of the roughness of the sliding surfaces (e.g. Brown and Scholz, 1985; Turcotte, 1989; Power and Tullis, 1991; Wang and Scholz, 1993). In fact, the surfaces involved in the process are the results of large scale fractures separating the moving crustal plates. Hence, such rough surfaces obey the fundamental properties of self-affine fractals (Mandelbrot, 1982; Mandelbrot et al., 1984) and recent studies on earthquake dynamics have already pointed out that the fracture mechanics of the stressed crust forms self-affine fault patterns with well-defined fractal dimensions of the contact areas (Barriere and Turcotte, 1991; Sahimi et al., 1992).

Since the times of Newton, an essential hypothesis which is put forward in the description of natural physics is that of differentiability. Smooth euclidean shapes have been adopted in almost all modellizations of the physical World. This hypothesis allowed physicists to write the equations of physics in terms of differential equations. The possibility of associating gradients and curvatures to euclidean surfaces implies the smoothness (or measurability) of the sets and therefore their scale-independence. However, there is no a priori principle which imposes the laws of physics to be differentiable. Multi-scale phenomena are nowadays successfully interpreted by means of fractal models. As a consequence, the non-integer Hausdorff dimension of the domains on which the physical quantities are defined assumes a profound significance.

In this framework, the *Renormalization Group Theory* introduced by Wilson (1971) has profitably been applied to determine synthetic scaling laws describing the mechanical behaviour of disordered materials with fractal boundaries. Pioneering studies in this research field have addressed the analysis of size-scale effects on tensile strength and fracture energy of brittle materials (Carpinteri, 1994a, b), and the characterization of size-scale effects in contact problems between numerically generated fractal surfaces (e.g. Borri-Brunetto et al., 1999; Carpinteri and Paggi, 2005).

Explicitly considering the fractal nature of the roughness of the plate boundary surfaces, in this study we apply the *Renormalization Group Theory* to determine the scaling laws for the nominal normal pressure, the nominal shear strength, the nominal friction coefficient, and the fracture energy of faults. None of these quantities is found to be scale-independent, as also experimentally evidenced in the literature. A rather good agreement between the proposed scaling laws and the experimental data ranging from the laboratory scale up to the scale of natural faults is achieved.

2. Size-Scale Dependence of the Nominal Normal Pressure

A straightforward interpretation of size-scale effects in contact problems between bodies with fractal boundaries can be gained as a direct consequence of the fractality of the contact domain C. Borri-Brunetto et al. (1999) have shown that, considering rough interfaces with a fixed fractal dimension, Δ, but at different resolutions, $s = 1/\delta$, the concept of area of true contact (Greenwood and Williamson, 1966) is not able to describe consistently (that is, in a scale-independent manner) the interface interactions. In fact, in correspondence to the same closure displacement of the two surfaces in contact, the real contact area A_r progressively decreases with increasing the resolution, ideally tending to zero in the theoretical limit of $\delta \to 0$ (see Fig. 1). This behaviour implies the lacunarity of the contact domain, and therefore the necessity of abandoning its euclidean description and moving to a fractal model, characterized by the noninteger dimension Δ_σ ($\Delta_\sigma \leqslant 2$) of the domain C. This observation suggests that larger contact domains (i.e. larger apparent areas A_0) are less dense in the euclidean sense, that is, the probability of the occurrence of large zones without contact increases with the size of the interface.

The *Renormalization Group Theory* introduced by Wilson (1971) can be profitably applied to determine synthetic scaling laws describing the mechanical behaviour of

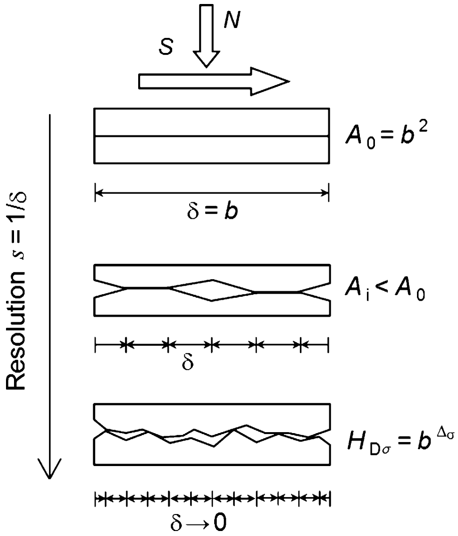

Fig. 1. Lacunarity of the contact domain observed by increasing the resolution $s = 1/\delta$. In the limit of $\delta \rightarrow 0$, the real contact area vanishes

