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Fracture mechanics and complexity sciences – Part I. Order emerging from complex systems: Fractals and renormalization group [1]

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Abstract. The so-called Complexity Sciences are a topic of fast growing interest inside the scientific community. Aim of this paper is to provide an insight into the role of complexity in the field of Materials Science and Fracture Mechanics. The paper is divided into two parts, which deal with the two opposite natural trends of composite systems: order and structure emerging from heterogeneous and random systems and the route towards instability and chaos arising from simple nonlinear rules. In this paper (Part I), the former trend will be illustrated by means of two fracture mechanics applications, whereas the latter trend will be treated in the companion paper. The first application concerns the occurrence of self-similarity and fractal patterns in material damage and deformation of heterogeneous materials, and the apparent scaling of the nominal mechanical properties of disordered materials. The second application deals with criticality in the acoustic emissions of damaged structures and with scaling in the time-to-failure.

1. Introduction

Complexity, as a discipline, generally refers to the study of large-scale systems with many interacting components, in which the overall system behaviour is qualitatively different from (and not encoded in) the behaviour of its components. Actually, researchers did not come to a definition of complexity, since it manifests itself in so many different ways. This field itself is not a single discipline, but rather a heterogeneous amalgam of different techniques of mathematics and science [1]. In fact, under the label of Complexity Sciences we comprehend a large variety of approaches: nonlinear dynamics, deterministic chaos theory, nonequilibrium thermodynamics, fractal geometry, intermediate asymptotics, complete and incomplete similarity, renormalization group theory, catastrophe theory, self-organized criticality, neural networks, cellular automata, fuzzy logic, etc.

Complex systems lie somehow in between perfect order and complete randomness – the two extreme conditions that are likely to occur only very seldom in nature – and exhibit one or more common characteristics, such as: sensitivity to initial conditions, pattern formation, spontaneous self-organization,

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emergence of cooperation, hierarchical or multiscale structure, collective properties beyond those directly contained in the parts, scale effects.

Complexity has two distinct and almost opposite meanings: the first goes back to Kolmogorov's reformulation of probability and his algorithmic theory of randomness via a measure of complexity, now referred to as Kolmogorov Complexity [1]; the second to the Shannon's studies of communication channels via his notion of information. In both cases, complexity is a synonym of disorder and lack of a structure: the more random a process is, the more complex it results to be. The second meaning of complexity refers instead to how intricate, hierarchical, structured and sophisticated a process is. In other words, complexity is in this sense an indication of how many layers of structure are embedded in the process or system.

Associated with these two almost opposite meanings, are two natural trends of composite systems, and two corresponding questions: how does order and structure emerge from large, complicated systems? And, conversely, how do randomness and chaos arise from systems with only simple constituents, whose behaviour does not directly encode randomness?

The former case is typical of all those phenomena which could be described through the concepts of scale invariance, phase transition, and with the use of power laws. They are associated with the emergence of fractal patterns and spontaneous self-organization; typical examples are earthquakes, avalanches in granular media, and material damage, as will be shown in this paper (Part I). The latter case, which will be discussed in Part II, is that of instability and bifurcations, as well as of dynamical systems showing chaotic attractors and transition to chaos: how can a simple nonlinear rule, which does not have built-in randomness or probability, generate a completely unpredictable dynamical behaviour?

2. The fractal interpretation of the size-scale effects

The first topic is concerned with the size-scale effects on the mechanical properties of heterogeneous disordered materials (Carpinteri [2,3]), that can be interpreted synthetically through the use of fractal sets. Fractal sets are characterized by non-integer dimensions (Mandelbrot [4], Feder [5]). For instance, the dimension α of a fractal set in the plane can vary between 0 and 2. Accordingly, increasing the measure resolution, its length tends to zero if its dimension is smaller than 1 or tends to infinity if it is larger. In these cases, the length is a nominal, useless quantity, since it diverges or vanishes as the measure resolution increases. A finite measure can be achieved only using noninteger units, such as meters raised to $\alpha \neq 1$. Analogously, if a stress and strain localization occurs in a fractal damaged zone, the nominal quantities (ultimate strength, critical strain, fracture energy) should depend on the resolution used to measure the set where stress, strain and energy dissipation take place. In the limit of a very high measure resolution, the stress and the strain should be infinite, while the dissipated energy should be zero. Finite values can be obtained only introducing fractal quantities, i.e., mechanical quantities with non-integer physical dimensions. On the other hand, if the measure resolution is fixed, the nominal quantities undergo size effects.

