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SCALING LAWS FOR FRAGMENTATION ENERGY DISSIPATION IN IMPACTS AND EXPLOSIONS

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Abstract: The result of this work aims at demonstrating and unifying the three fundamental laws regarding energy dissipation during comminution by showing their validity also as far as impact and explosive natural and artificial phenomena are concerned and extending their import to similar events occurring in one and two-dimensional objects. In the three-dimensional case, it emerges that energy dissipation takes place not according to an Euclidean domain but according to a fractal one, which, surprisingly enough, is always included within Euclidean surfaces and volumes. It is interesting to point out how borderline cases correspond to the predictions of classical theories, namely traditional *Fracture Mechanics* on the one hand and *Plasticity* on the other.

Euclidean Geometry greatly contributed to the development of Natural Sciences, Mathematics and Physics. However, the 20th century clearly showed its limits, leading scientists to lay aside the old notion according to which Euclidean Geometry was suitable to represent the real Geometry of Nature. Pseudo Euclidean Geometry and Minkowski's spaces provided a basis for Restricted Relativity and likewise Gauss-Riemann's geometry provided a similar scientific fundation for the space-time description in General Relativity.

Gravity does not depend on scale. Therefore this has brought about the formation of cosmic structures, which often do not seem to have a random nature, but - on the contrary - possess a sponge-like topology, so as to influence consequently the selfsame space-time geometry. Such topology has been well described by the recent Fractal Geometry[1,2] which studies structures such as e.g. sponge pores, that appear self-similar at all scales. *Therefore Fractality is linked to Scale Relativity, making it impossible to establish an absolute scale*.

Self-similar objects at all scales are actually well known in Nature[3-7]; hence the use - and sometimes the misuse - of fractals can be found in the most diversified fields (e.g., energies, sizes and durations of solar flares, magnitudes of earthquakes, sizes of lakes, sizes of impact craters on moons, frequencies of usage of words, lifetime of biological taxa, fragments of coal, size of asteroids, particles in rings of Saturn, energy dissipation of warm-blooded animals, distribution of scales of coastlines etc.). For instance, the problem concerning the measurement of the length of Great Britain's[3] coastline showed the fractal nature of the coastline indentation. As a matter of fact the Euclidean method for the measurement adopted at the beginning led to conflicting results since the more accurate the measurement was the more the coastline diverged. Likewise, objects with different dimensions, even belonging to various magnitude sizes (e.g. a micro fracture caused by fatigue in a metal and a macro fracture on the earth's crust originating from an earthquake) seem to be topologically similar. In other words, the morphology of a fracture appears self-similar at all scales: paradoxically this does not allow researchers to understand whether the photograph of a fracture was taken using a microscope or from a satellite.

With a view to pursue such analysis in the field of fractures, a further consideration of fundamental purport is that at small scales the self-similarity phenomenon must fade away owing to quantization (a phenomenon which Quantum Mechanics has made researchers familiar with). Namely it is to be expected that at nano-scale the Continuum should die down and be replaced by Fracture Quanta[8]. Nowadays this issue is becoming more and more relevant due to the steady growth of the development of nanotechnologies, which has considerably increased the interest in the investigation into damages at small scale levels[9,10]. As far as a fracture is concerned, even if such fracture is extreme (e.g. in crushing processes), it is therefore impossible to produce matter particles below a certain dimensional threshold (*Material Quantum*) because of a drastic energy increase to be spent in the process[11]; hence the serious technological difficulty in carrying out a

fine comminution, which is essential in a vast number of processes (e.g. sintering, medicine production …).

By combining both concepts, i.e. *Scale Relativity* or Fractal Geometry on the one hand, and the hypothesis of the existence of a *Material Quantum* on the other, researchers have recently formulated some simple Universal Laws (one, two and three-dimensional) in order to calculate the energy involved in the crushing of matter as a result of impacts and explosions[12]. Such laws comply with thermodynamics principles. Actually, some researchers already realized as far as ten years ago that there was a kind of universal pattern underlying the crushing processes which implied different fracture systems and different materials[13]. In the three-dimensional case, for example, it emerges that energy dissipation takes place not according to a Euclidean domain but according to a fractal one, which, surprisingly enough, is always included within Euclidean surfaces and volumes. It is interesting to point out how borderline cases correspond to the classical cases of structural collapse, namely traditional *Fracture Mechanic*s on the one hand and *Plasticity* on the other[14]. In addition, it is relevant to this research to highlight how the three-dimensional Universal Law succeeds in unifying the three main theories about comminution (i.e. the *Surface Theory*[15], the *Volume Theory*[16] and the *Third Comminution Theory*[17]; see[18]), which, however, were developed on the basis of deeply different and inherently experimental concepts so as to ignore the fragment dimensional distribution.