disordered materials with fractal boundaries (Carpinteri, 1994a, b; Carpinteri and Chiaia, 1996; Borri-Brunetto et al., 1999; Carpinteri and Pugno, 2005). Regarding the normal contact problem, the following Renormalization Group can be considered for the applied normal load, N:

$$N = \sigma_0 A_0 = \sigma_1 A_1 = \cdots = \sigma_i A_i = \cdots = \sigma^* H_{D_\sigma}, \tag{1}$$

where N is considered as a scale-invariant quantity, whereas A_i and σ_i denote, respectively, the nominal contact area ($[L]^2$) and the nominal mean pressure ($[F][L]^{-2}$) measured at the i-th prefractal scale. In the theoretical limit of the highest resolution, the euclidean description looses its significance and leaves place to the Hausdorff measure H_{D_σ} of the contact domain, D_σ (defined by the noninteger dimension $[L]^{\Delta_\sigma}$, where Δ_σ is the Hausdorff dimension of the contact domain). The fractal mean pressure, σ^*, defined by the anomalous physical dimensions $[F][L]^{-\Delta_\sigma}$, results to be scale-independent. Moreover, it is possible to obtain a scaling law which yields the dependence of the nominal pressure, $\sigma_0 = N/A_0$, on the characteristic linear size of the specimen, b. By equating the second and the last term in Eq. (1) and taking the logarithm of both sides, we obtain:

$$\log(\sigma_0 A_0) = \log(\sigma^* H_{D_\sigma}), \tag{2}$$

$$\log(\sigma_0 b^2) = \log(\sigma^* b^{\Delta_\sigma}), \tag{3}$$

Equation (3) can be rearranged and the following scaling law which states the dependence of the nominal pressure, σ_0, on the characteristic linear size of the specimen, b, can be provided (see Fig. 2a for a graphic representation of this relationship):

$$\log \sigma_0 = \log \sigma^* - (2 - \Delta_\sigma) \log b. \tag{4}$$

Another fundamental aspect to be highlighted is the dimensional evolution of the contact domain C, which is initially very rarefied and progressively increases its den-

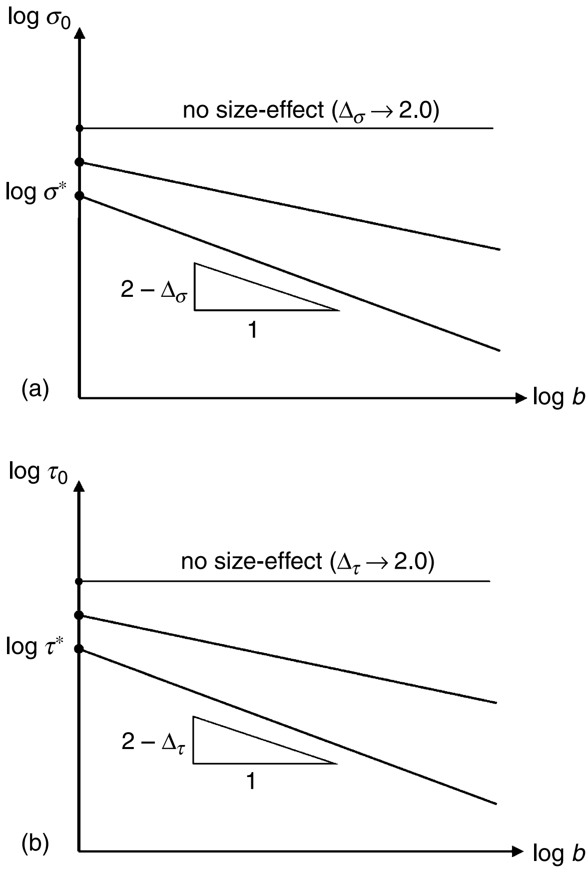

Fig. 2. Fractal scaling laws for (a) the nominal contact pressure and (b) the nominal tangential stress as functions of the characteristic structural size, b

sity at larger loads. The total saturation of the contact domain C (or, at least, of some islands) would imply $\Delta_\sigma = 2$. This value, in real materials, can be attained only under very high normal loads. In this limit case, the size-scale effect would disappear (see Eq. (4)) and the euclidean description would be consistent and the physical quantities would retain their usual integer dimensions.