Without entering the details, we wish to emphasize that the hypothesis of the fractal damage domain in quasibrittle material failure is not a mathematical abstraction, since fractal patterns have been detected in several experiments (see, for instance: Kleiser and Bocek [6], Mandelbrot et al. [7], Bouchaud et al. [8], Måloy et al. [9] and Imre et al. [10] for metals, Bouchaud et al. [11] for intermetallic alloys, Mecholsky et al. [12] and Måloy et al. [9] for ceramics, Engøy et al. [13]; Morel et al. [14] for wood, Schmittbuhl et al. [15,16] for rocks and Stroeven [17], Carpinteri and Ferro [18] and Carpinteri et al. [19,20] for

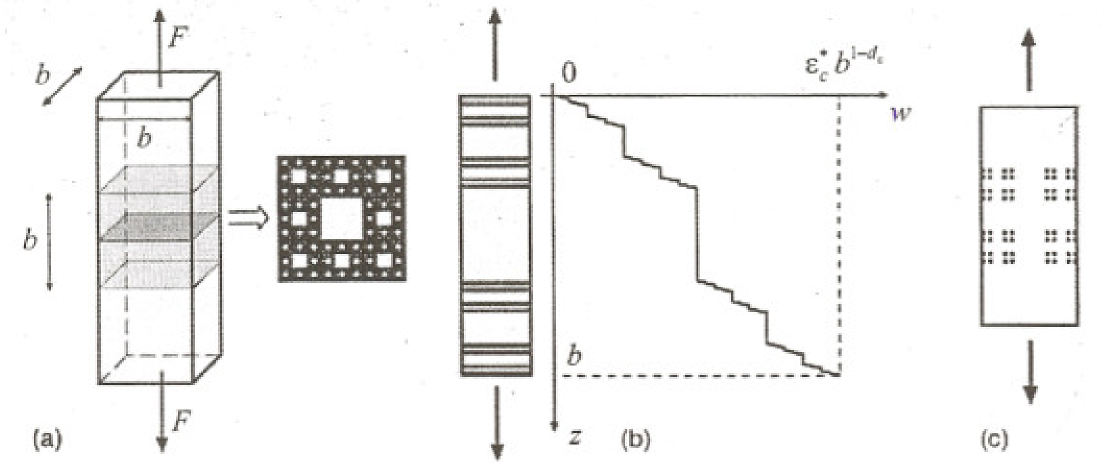

Fig. 1. A concrete specimen subjected to tension. Fractal localization of the stress upon the resistant cross section (a); fractal localization of the strain (b) and of the energy dissipation inside the damaged band (c).

concrete). Furthermore, for what concerns the concrete case, another explanation of the fractal features of the damaged zones has been recently derived from the analysis of the aggregate size distribution (Carpinteri et al. [21], Carpinteri and Cornetti [22]). Since the flaw distribution in quasi-brittle materials is often self-similar (i.e., it looks the same at different magnification levels), the microstructure can be correctly modelled by fractal sets.

In other words, the complex phenomena occurring at different scales of the random material microstructure in heterogeneous and disordered materials finds a simple and synthetic interpretation in fractal sets. The assumption of fractality and self-similarity in deformation and damage of heterogeneous materials is thus the key for interpreting the apparent scaling on the nominal mechanical properties of disordered materials.

Let us start analyzing the size effect upon the tensile strength and considering a concrete specimen subjected to tension (Fig. 1a). A consistent modelling of damage in concrete can be achieved by assuming that the rarefied resisting sections in correspondence of the critical load can be represented by stochastic lacunar fractal sets with dimension $2 - d_\sigma$ ($d_\sigma \geqslant 0$) [2,3,20,22].