The starting point in the research involved assuming of a fractal probability density function for the fragment size distribution[19] $p(r) = D\frac{r_{min}^{D}}{r^{D+1}}$, where r_{min} is the typical dimension of the smallest fragment (matter quantum), r is the typical dimension of a generic fragment (proportional to the cube root of the fragment's volume) and D is the so called fractal dimension of the distribution. Theoretically such fractal exponent is positive. Experimentally it is possible to observe that in the vast majority of cases involving the crushing of three dimensional objects

such exponent is comprised between values ranging from 2 to 3 (e.g. disaggregated gneiss D=2.13, disaggregated granite D=2.22, broken coal D=2.50, projectile fragmentation of quartzite D=2.55, projectile fragmentation of basalt D=2.56, fault gouge D=2.60, sandy clays D=2.61, terrace sands and gravels D=2.82, glacial till D=2.88)[19]. Theoretically this is equivalent to a crushing in which the smallest fragments provide the main contribution to the creation of the fracture surface while the largest ones contribute to defining its volume. All the same, fractal exponent values outside (i.e. below or above) this interval can be detected in a few cases such as artificially crushed quartz (D=1.89) or ash and pumice (D=3.54)[19]. As far as two-dimensional objects are concerned, the radius which characterizes the fractal size distribution $p(r)$ is proportional to the square root of the typical surface (of constant thickness) of the fragment. For the most frequent crushing, in which the smallest fragments provide the main contribution to the creation of the fracture perimeter while the largest ones define the area, the two-dimensional fractal exponent is comprised between values ranging from 1 to 2. This is experimentally substantiated: in fact several texts regarding ice floe fragmentation set the values of D as 1.7-1.8, 1.36. and 1.56[20]. Likewise a value ranging from 0 to 1 can be expected for the one-dimensional fractal exponent.

Some simple models[19] show how D is proportional to the logarithm of the crushing probability, i.e. the probability a certain block has to be crushed into sub-blocks and then into even smaller ones and so on. So, assuming a fractal law (D=const) means assuming a constant crushing probability at all scales: for the starting block, for sub-blocks and so forth. If the crushing probability increases, also the fractal exponent becomes bigger. For instance: in passing from one dimensional to three-axial compression, the fractal exponent rises, starting from an initial value close to 2, due to the greater confinement to which a greater crushing probability corresponds[20]. The crushing is therefore finer and the fractal exponent bigger. Another example

can be found in the case of ice floes, where the fractal exponent seems to increase towards the ice-sea interface owing to the major confinement and stress set by waves and wind (*D* increases starting from 1 for the internal interface areas and reaches 1.8 for the external ones)[20]. To sum up, it is possible to state that, given an equal space dimension (1, 2 or 3) to be taken into consideration, inferior values can be expected for less confined phenomena - such as explosions; whereas greater values have to be expected for the more confined events - such as impacts, especially repeated ones. All this conforms to the full to experimentation. In addition to this experimental corroboration, some authors have demonstrated that the fractal distribution law is theoretically a consequence of the Maximum Entropy Principle[21].

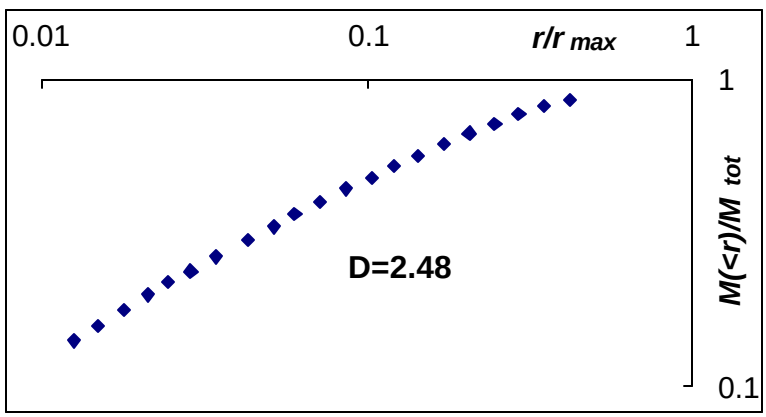

Figure 1: Statistical analysis of mass particle size distribution.