3. Size-Scale Dependence of the Nominal Shear Strength

A multiscale analysis of the domains where the shear resistance is activated was also proposed by Borri-Brunetto et al. (2001) and Chiaia (2002), considering the following Renormalization Group for the applied shear load, S, which is equilibrated by the friction resistance, R:

$$S = \tau_0 A_0 = \tau_1 A_1 = \cdots = \tau_i A_i = \cdots = \tau^* H_{D_\tau}, \tag{5}$$

where S is considered as a scale-invariant quantity. In the theoretical limit of the highest resolution, the Hausdorff measure H_{D_τ} of the contact domain, D_τ, where the effective shear strength is activated, defined by the noninteger dimension $[L]^{\Delta_\tau}$, holds. The fractal shear strength, τ^*, defined by the anomalous physical dimensions $[F][L]^{-\Delta_\tau}$, is now scale-independent. By equating the second and the last term in Eq. (5) and taking the logarithm of both sides, we obtain:

$$\log(\tau_0 A_0) = \log(\tau^* H_{D_\tau}), \tag{6}$$

$$\log(\tau_0 b^2) = \log(\tau^* b^{\Delta_\tau}), \tag{7}$$

Equation (7) can be rearranged and the following scaling law which states the dependence of the nominal shear strength, τ_0, on the characteristic linear size of

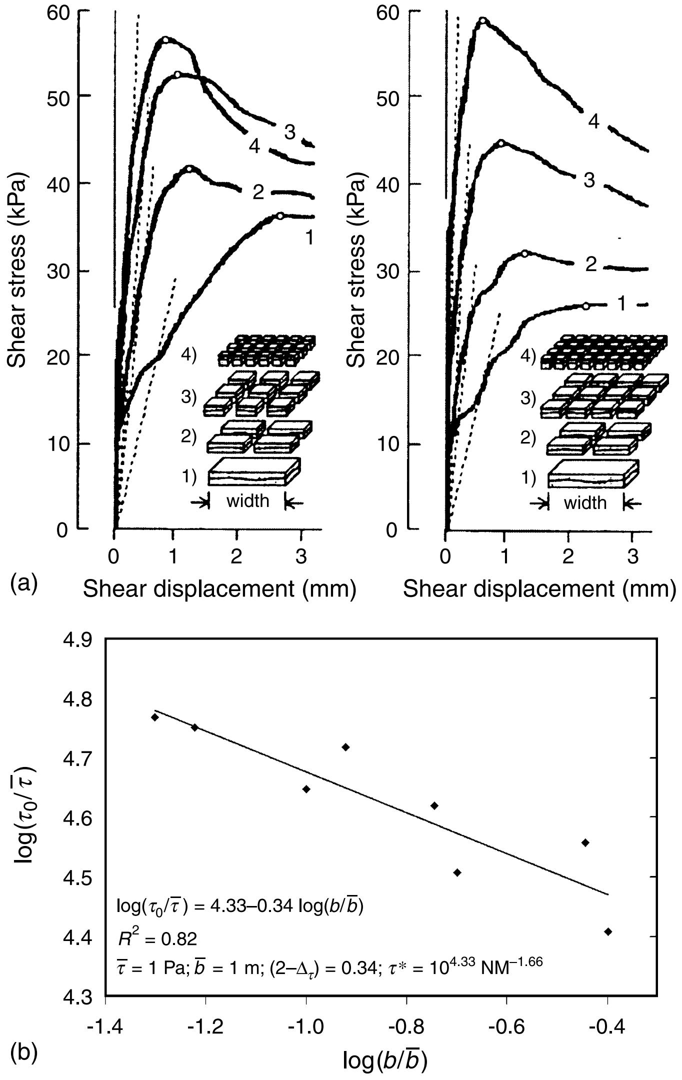

Fig. 3. Size-scale effects on the peak shear strength. (a) Results of the direct Shear tests by Bandis et al. (1981). (b) Interpretation of these experimental data according to Eq. (8)

the specimen, b, can be provided (see Fig. 2b for a graphic representation of this relationship):

