Time scale effects on acoustic emission due to elastic waves propagation in monitored cracking structures*

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In this paper we propose a fractal theory for predicting the time effects on the damage evolution in cracking solids. By means of the acoustic emission technique, we have analyzed the evolution of damage in several structures by an extensive experimental analysis in time. Theory and experiments agree closely. Consequently, the life-time predictions of monitored solid structures can be estimated; as an example two viaduct pilasters are investigated.

1. Introduction

The evaluation of safety and reliability for reinforced concrete structures, like bridges and viaducts, represents a complex task at the cutting edge of scientific research. Due to these reasons, the diagnosis and monitoring techniques are assuming an increasing importance in the evaluation of structural conditions and reliability. Among these methods, the nondestructive methodology based on Acoustic Emission (AE) proves to be very effective [1–3].

Some applications of AE technique to construction monitoring are described by Carpinteri and Lacidogna [4, 5]. In addition, strong space effects are clearly observed on energy density dissipated during fragmentation. Recently, a multiscale energy dissipation process has been shown to take place in fragmentation, from a theoretical and fractal viewpoint as proposed by Carpinteri and Pugno [6, 7].

This fractal theory takes into account the multiscale character of energy dissipation and its strong space effects. Such a approach for the space-scaling of the energy density has been experimentally verified by AE technique [8]. Here we focus the attention on the complementary effects, related to the time effects. The understanding of the space-time effects make it possible to introduce a useful energetic damage parameter for structural assessment based on a correlation between AE activity in a structure and the corresponding activity recorded on a small specimen extracted from the structure and tested to failure. Moreover, by our findings on space-time effects, the safety of structures undergoing damage and degradation processes can be efficiently evaluated in exercise conditions and in situ.

2. Time effects during damage evolution: a fractal theory

Each acoustic emission event, due to crack and elastic wave propagations in the damaging solids, has a characteristic duration τ that, according to the experimental evidence on earthquakes [9, 10], we assume as a fractal (i.e., self-similar). Accordingly, distribution of the event durations, must be the following power law:

$$P(<\tau) = \frac{N(<\tau)}{N_{\max}} = 1 - \left(\frac{\tau_{\min}}{\tau}\right)^{D_T}, \qquad (1)$$

where $N(<\tau)$ is the number of events with duration smaller than τ, $N_{\max}$ is the total number of events, $\tau_{\min}$ ($\ll \tau_{\max}$) is the minimum duration, and D_T (> 0) is the fractal dimension of the self-similar time distribution of events.

The probability density function $p(\tau)$ is provided by derivation of the cumulative distribution function (1):

$$p(\tau) = D_T \frac{\tau_{\min}^{D_T}}{\tau^{D_T+1}}. \qquad (2)$$

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During fragmentation, the energy dissipation W in a volume V is [6]:

$$W \propto V^{D_S/3}, \quad (3)$$

where D_S (comprised between 2 and 3) is the fractal dimension of the self-similar space distribution of fragments (assumed to follow the fractal size distribution of Eq. (2), replacing the duration of the event with the size of the fragment). Accordingly, the infinitesimal energy dW dissipated during a single event will follow Eq. (3), in which V represents the volume involved by the associated wave propagation. For isotropic three-dimensional wave propagation, along each axis the characteristic length of the event is $c\tau$, with c sound speed, so that $V \propto \tau^3$.

Thus locally:

$$\mathrm{d}W \propto \tau^{D_S}. \quad (4)$$

For two- or one-dimensional objects having characteristic size A or L respectively, the result is the same, since instead of Eq. (3) we have $W \propto A^{D_S/2}$ or $W \propto L^{D_S}$ with $A \propto \tau^2$ or $L \propto \tau$ and thus again Eq. (4).