From fractal geometry, we know that the area of lacunar sets is scale-dependent and tends to zero as the resolution increases: the tensile strength should be infinite, which is meaningless. The assumption of euclidean domain characterizing the classical continuum theory states that the maximum load F is given by the product of the strength σ_u times the nominal area $A_0 = b^2$, whereas, in our model, F equals the product of the (Hausdorff) fractal measure $A^*_{\rm res} = b^{(2-d_\sigma)}$ of the lacunar fractal times the fractal tensile strength σ_u (Carpinteri [2]):

$$F = \sigma_u A_0 = \sigma^*_u A^*_{\rm res}, \tag{1}$$

where σ^*_u presents the anomalous physical dimensions $[F][L]^{(2-d_\sigma)}$. The fractal tensile strength is the true material constant, i.e., it is scale-invariant. From Eq. (1) we obtain the scaling law for tensile strength:

$$\sigma_u = \sigma^*_u b^{-d_\sigma}, \tag{2}$$

i.e. a power law with negative exponent $-d_\sigma$. Equation (2) represents the negative size effect on tensile strength, experimentally revealed by several authors.

Analogously, if we turn our attention to the deformation inside the zone where damage localizes (the so-called damaged band), we can assume that the strain field presents fractal patterns. This could appear strange at a first glance; on the contrary, fractal strain distributions are rather common in material science. For instance, in some metals, the so-called slip-lines develop with typical fractal patterns (Kleiser and Bocek [6]). Fractal crack networks develop also in dry clay or in old paintings under tensile stresses due to shrinkage. Thus, in the case of a bar subjected to tension (Fig. 1b), dilation strain tends to concentrate into different softening regions, while the rest of the body undergoes elastic unloading. Assume, for instance, that the strain is localized at cross-sections whose projections onto the longitudinal axis are provided by a fractal lacunar set, whose dimension is $1 - d_\varepsilon$ ($d_\varepsilon \geqslant 0$) [21].

According to the fractal measure of the damage line projection, the total elongation w_c of the band at rupture must be given by the product of the Hausdorff measure $b^* \sim b^{(1-d_\varepsilon)}$ of the lacunar set times the critical fractal strain ε_c^*, while in the classical continuum theory it equals the product of the length b times the critical strain ε_c, where ε_c^* has the anomalous physical dimension $[\mathrm{L}]^{d_\varepsilon}$. The fractal critical strain is the true material constant, i.e. it is the only scale-invariant parameter governing the kinematics of the damaged band. On the other hand, the scaling of the critical strain is described by a power law with negative exponent $-d_\varepsilon$:

$$\varepsilon_c = \varepsilon_c^* b^{-d_\varepsilon}. \tag{3}$$

Eventually, let us consider the work W necessary to break a concrete specimen of cross section b^2 (Fig. 1c). It is equal to the product of the fracture energy $\mathcal{G}_F$ times the nominal fracture area $A_0 = b^2$. On the other hand, the surface where energy is dissipated is not a flat cross-section: it is a crack surface, whose area A_{dis}^* diverges as the measure resolution tends to infinity because of its roughness at any scale. Therefore, the fracture energy should be zero, which is physically meaningless. Finite values of the measure of the set where energy is dissipated can be achieved only via non-integer fractal dimensions.

The fractal dimension of this invasive domain is $2 + d_\mathcal{G}$ ($d_\mathcal{G} \geqslant 0$). The classical cohesive crack model states that the failure work W is given by the product of the fracture energy $\mathcal{G}_F$ times the nominal area $A_0 = b^2$, whereas, in the present model, W equals the product of the fractal (Hausdorff) measure $A_{\text{dis}}^* \sim b^{2+d_\mathcal{G}}$ times the fractal fracture energy $\mathcal{G}_F^*$, which is the true scale invariant material parameter, whereas the nominal value $\mathcal{G}_F$ is subjected to a scale effect described by a positive power law, with exponent $d_\mathcal{G}$.

$$\mathcal{G}_F = \mathcal{G}_F^* b^{d_\mathcal{G}}. \tag{4}$$

The three size effect laws (2), (3) and (4) of the cohesive law parameters are not completely independent of each other. In fact, there is a relation among the scaling exponents that must be always satisfied. This means that, when two exponents are given, the third follows from the first two. In order to get this relation, the simplest path is to consider the damage domain in Fig. 1c as the cartesian product of those in Figs 1a and 1b. As a result, we obtain that the sum of the three scaling exponents is always equal to one:

$$d_\sigma + d_\varepsilon + d_\mathcal{G} = 1. \tag{5}$$

According to these definitions, we call the σ^*–ε^* diagram the fractal or scale-independent cohesive law. Contrarily to the classical cohesive law, which is experimentally sensitive to the structural size, this curve