$$\log \tau_0 = \log \tau^* - (2 - \Delta_\tau) \log b. \tag{8}$$

In the field of rock mechanics, size-scale effects on shear strength were experimentally detected by Bandis et al. (1981), who observed that the peak shear stress before sliding increases by reducing the size of the tested specimens. They cast 360 to 400 mm long replicas of eleven natural joint surfaces with a wide range of different roughnesses, using artificial rock material. For each of the natural joint surfaces considered, several specimens were prepared which, for practical purposes, could be considered identical. For each natural joint, a full size replica was subjected to direct shear testing under constant compressive stress. Then, another replica of the same joint was sawn into four parts with each part being subjected to shear testing under the same nominal compressive stress. On the remaining samples, further subdivisions were created and tested. Shear stress vs. shear displacement results are shown in Fig. 3a, in which the size-scale effect on the peak shear stress is clearly evidenced. Plotting the values of the peak shear stress vs. the characteristic specimen size, the fractal dimension of the contact domain where the shear strength is activated can be computed and it turns out to be equal to $\Delta_\tau \cong 1.66$ (see Fig. 3b, where $2 - \Delta_\tau \cong 0.34$).

4. Size-Scale Dependence of the Nominal Friction Coefficient

A satisfactory understanding of how large is the friction resistance of faults during earthquakes is one of the major research topics in geophysics and has enormous implications for the dynamics of seismic rupture. For about 20 years, engineers and geophysicists had different opinions about the fundamental question on the magnitude of the shear stress resisting slip along the major faults, like the San Andreas fault in southern California. In fact, although recent *in situ* experimental results indicate that these faults support a low frictional strength (Townend and Zoback, 2004), these observations are in contrast with the values of the friction coefficient determined at the laboratory scale (Lachenbruch and Sass, 1980). Currently, a possible explanation of this phenomenon has been attributed to either the slip-weakening effect (Ida, 1972; Andrews, 1976), or to a rate- and state-dependent friction law (Dieterich, 1981; Ruina, 1983). More recently, melt lubrication has been indicated as a possible cause of low frictional strength (Di Toro et al., 2006), although increases in heat flow have not been found near active faults (Lachenbruch and Sass, 1992).

According to the fractal analysis of the contact domain previously summarized, a fractal friction coefficient, f^*, which takes into account the dimensional disparity between normal and tangential tractions and represents the scale-invariant property of the interface, can be introduced to explain the size-scale effects on the nominal friction coefficient (see also Carpinteri and Paggi, 2005). Postulating a fractal Coulomb law to link the fractal normal and tangential tractions, we have:

$$\tau^* = f^* \sigma^*. \tag{9}$$

In particular, at each scale the Coulomb law can be written as follows:

$$\log f_0 = \log\left(\frac{\tau_0}{\sigma_0}\right) = \log \tau_0 - \log \sigma_0, \tag{10}$$

$$\log f^* = \log\left(\frac{\tau^*}{\sigma^*}\right) = \log \tau^* - \log \sigma^*. \tag{11}$$

By introducing Eqs. (4) and (8) into Eq. (10), we obtain the following relationship:

$$\log f_0 = \log \tau^* - (2 - \Delta_\tau) \log b - \log \sigma^* + (2 - \Delta_\sigma) \log b. \tag{12}$$

Finally, introducing Eq. (11) into Eq. (12), a scaling law which states the dependence of the apparent friction coefficient on the characteristic nominal size of the specimen, b, is provided (see Fig. 4a for a graphic representation of this scaling law):