In order to show an example of fragment fractal distribution, Figure 1 illustrates the mass distribution of fragments (analyzed by employing instruments which exploited diffraction) following an artificial crushing of heterogeneous material (concrete)[22]. The fractal hypothesis provides, by means of a simple integration originating from the distribution function $p(r)$, a prediction about the total fragment mass with a typical radius minor of a certain value *r*, such as: $\log \frac{M(<r)}{M_{tot}} \cong (3-D)\log \frac{r}{r_{max}}$, namely a linear pattern in a bi-logarithmic plane with a slope linked

to a fractal exponent which is expected to range between the aforesaid values 2 to 3 in such an instance of three-dimensional crushing. The statistical analysis shown in Figure 1 fit in with the fractal hypothesis (alignment on a straight line with D=2.48). Fragments are reported in Fig. 2.

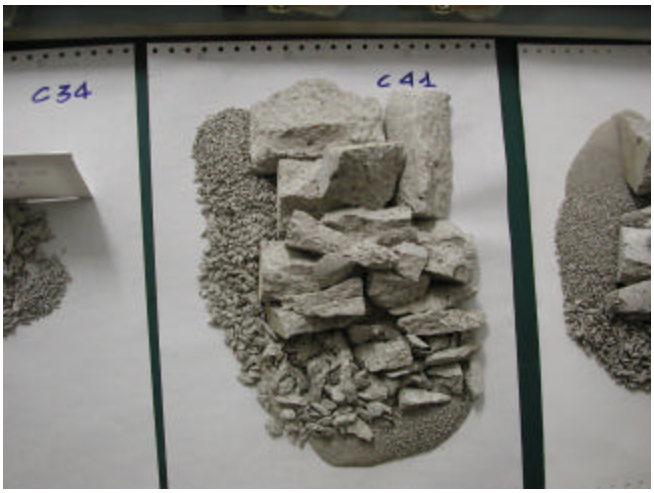

Figure 2: Experimental mass particle size distribution.

Resuming the main issue concerning the process energetics, it is surprising to see that the main dissipations involved in fragmentation are, all the same, proportional to the free surface of the fragments produced. In addition to the energy dissipated during the breaking off of the chemical bonds, which – according to Griffith[23] – is proportional to the fragment surface, and actually far inferior to the dissipation causes which will be mentioned below in the following paragraph, it is possible to detect, in fragmentation under compression (impacts), a high heat production due to friction among particles: such friction may be assumed to be proportional to their free surface[24]. In fragmentation under traction (explosions), energy is substantially dissipated via the fragments' kinetic energy. Such dissipation appears to be proportional to the fragments' surface only in the specific instance when the ejection velocity of such ballistic projectiles is proportional to the reciprocal of the square root of their typical linear dimension. Surprisingly enough, this is the case[25]. This involves a consequence of paramount importance,

namely that energy dissipation in matter crushing due to impacts and explosions may be unified and considered proportional to the total free surface created.

Whenever a three-dimensional case is analyzed, such a surface has obviously a fractal nature, thanks to the dimensional self-similar nature of the particles, and is always comprised within an Euclidean surface and volume. The theoretical result of this work aims at demonstrating and unifying - simply but rigorously - the three fundamental laws regarding comminution by showing their validity also as far as explosive phenomena are concerned and extending their import to similar events occurring in one and two-dimensional objects. As a conclusion, the result of the research can be simply summarized as follows for the three-dimensional case: $W \propto V^{\overline{D}/3}$, being W the energy dissipated, V the fragmented volume and $\overline{D}$ the fractal exponent of the particle size distribution set at values 2-3. i.e. $\overline{D} = 2$ for $D < 2$, $\overline{D} \equiv D$ for $2 \le D \le 3$ and $\overline{D} = 3$ for $D > 3$. Similar correspondences can be obtained if two-dimensional objects with A surface and one-dimensional objects with L length are crushed.