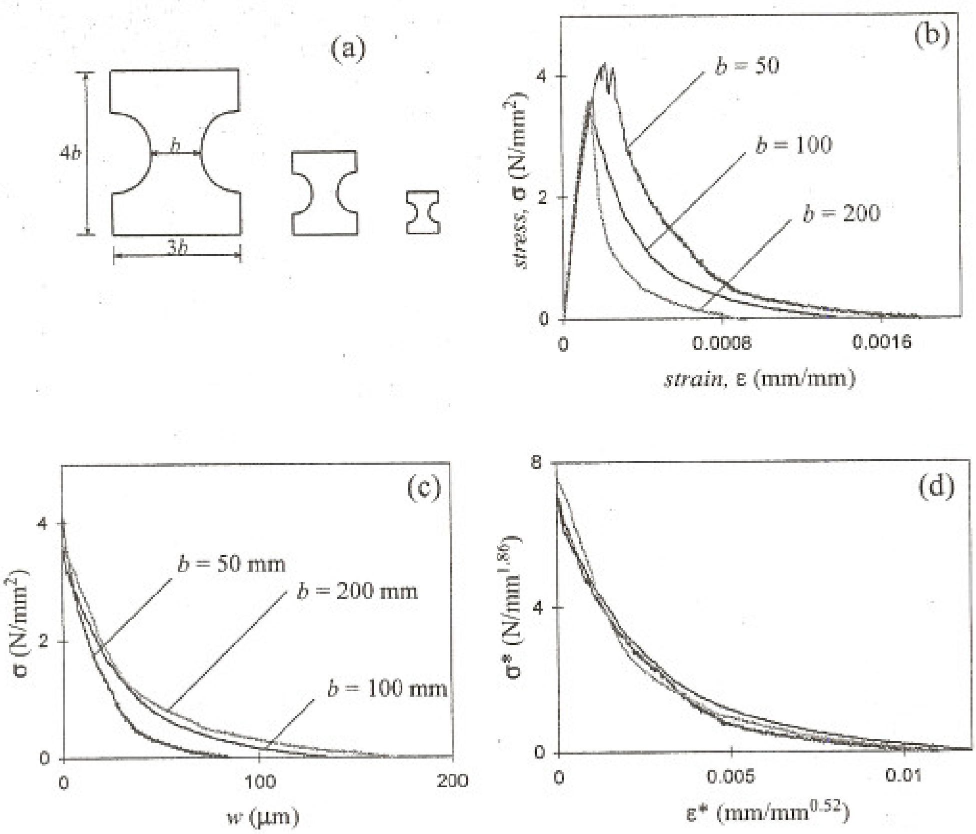

Fig. 2. Tensile tests on dog-bone shaped specimens (a) by Carpinteri and Ferro [46]: stress–strain diagrams (b), cohesive law diagrams (c), fractal cohesive law diagrams (d).

is an exclusive property of the material since it is able to capture the fractal nature of the damage process. The area below the softening fractal stress–strain diagram represents the fractal fracture energy $\mathcal{G}_F^*$.

In order to validate the model, it has been applied to the data obtained in 1994 by Carpinteri and Ferro [18,23] for tensile tests on dog-bone shaped concrete specimens of various sizes under controlled boundary conditions (Fig. 2a). They interpreted the size effects on the tensile strength and the fracture energy by fractal geometry. Fitting the experimental results, they found the values $d_\sigma = 0.14$ and $d_\mathcal{G} = 0.38$. Some of the σ–ε (stress vs. strain) and σ–w diagrams are reported respectively in Fig. 2b and 2c, where w is the displacement localized in the damaged band, obtained by subtracting, from the total one the displacement due to elastic and inelastic pre-peak deformation. Equation (5) yields $d_\varepsilon = 0.48$, so that the fractal cohesive laws can be plotted in Fig. 2d. As expected, all the curves related to the single sizes tend to merge in a unique, scale-independent cohesive law. The overlapping of the cohesive laws for the different sizes proves the soundness of the fractal approach to the interpretation of concrete size effects.