$$\log f_0 = \log f^* - (\Delta_\sigma - \Delta_\tau) \log b, \tag{13}$$

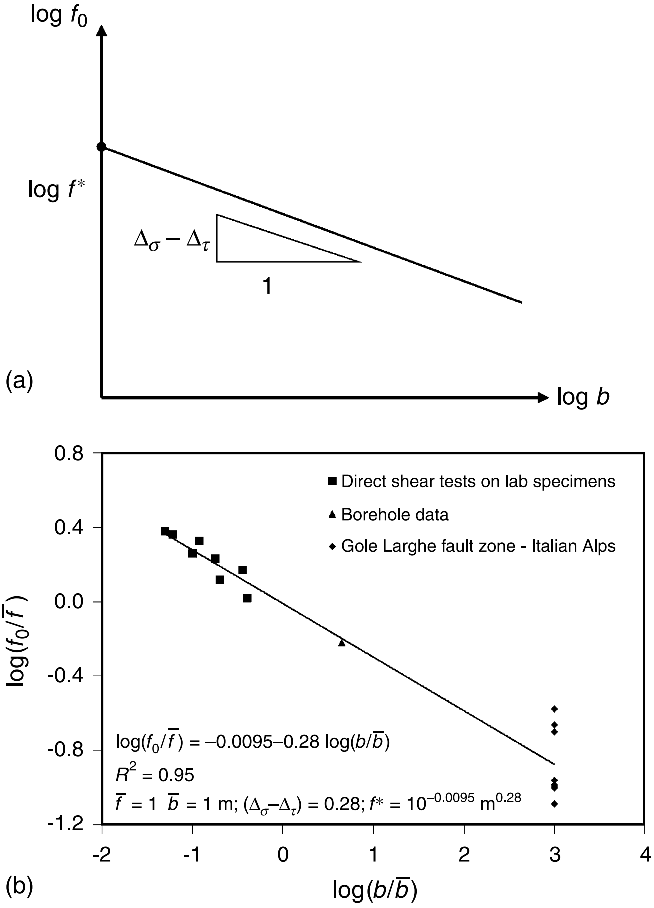

Fig. 4. Size-scale effects on the friction coefficient. (a) Schematic representation of the scaling law in Eq. (13). (b) Interpretation of experimental data according to Eq. (13)

where the difference $(\Delta_\sigma - \Delta_\tau)$ is generally a positive quantity. The fractal friction coefficient, f^*, defined by the anomalous physical dimensions $[L]^{\Delta_\sigma - \Delta_\tau}$, is a scale-independent quantity.

This scaling law can be profitably applied to interpret the phenomenon of size-scale effect on the friction coefficient. More specifically, shear test data by Bandis et al. (1981) can be considered for the laboratory scale (b ranging from 50 to 400 mm), whereas the natural data by Di Toro et al. (2005, 2006) for the Gole-Larghe fault zone in the Italian Alps are assumed to be representative of large faults ($b \sim 1$ km). Borehole measures performed by Brudy et al. (1997) and Zoback et al. (1993) can also be included in the analysis to characterize the intermediate scale range (b ranging from 2 to 10 m). The log–log plot of the friction coefficient vs. the characteristic linear size, b, is reported in Fig. 4b, clearly showing the aforementioned decrease in the friction coefficient revealed in large faults.

Di Toro et al. (2006) suggested that melt produced by friction during earthquakes may act as a coseismic fault lubrication as evidenced in the high-velocity frictional experiments, thus reducing the friction coefficient. It has to be argued that this interpretation which completely refuses the existence of size-scale effects is highly questionable at least for two reasons: (i) increases in heat flow proving melting have not been found near active faults and (ii) borehole data in Fig. 4b cannot be explained according to this theory. On the contrary, our proposed fractal interpretation seems to be fully consistent with the whole scaling range. The difference $(\Delta_\sigma - \Delta_\tau)$ in Eq. (13) can be computed according to a best-fitting procedure on the frictional data and it turns out to be equal to 0.28 (see Fig. 4b). The noninteger dimension Δ_σ of the normal contact domain C should be then equal to $\Delta_\sigma = 1.94$, since from Fig. 3b we know that $\Delta_\tau \cong 1.66$.

5. Size-Scale Dependence of the Nominal Fracture Energy

Size-scale effects on the fracture energy can also be interpreted in the framework of the slip-weakening model incorporating the size-scale effects on the peak shear stress. Slip along the fault associated to the stick-slip event is in fact usually followed by a large and rapid drop in shear stress from the peak value τ_0. Sliding continues at a lower, relatively constant residual shear stress level, τ_r, until the end of sliding (Okubo and Dieterich, 1984). This observed fault slip weakening has been recently interpreted in terms of an apparent fracture energy required for continued fault growth, or energy release rate, G. An idealized model of fault slip-weakening behaviour proposed by Ida (1972) and Andrews (1976), in which the decrease in shear stress proceeds linearly with increasing fault slip, is shown in Fig. 5a, where the fracture energy is given by the area of the shaded region.