The three *Scaling Laws* for energy dissipation during fragmentation under impacts or explosions are the following[12]:

$$
\begin{array}{lll}
W \propto L^{\overline{D}} & \left(0 \le \overline{D} \le 1\right) & 1-\text{dimensional} \\
W \propto A^{\overline{D}/2} & \left(1 \le \overline{D} \le 2\right) & 2-\text{dimensional} \\
W \propto V^{\overline{D}/3} & \left(2 \le \overline{D} \le 3\right) & 3-\text{dimensional.}
\end{array}
$$

Applications. As a first example, we can apply the three-dimensional law to the prediction of the devastated area due to asteroid impacts[26,27], as a function of the energy released in the collision. The comparison with the experimental Steel's law[28], based on nuclear weapon tests, shows a good correspondence. Assuming that the destroyed zones (or fragmented volumes V) are self-

similar at each scale, the area W devastated by an impact is proportional to $V^{2/3}$ and, being $W \propto V^{\overline{D}/3}$, the theoretical prediction for the devastated area will be $W \propto W^{2/\overline{D}}$. Steel[28] provided the following formula, based on nuclear weapon tests, for estimating the area of destruction due to asteroid impacts $W = 400W^{0.67}$, $[W] = [\mathrm{km}^2]$, $[W] = [\mathrm{Megatons}]$. It appears in good agreement with the theoretical prediction and, if we assume $\overline{D} \approx 3$, they practically coincide.

As a second example of application we can considered the size and shape effects on material properties in compression. We can evaluate the dissipated strain energy density $Y = \frac{W}{V}$ during the compression of a specimen as a function of its characteristic length l: $\frac{Y}{Y_0} = \left(\frac{l}{l_0}\right)^{\overline{D}-3}$ where Y_0, l_0 are related to a reference specimen. In addition, if we suppose that the size-effects on the compressive strength s_C can be estimated assuming $Y \propto s_C^2$, we have: $\frac{s_C}{s_C^0} = \left(\frac{l}{l_0}\right)^{\frac{\overline{D}-3}{2}}$. Size-effects on strain energy density and on material strength, described by straight lines in a bi-logarithmic diagram, are experimentally confirmed[29]. The fractal exponent, obtained as a best-fit parameter by fitting the experimental data, has been found close to 2. Fractal exponents around two are in fact observed for particle size distribution obtained under uniaxial compression[30]. On the other hand – regarding shape effects in compression – considering specimens with constant base area l^2, the specimen slenderness (height h over base side l) can be obtained as $l = \frac{V}{V_{l=1}} = \frac{l^2 h}{l^3} = \frac{h}{l}$ and the shape-effects become $\frac{Y}{Y_{l=1}} = l^{\frac{D-3}{3}}$, $\frac{s_C}{s_C(l=1)} = l^{\frac{D-3}{6}}$. They predict the shape effects on dissipated energy density and strength for specimens under compression. The fractal exponent, obtained as best-fit parameter[29] by fitting the experimental data, has been found again close to 2.

As third and last example of application we can considered the power balance for drilling comminution. Considering a drilling tool and the applied vertical thrust force F (typically supplied by the operator), the torque M_t (typically supplied by the engine) and their dual displacements d and j, the power balance for drilling comminution can be written as $F\dot{d} + M_t\dot{j} = \dot{W} + \dot{W}_f$, where W represent the global work dissipated (by fracture and by internal friction) in comminution processes, W_f is the heat production by external friction and the dot over the symbols represents time derivation[22]. It is important to observe how the internal heat production (substantially by friction) in the comminution process is included in W. Such quantitiy should be proportional to the new produced free surface and absolutely prevailing over the fracture work – only about 3% of the total[22]. Evaluating the power $\dot{W}$ by the fragmentation law and $\dot{W}_f$ as dissipated by the external friction forces, the balance can be rewritten as $F\dot{d} + M_t\dot{j} = G\left(A_{bit}\dot{d}\right)^{\overline{D}/3} + mFR\dot{j}$, where G is the energy dissipated on the fractal free surface of the material (by fracture and internal friction); A_{bit} is the effective area of the tool ring, m is the friction coefficient between the two materials and R the mean radius of the drilling tool. From experiments on concrete we know that the fractal exponent for drilling detritus is $\overline{D} \cong 2$ and the fractal drilling strength $G \cong 15\text{MN}/\left(\text{ms}^{1/3}\right)$. So, we can describe the process from a global point of view and predict the drilling velocity[22,31].

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