The total energy dissipated will be consequently:

$$W \propto \int_{\tau_{\min}}^{\tau_{\max}} \tau^{D_S}\,\mathrm{d}N = \int_{\tau_{\min}}^{\tau_{\max}} N_{\max}\,p(\tau)\mathrm{d}\tau \propto$$

$$\propto N_{\max}\frac{D_T}{D_S - D_T}\tau_{\min}^{D_T}(\tau_{\max}^{D_S-D_T} - \tau_{\min}^{D_S-D_T}) \cong$$

$$\cong \begin{cases} N_{\max}\dfrac{D_T}{D_S - D_T}\tau_{\min}^{D_T}\tau_{\max}^{D_S-D_T}, & D_T < D_S, \\ N_{\max}\dfrac{D_T}{D_T - D_S}\tau_{\min}^{D_S}, & D_T > D_S. \end{cases} \quad (5)$$

On the other hand, the total (monitoring) time, is given by:

$$t \propto \int_{\tau_{\min}}^{\tau_{\max}} \tau\,\mathrm{d}N = \int_{\tau_{\min}}^{\tau_{\max}} N_{\max}\,\tau p(\tau)\mathrm{d}\tau \propto$$

$$\propto N_{\max}\frac{D_T}{1 - D_T}\tau_{\min}^{D_T}(\tau_{\max}^{1-D_T} - \tau_{\min}^{1-D_T}) \cong$$

$$\cong \begin{cases} N_{\max}\dfrac{D_T}{1 - D_T}\tau_{\min}^{D_T}\tau_{\max}^{1-D_T}, & D_T < 1, \\ N_{\max}\dfrac{D_T}{D_T - 1}\tau_{\min}, & D_T > 1. \end{cases} \quad (6)$$

Here, according to the experimental acoustic emission monitoring, the events are assumed to be in series rather than in parallel. On the other hand, since a symbol of proportionality and not of equality is required in Eq. (6) for the definition of the monitoring time t, parallel events would be in principle allowed.

In addition, let us assume a duration "quantum" of size $\tau_{\min}$ = const and make a statistical hypothesis of self-similarity, i.e. $\tau_{\max} \propto t$ (the larger the monitoring time, the larger the largest event). Accordingly, eliminating $N_{\max}$ from Eqs. (5) and (6) we have:

$$\text{if } D_S \geq 1, \; W \propto \begin{cases} t^{D_S}, & D_T < 1, \\ t^{1+D_S-D_T}, & 1 \leq D_T \leq D_S, \\ t, & D_T > D_S, \end{cases} \quad (7.1)$$

$$\text{if } D_S < 1, \; W \propto \begin{cases} t^{D_S}, & D_T < D_S, \\ t^{D_T}, & D_S \leq D_T \leq 1, \\ t, & D_T > 1. \end{cases} \quad (7.2)$$

We have found that the same time scaling hold for the standard deviation σ_W of the energy if we formally replace in Eqs. (7) W with σ_W and D_S with $2D_S$. A similar fractal approach on size scaling rather than on time has already been proposed for predicting the size effects on the mean values and on the standard deviations for the main mechanical properties of materials [7], starting from the space scaling of the energy [6].

Note that usually $D-1 < D_S < D$ with $D = 1, 2, 3$ object dimension [6]. From Eqs. (7), $W \propto t^{\beta_t}$ with $1 \leq \beta_t \leq D_S$, if $D_S \geq 1$, or $D_S \leq \beta_t \leq 1$, if $D_S < 1$, i.e., in general:

$$W \propto t^{\beta_t}, \; 0 \leq \beta_t \leq 3. \quad (8)$$

The corresponding fractal size-scaling on acoustic emission during cracking of solids has already been proposed by the same authors [8], on the basis of the fractal fragmentation law [6]. The experimental validation of the time scaling of Eq. (8) represents the aim of the next section.

3. Acoustic emission monitoring: experimental evidence

The AE method, which is called Ring-Down Counting or Event Counting, considers the number of waves beyond a certain threshold level (measured in Volts) and is widely used for defect analysis [11–13]. As a first approximation, in fact, the cumulative number of counts N can be compared with the amount of energy released during the loading process, assuming that both quantities increase with the extent of damage (i.e., $W \propto N$, and the energy must be additive).

By means of this technique, we have analysed the evolution of cracks and estimated the released strain energy during their propagation in structural members. In particular, the damage evolution in several structures by an extensive experimental analysis in space and time have been investigated. Among these structures we also analysed the damage evolution in two pilasters sustaining a viaduct along an Italian highway built in the 1950s. From the pilasters we drilled some concrete cylindrical specimens in order to detect the mechanical properties of the material under compression and to evaluate the scale effects on AE activity in size [8] and time. For details on test specimens, machine and conditions the reader should refer to [8].