The problem of scaling of G has been addressed in the past and it has been concluded that the fracture energy measured for earthquakes is several orders of magnitude larger than laboratory scale measurements (e.g. Chester et al., 2005; Li, 1987). Moreover, field data have shown that the critical fault displacement w_c in Fig. 5a is positively correlated with the fault length: $w_c \propto b^{1.5}$ according to Marrett and Allmendinger (1991) and Gillespie et al. (1992), and $w_c \propto b^2$ according to Watterson (1986). More recently, Scholz et al. (1993) and Scholz (2005) have found

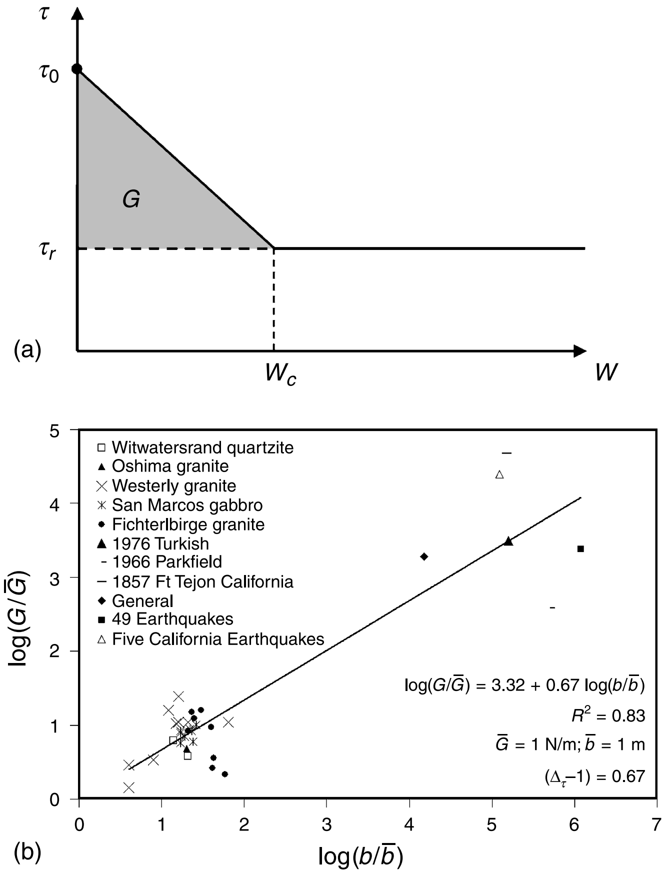

Fig. 5. Size-scale effects on the fracture energy. (a) Schematic representation of the slip-weakening model. (b) Interpretation of the experimental data in Li (1987) according to Eq. (14)

that the critical fault displacement increases linearly with b over seven orders of magnitude. If the peak shear stress were a scale-invariant quantity, this result would imply that the fracture energy is also linearly correlated with b, as recently suggested by Scholz (2005). However, if the power-law scaling relationship for the shear strength is considered according to Eq. (8), and if we assume $w_c \propto b$ according to the correlation by Scholz, then we expect the following power-law scaling for the fracture energy:

$$G \cong \frac{1}{2}(\tau_0 - \tau_r) w_c \propto b^{\Delta_\tau - 1}. \tag{14}$$

Since $\Delta_\tau \cong 1.66$, we expect a positive scaling with an exponent equal to 0.66. An experimental assessment of this relationship is provided in Fig. 5b using the data collected in Li (1987), including both laboratory and large scale measurements of fracture energy. A regression analysis on these data provides an exponent equal to 0.67, in close agreement with our predicted value. It has to be remarked that this value is also in agreement with that proposed by Chambon et al. (2005), who determined an exponent equal to 0.60 for rotary gouge friction experiments. A higher value equal to

1.28 was instead determined by Abercrombie and Rice (2005), although their finding was based on data in the slip range from 0.2 mm to 0.2 m.

6. Conclusion

It is well-known that the conventional statistical methods present some limits to describe the multi-scale features of real surface geometries. For these reasons, the ability of fractal geometry to give a scale-independent description of reality has exerted a strong appeal to many researchers, who applied the fractal concepts to various branches of mechanics. Many forms of scaling invariance appear in seismic phenomena. The most impressive feature is the well-known Gutenberg-Richter law for the magnitude distribution of earthquakes. In this case there is experimental evidence suggesting that the epicentre distribution is self-similar both in space and in time. The concept of self-organized criticality and fractal geometry are then profitably applied to capture the basic features of magnitude distribution of earthquakes (e.g. Hallgass et al., 1997; Chakrabarti and Stinchcombe, 1999).