The experiments show that the fractal scaling of σ_u and $\mathcal{G}_F$ is strictly valid only in a limited scale range, where the fractal dimensions of the supporting domains can be considered to be constant. As the size increases, in fact, the concept of geometrical multifractality or self-affinity (Carpinteri [3]) implies the progressive vanishing of fractality ($d_\sigma \to 0, d_\mathcal{G} \to 0$) with a corresponding homogenization of the domains. Observe that the geometrical multifractality is strictly connected with the characteristics of self-affine fractals: its meaning differs from the one traditionally found in the literature (where multi-fractal denotes set with a spectrum of fractal dimensions, see [4]). The scaling laws previously derived

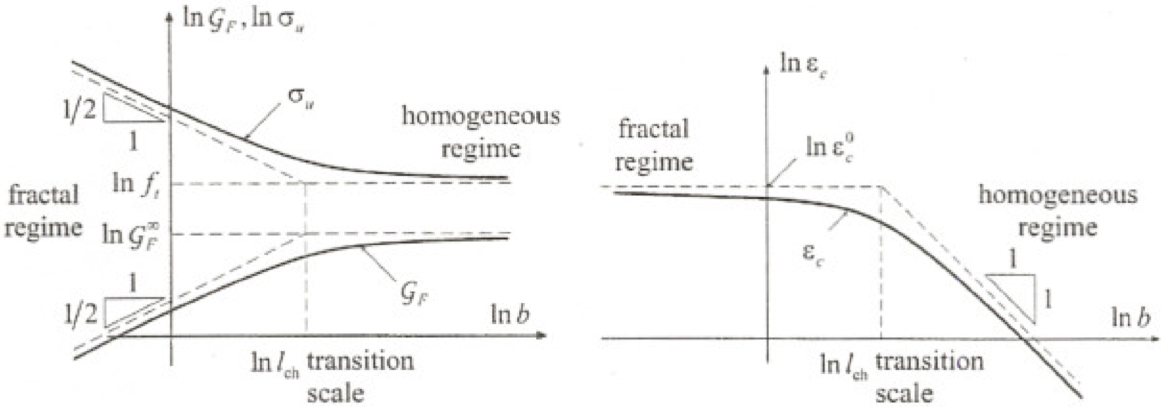

Fig. 3. Multifractal scaling laws.

have been therefore extended to the self-affine case, leading to the definition of the multifractal scaling laws [3,21,24–30].

Intuitively, since the microstructure of a disordered material is the same, independently of the macroscopic specimen size, the influence of disorder on the mechanical properties essentially depends on the ratio between a characteristic material length l_{ch} and the external size b of the specimen. Therefore, the effect of microstructural disorder on the mechanical behaviour of materials becomes progressively less important at the largest scales. At the smallest scales, Carpinteri [3] observed that a Brownian disorder seems to be the highest possible, yielding, respectively for invasive and lacunar morphologies, fractal scaling exponents equal to $+1/2$ and $-1/2$. These results have been confirmed also by a stereological analysis of the concrete microstructure; these values could be seen as an upper bound to concrete surface roughness and lacunarity and characterize the slope, in the bi-logarithmic diagram, of the scaling laws for the fracture energy and the tensile strength, respectively (see Fig. 3a).

In analytical form, these first two Multifractal scaling laws can be written as:

$$\mathcal{G}_F = \mathcal{G}_F^{\infty}\left[1 + \frac{l_{\mathrm{ch}}}{b}\right]^{-1/2}, \tag{6}$$

$$\sigma_u = f_t\left[1 + \frac{l_{\mathrm{ch}}}{b}\right]^{1/2}. \tag{7}$$

In perfect analogy with the MFSLs for tensile strength and fracture energy – see Eqs (6) and (7) – a new Multifractal Scaling Law has been proposed for the critical strain ε_c. The lack of complete similarity implies that an internal length l_{ch} is present also in the kinematics of damage. Thereby, the previous (monofractal) scaling relation (3) has to be modified by considering the exponent d_ε smoothly variable with the structural size. Since the influence of microstructural disorder is higher for smaller sizes, it seems reasonable to put d_ε equal to zero for very short bars (maximum disorder, that is, damage diffused throughout the volume, corresponding to ductile behaviour), and $d_\varepsilon = 1$ for very long bars (maximum order, that is, localization of fracture on a single cross-section, corresponding to brittle behaviour). The Multifractal Scaling Law for the critical strain can thus be written as:

$$\varepsilon_c = \varepsilon_c^0\left[1 + \frac{b}{l_{\mathrm{ch}}}\right]^{-1}. \tag{8}$$