According to Eq. (8) and to $W \propto N$, an energy damage parameter η during the specimen testing, can be defined as:

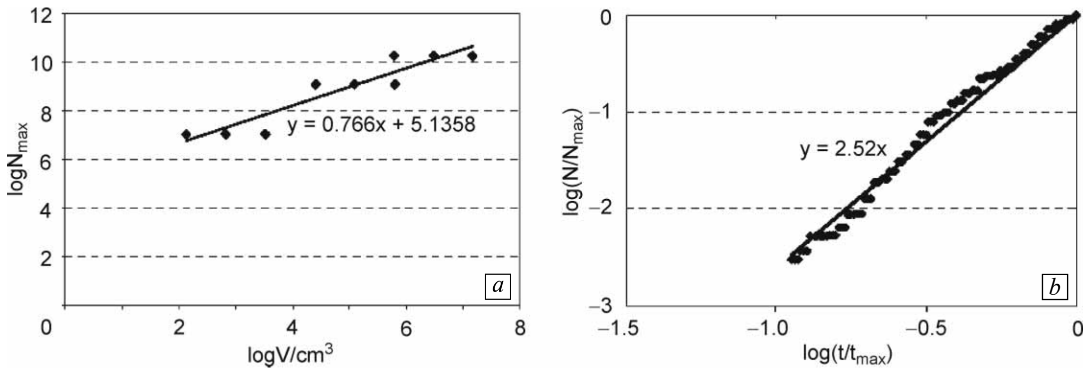

Fig. 1. Space-time scaling in damage evolution

$$\eta \equiv \frac{W}{W_{\max}} = \frac{N}{N_{\max}} = \left(\frac{t}{t_{\max}}\right)^{\beta_t}, \qquad (9)$$

where "max" refers to the reaching of the maximum stress (that we chose as the critical condition). From Eq. (9) the experimental values of β_t, describing the time scaling of the energy dissipated or released can be deduced (according to the fractal theory, it is expected not to be not strongly dependent on test conditions).

As example of experimental space-time effects are given in Fig. 1. After an initial transient period ($0 < t/t_{\max} < 0.4$) [14], a true power-law for the time-scaling is observed. From the best-fitting in the bilogarithmic plane (Fig. 1(*b*)), for the tested specimen ($d = 59$ mm, $\lambda = 1$) we obtain the slope $\beta_t = 2.52$. Similar results can be observed for stress and strain dependencies. The size scaling on $N_{\max}$ is also represented as a function of the volume specimen (Fig. 1(*a*)), fitted to experimental data. A slope in the log-log plane between $2/3$ and 1 (experimentally close to 0.77) emphasizes that the energy dissipation occurs in a fractal domain, intermediate between a surface and a volume (for details see [6, 8]). The β_t values plotted versus the specimen diameter are reported in Fig. 2. The observed trend is an increase of the β_t values by increasing the specimen diameter. The experimental time scaling agree with the fractal law of Eq. (8), giving exponent in the range (0, 3). The experimental results are summarized in Table 1.

The tested specimens come from two pilasters, monitored them self utilizing the described AE data acquisition sys-