As far as the shear strength, the friction coefficient and the fracture energy of faults concern, experimental observations reveal that they cannot be considered as universal, scale-independent, mechanical properties. At present, these anomalous features have not yet been described satisfactorily in a systematic manner.

Our proposed interpretation of size-scale effects on the shear strength, on the friction coefficient and on the fracture energy of faults aims at fostering the use of fractal geometry also for the analysis of the aforementioned size-scale effects. This contribution sheds a new light on the non-linear properties of friction and on the understanding the fundamental physics governing the scaling of the mechanical properties in geophysics from the laboratory to a planetary scale.

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References

Abercrombie, R. E., Rice, J. R. (2005): Can observations of earthquake scaling constrain slip weakening? Geophys. J. Int. 162, 406–424.

Andrews, D. J. (1976): Rupture velocity of plane-strain cracks. J. Geophys. Res. 81, 5679–5687.

Bandis, S., Lumsden, A. C., Barton, N. R. (1981): Experimental studies of scale effects on the shear behaviour of rock joints. Int. J. Rock Mech. Min. Sci. Geomech. Abstr. 18, 1–21.

Barriere, B., Turcotte, D. L. (1991): A scale-invariant cellular automata model for distributed seismicity. Geophys. Res. Lett. 18, 2011–2014.

Borri-Brunetto, M., Carpinteri, A., Chiaia, B. (1999): Scaling phenomena due to fractal contact in concrete and rock fractures. Int. J. Fract. 95, 221–238.

Borri-Brunetto, M., Chiaia, B., Ciavarella, M. (2001): Incipient sliding of rough surfaces in contact: a multi-scale numerical analysis. Comput. Methods Appl. Mech. Eng. 190, 6053–6073.

Brown, S. R., Scholz, C. H. (1985): Broad bandwith study of the topography of natural rock surfaces. J. Geophys. Res. 90, 12575–12582.

Brudy, M., Zoback, M. D., Fuchs, K., Rummel, F., Baumgartner, J. (1997): Estimation of the complete stress tensor to 8 km depth in the KTB scientific drill holes – implications for crustal strength. J. Geophys. Res. 102, 453–475.

Carpinteri, A. (1994a): Fractal nature of materials microstructure and size effects on apparent mechanical properties. Mech. Mater. 18, 89–101.

Carpinteri, A. (1994b): Scaling laws and renormalization groups for strength and toughness of disordered materials. Int. J. Solids Struct. 31, 291–302.

Carpinteri, A., Chiaia, B. (1996): Power scaling laws and dimensional transitions in solid mechanics. Chaos Soliton. Fract. 7, 1343–1364.

Carpinteri, A., Paggi, M. (2005): Size-scale effects on the friction coefficient. Int. J. Solids Struct. 42, 2901–2910.

Carpinteri, A., Pugno, N. (2005): Are scaling laws on strength of solids related to mechanics or to geometry? Nat. Mater. 4, 421–423.

Chakrabarti, B. K., Stinchcombe, R. B. (1999): Stick-slip statistics for two fractal surfaces: a model for earthquakes. Physica. A 270, 27–34.

Chambon, G., Schmittbuhl, J., Corfdir, A. (2005): Non-linear slip-weakening in a rotary gouge friction experiment. In: Proc. 11th Int. Conf. Fracture ICF 11, Torino, Italy.

Chester, J. S., Chester, F. M., Kronenberg, A. K. (2005): Fracture surface energy of the Punchbowl fault, San Andreas system. Nature 437, 133–136.

Chiaia, B. (2002): On the sliding instabilities at rough surfaces. J. Mech. Phys. Solids 50, 895–924.

Dieterich, J. H. (1981): Constitutive properties of faults with simulated gouge. In: Carter, N. L., Friedman, M., Logan, J. M., Stearns, D. W. (eds.), Mechanical behavior of crustal rocks. American geophysical union geophysical monograph, vol. 24. AGU, Washington, DC, 103–120.