In conclusion, the fundamental relation among the three fractional exponents, Eq. (5), specialized at the various scales, can be written as:

$$\text{MACROSCALE:} \quad d_\sigma \approx d_\mathcal{G} \approx 0, \quad d_\varepsilon \approx 1, \tag{9}$$

$$\text{MESOSCALE:} \quad d_\sigma + d_\mathcal{G} + d_\varepsilon = 1, \tag{10}$$

$$\text{MICROSCALE:} \quad d_\sigma \approx d_\mathcal{G} \approx 1/2, \quad d_\varepsilon \approx 0. \tag{11}$$

Regarding the fractal parameters, namely ε_c^*, σ_u^* and $\mathcal{G}_F^*$, observe that at the larger scales they coincide with the cohesive law parameters, i.e. the critical crack opening displacement, the tensile strength and the fracture energy, whereas at the smaller scales their physical dimensions change.

3. The fractal interpretation of multiscale cracking phenomena

The second topic deals with the criticality of the complex multiscale cracking phenomena in heterogeneous and disordered materials, evaluated by means of the Acoustic Emission (AE) technique. As in the previous example, the complex phenomena will find a simple and synthetic interpretation through the use of fractal concepts.

Acoustic Emission (AE) is represented by the class of phenomena whereby transient elastic waves are generated by the rapid release of energy from localized sources within a material. All materials produce AE during both the generation and propagation of cracks. The elastic waves move through the external solid surface, where they are detected by sensors. These sensors are transducers that convert the mechanical waves into electrical signals. In this way, information about the existence and location of possible damage sources is obtained. This is similar to seismicity, where seismic waves reach the station placed on the earth surface (Richter [30], Chakrabarti and Benguigui [31]). Therefore, among the structural nondestructive tests, the AE monitoring technique is the only one able to detect a damage process at the same time as it occurs.

With regard to the basis of AE research in concrete, the early scientific papers were published in the 1960s. Particularly interesting are the contributions by Rusch [32], L'Hermite [33] and Robinson [34]. They discussed the relation between fracture process and volumetric change in the concrete under uniaxial compression. The most important applications of AE to structural concrete elements started in the late 1970s, when the original technology developed for metals was modified to suit heterogeneous materials (McCabe et al. [35], Niwa et al. [36]). Regarding the signal source location, or the determination of the defects position and orientation in the material, research has been growing at a fast rate in the last decade; among the researchers who studied this methodology we find Shah and Zongjing [37] and Ohtsu [38].

In the last few years the AE technique has been applied to identify defects and damage in reinforced concrete structures and masonry buildings (Carpinteri and Lacidogna [39–42]). By means of this technique, a particular methodology has been put forward for crack propagation monitoring and crack stability assessment in structural elements under service conditions. This technique permits to estimate the amount of energy released during fracture propagation and to obtain information on the criticality of the ongoing process [43,44]. A statistical and fractal analysis of laboratory experimental data was performed, considering the multiscale aspect of cracking phenomena. This approach shows that the energy dissipation, detected by AE, occurs in a fractal domain. Consequently, based on Fracture Mechanics concepts, a fractal or multiscale methodology to predict damage evolution and time to structural collapse was proposed.

Recent developments in fragmentation theories (Carpinteri and Pugno [45–47]) have shown that the energy dissipation E during microcrack propagation occurs in a fractal domain comprised between a surface and the specimen volume V. The result is that the total dissipated energy $E_{\max}$ after fragmentation scales as:

$$E_{\max} = \Gamma V^{D/3}, \tag{12}$$

where Γ is the critical value of fractal energy density and D is the so-called fractal exponent, comprised between 2 and 3 [45]. As a consequence, the energy density scales as:

$$\Psi = \frac{E_{\max}}{V} = \Gamma V^{(D-3)/3}. \tag{13}$$

This implies that not the energy density but the fractal energy density (having anomalous physical dimensions) can be considered as a size-independent quantity:

$$\Gamma = \frac{E_{\max}}{V^{D/3}}. \tag{14}$$

On the other hand, during microcrack propagation, acoustic emissions can be clearly detected. The dissipated energy E is proportional to the number N of acoustic emission events (with intensity proportional to $\Delta t \, \Delta N$, where t is the time). Accordingly to the energy dissipation over a fractal domain – as described by Eq. (14) – the critical number of acoustic emissions $N_{\max}$, not over a volume but over a fractal domain, can be considered as a size-independent parameter:

$$\Gamma_{\mathrm{AE}} = \frac{N_{\max}}{V^{D/3}}, \tag{15}$$

where Γ_{AE} is the critical value of fractal acoustic emission density. This fractal criterion predicts a volume-effect on the maximum number of acoustic emission events, that, in a bilogarithmic diagram, would appear as:

$$\log N_{\max} = \log \Gamma_{\mathrm{AE}} + \frac{D}{3} \log V \tag{16}$$

with a slope equal to $D/3$. Experiments carried out by Carpinteri et al. [44] confirm the soundness of the proposed approach. Cylindrical concrete specimens (see Fig. 4) were tested in compression under displacement control. A low displacement rate equal to 10^{-4} mm/s for all specimens was chosen, in order to obtain a very slow-crack growth and to detect all possible AE signals, capturing also the softening branch of the stress–strain diagrams. Regarding the specimen sizes, three different specimen diameters d were considered, in a maximum scale range of $1:3.4$. The specimens presented three different slendernesses: $\lambda = h/d = 0.5$, 1.0 and 2.0, thus nine geometries were tested, with a maximum volume ratio of about $1:156$.

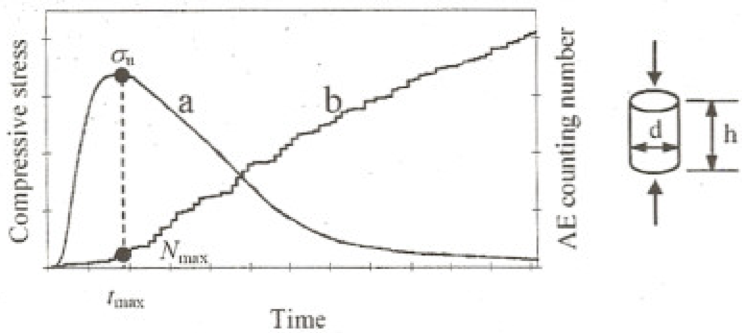

Fig. 4. Stress and cumulated event number versus time. a – Compressive stress, b – AE counting number.

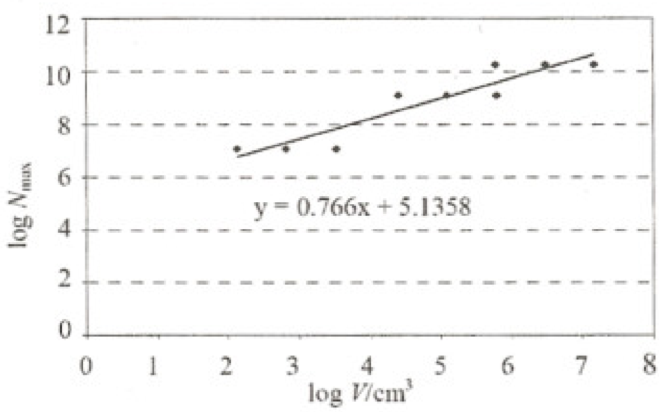

Fig. 5. Volume effect on the maximum number of acoustic emissions.

For all the tested specimens, the critical number of acoustic emissions $N_{\max}$ was evaluated in correspondence to the peak-stress σ_u (see Fig. 4). The compression tests show an increase in AE cumulative event number by increasing the specimen volume. More in detail, subjecting the average experimental data to a statistical analysis, the parameters D and Γ_{AE} in Eq. (16) were quantified. From the best-fitting, reported graphically in Fig. 5, the estimated value of the slope was computed as $D/3 \cong 0.766$, so that the fractal exponent results, as predicted by the fragmentation theories, to be comprised between 2 and 3 ($D \cong 2.3$). This result is a confirmation of the fact that the energy dissipation, measured by the number of acoustic emissions N, occurs over a fractal domain.