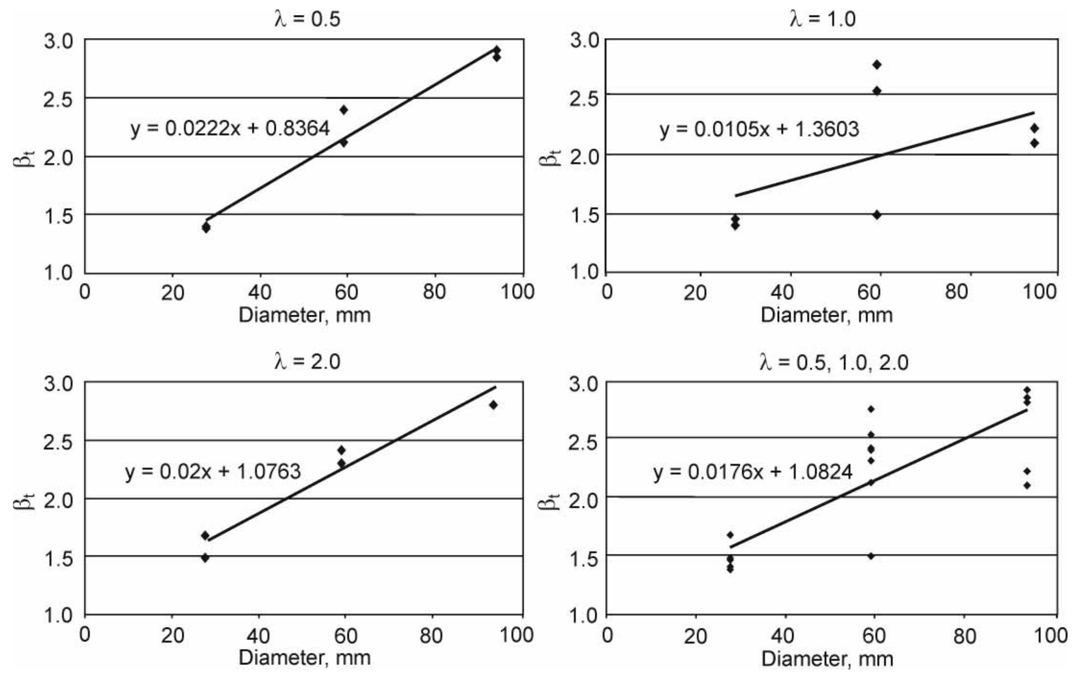

Fig. 2. Life-time exponent β_t plotted versus specimen diameter

Table 1

Average values for the specimens obtained form pilasters P_1 and P_2

Specimen number	Diameter d, mm	$\lambda = h/d$	Pilaster P_1			Pilaster P_2		
			Peak stress σ_u, MPa	N_{max} at σ_u	β_t	Peak stress σ_u, MPa	N_{max} at σ_u	β_t
1	27.7	0.5	91.9	1 186	1.40	84.7	1 180	1.38
2	27.7	1.0	62.8	1 191	1.41	46.7	1 181	1.46
3	27.7	2.0	48.1	1 188	1.48	45.8	1 186	1.67
4	59.0	0.5	68.1	8 936	2.12	57.5	8 924	2.39
5	59.0	1.0	53.1	8 934	1.49	41.7	8 930	2.52
6	59.0	2.0	47.8	8 903	2.30	38.2	8 889	2.41
7	94.0	0.5	61.3	28 502	2.90	45.2	28 484	2.84
8	94.0	1.0	47.8	28 721	2.09	38.2	28 715	2.21
9	94.0	2.0	44.1	28 965	2.80	38.1	28 956	2.92

tem. During the observation period (172 days), we obtained a number of events $N \cong 2\cdot10^5$ for the more damaged pilaster P_1, and $N \cong 8\cdot10^4$ for the less damaged P_2, respectively. Since the volume of each pilaster is about $2\cdot10^6$ cm^3, extrapolating from Fig. 1(*a*), we estimate the critical number of AE for the pilasters equal to $N_{max} \cong 11.51\cdot10^6$.

Inserting the values of N and N_{max} into Eq. (9), and assuming an exponent $\beta_t = 2.52$ (a more conservative choice would be 3), we obtain $t/t_{max} \cong 0.2$ for pilaster P_1, and $t/t_{max} \cong 0.14$ for pilaster P_2. The lifetime of these structural elements is therefore defined, corresponding to the achievement of the maximum number of events, at respectively 2.4 and 3.4 years considering the time origin from the instant in which the specimens have been drilled.

4. Conclusions

In this paper a fractal theory for predicting the time scaling of the damage evolution in cracking solids has been presented and investigated experimentally by acoustic emission technique. The analytical result, summarized in Eq. (8), seems to be confirmed by the experimental evidence on acoustic emission, showing power law damage evolution with fractal exponents β_t comprised between 0 and 3. Coupling space-time effects, the life time predictions for structures can be estimated in exercise conditions and in situ.

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