Di Toro, G., Nielsen, S., Pennacchioni, G. (2005): Earthquake rupture dynamics frozen in exhumed ancient faults. Nature 436, 1009–1012.

Di Toro, G., Hirose, T., Nielsen, S., Pennacchioni, G., Shimamoto, T. (2006): Natural and experimental evidence of melt lubrication of faults during earthquakes. Science 311, 647–649.

Gillespie, P., Walsh, J. J., Watterson, J. (1992): Limitations of displacement and dimension data from single faults and the consequences for data analysis and interpretation. J. Struct. Geol. 14, 1157–1172.

Greenwood, J. A., Williamson, J. B. P. (1966): Contact of nominally flat surfaces. Proc. R. Soc. A 295, 300–308.

Hallgass, R., Loreto, V., Mazzella, O., Paladin, G., Pietronero, L. (1997): Earthquake statistics and fractal faults. Phys. Rev. E 56, 1346–1356.

Ida, Y. (1972): Cohesive force across the tip of a longitudinal shear crack and Griffith's specific surface energy. J. Geophys. Res. 77, 3796–3805.

Lachenbruch, A., Sass, J. (1980): Heat flow and energetics of the San Andreas fault zone. J. Geophys. Res. 85, 6185–6223.

Lachenbruch, A., Sass, J. (1992): Heat flow from Cajon Pass, fault strength and tectonic implications. J. Geophys. Res. 97, 4995–5015.

Li, V. C. (1987): Mechanics of shear rupture applied to earthquake zones. In: Atkison, B. (ed.) Fracture mechanics of rock. Academic Press, London, 351–428.

Mandelbrot, B. B. (1982): Fractals geometry of nature. Freeman, New York.

Mandelbrot, B. B., Passoja, D. E., Paullay, A. J. (1984): Fractal character of fracture surfaces of metals. Nature 308, 721–722.

Marrett, R. A., Allmendinger, R. W. (1991): Estimates of strain due to brittle faulting: sampling of fault populations. J. Struct. Geol. 13, 735–738.

Okubo, P. G., Dieterich, J. H. (1984): Effects of physical fault properties on frictional instabilities produced on simulated faults. J. Geophys. Res. 89, 5817–5827.

Power, W. L., Tullis, T. E. (1991): Euclidean and fractal models for the description of rock surface roughness. J. Geophys. Res. 96, 415–424.

Ruina, A. (1983): Slip instability and state variable friction laws. J. Geophys. Res. 88, 10359–10370.

Sahimi, M., Robertson, M. C., Sammis, C. G. (1992): Relations between the earthquake statistics and fault patterns, and fractals and percolation. Physica. A 191, 57–68.

Scholz, C. H. (2002): The mechanics of earthquakes and faulting, 2nd edn. Cambridge Univ. Press, Cambridge.

Scholz, C. H. (2005): The scaling of geological faults. In: Proc. 11th Int. Conf. Fracture ICF 11, Torino, Italy.

Scholz, C. H., Dawers, N. H., Yu, J.-Z., Anders, M. H. (1993): Fault growth and fault scaling laws: preliminary results. J. Geophys. Res. 98, 21951–21961.

Scholz, C. H., Mandelbrot, B. B. (1989): Fractals in geophysics. Birkhäuser Verlag, Basel.

Townend, J., Zoback, M. D. (2004): Regional tectonic stress near the San Andreas fault in central and southern California. Geophys. Res. Lett. 31, L15S11.

Turcotte, D. L. (1989): Fractals and chaos in geology and geophysics. Cambridge Univ. Press, Cambridge.

Wang, W., Scholz, C. (1993): Scaling of constitutive parameters of friction for fractal surfaces. Int. J. Rock Mech. 30, 1359–1365.

Watterson, J. (1986): Fault dimensions, displacements and growth. Pure Appl. Geophys. 124, 365–373.

Wilson, K. G. (1971): Renormalization group and critical phenomena. Phys. Rev. B 4, 3174–3205.

Zoback, M. D., Apel, R., Baumgaertner, J., Brudy, M., Emmermann, R., Engeser, B., Fuchs, K., Kessels, W., Rischmueller, H., Rummel, F., Vernik, L. (1993): Strength of continental crust and the transmission of plate-driving forces: implications of in situ stress measurements in the KTB scientific borehole. Nature 365, 633–635.

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