Interestingly, the criticality of the cracking phenomena does appear not only in space, but also in time. Let us start by considering that the damage level of a structure can be obtained from AE data of a reference specimen (pedex r) extracted from the structure and tested up to rupture. From Eq. (15) we have:

$$N_{\max} = N_{\max r}\left(\frac{V}{V_r}\right)^{D/3} \tag{17}$$

from which we can obtain the total number of acoustic emission events $N_{\max}$ that the structure may provide before achieving the collapse. An energy parameter describing the damage level of the structure can be defined as the following ratio:

$$\eta = \frac{E}{E_{\max}} = \frac{N}{N_{\max}}, \tag{18}$$

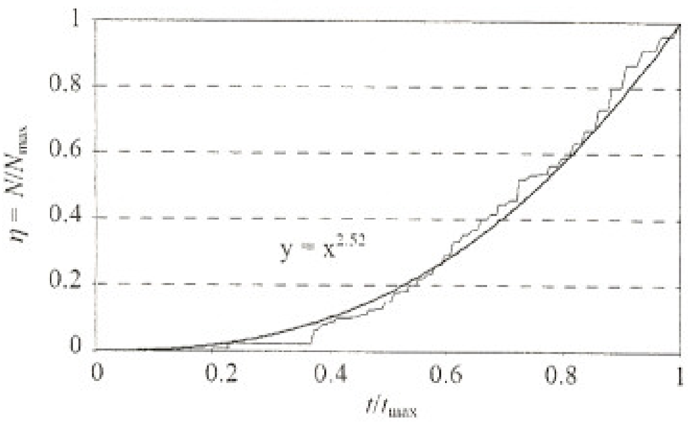

Fig. 6. Damage evolution in time.

N being the monitored cumulative number of acoustic emissions. The damage level can be expressed [43,44] as a function of different parameters, i.e., stress σ, strain ε or time t:

$$\eta = \frac{N}{N_{\max}} = \left(\frac{\sigma}{\sigma_{\max}}\right)^{\beta_\sigma} = \left(\frac{\varepsilon}{\varepsilon_{\max}}\right)^{\beta_\varepsilon} = \left(\frac{t}{t_{\max}}\right)^{\beta_t}, \tag{19}$$

where the exponents β can be obtained from the AE data of a reference specimen. An example of the $N/N_{\max}$ vs. time dependence is given in Fig. 6 for one of the tested specimens. After an initial transient period ($0 < t/t_{\max} < 0.4$), a power-law time scaling is observed (Shcherbakov and Turcotte [48]). From the best fitting procedure, the exponent $\beta_t = 2.52$, is obtained. Similar results can be observed for stress and strain dependencies.

The fractal multiscale criterion of Eq. (19) is a fundamental result, since it allows to predict the damage evolution also in large concrete structural elements. Monitoring the damage evolution by AE, it is therefore possible to evaluate the damage level as well as the time to final collapse [43].

4. Conclusions

The so-called "Complexity Sciences" represent a subject of fast-growing interest in the Scientific Community. They have entered also our more circumscribed Communities of Material Science and Material Strength, as the proposed examples in these two companion papers may confirm. Under the label of "Complexity Sciences" we usually comprehend a large variety of phenomena, theories, approaches and techniques: nonlinear dynamics, deterministic chaos, nonequilibrium thermodynamics, fractal geometry, intermediate asymptotics, renormalization group theory, catastrophe theory, self-organised criticality, neural networks, cellular automata, fuzzy logic, etc.

Complex systems lie somehow in between order and randomness and exhibit some common characteristics, such as: sensitivity to initial conditions, pattern formation, spontaneous self-organisation, emergence of cooperation and collective properties, hierarchical or multiscale meso-structures, scaling and size effects. We could try to summarize by saying that the nonlinearity in the constitutive laws may produce complex structures and scale-dependent behaviours.

Aim of this paper is that of providing insight into the role of complexity in the fields of Material Strength and Fracture Mechanics. The topics presented in this paper (Part I) are concerned with order emerging from complex systems; the proposed examples deal with the occurrence of fractal patterns and geometrically self-similar morphologies in deformation, damage and fracture of heterogeneous materials, the apparent scaling in the nominal mechanical properties of disordered materials and the acoustic emission criticality in progressive structural collapses. As shown in these examples, the most interesting behaviours and phenomena can be synthetically interpreted only through the use of new and refined conceptual tools in the framework of "Complexity Sciences".

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