Size Effects on Concrete Tensile Fracture Properties: An Interpretation of the Fractal Approach Based on the Aggregate Grading

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ABSTRACT

The fractal approach to the mechanics of materials with disordered microstructure allows an elegant and unified explanation of the size effects the cohesive model parameters (i.e. the tensile strength and the fracture energy) are subjected to. Since, in normal strength concrete, the weakest link is represented by the interfaces between the aggregate and the cement matrix, it seems reasonable to connect the localization of the damage with the aggregate distribution inside concrete. The aim of this work is to show how fractal patterns in the tensile failure of concrete specimens can be derived by the grain size distribution of the aggregates composing the concrete. The most common aggregate grading, the Füller sieve curve, will be considered and expressed in terms of probability density function of the diameter of the grains. It is found that the surface of the aggregate inside the damaged band where the main crack will grow in the last stage of the tensile failure can be modelled by an invasive fractal set of dimension 2.5. By stereology concepts, we obtain also the probability density function of the diameters of the circles interception of the grains with a plane cutting the damaged band; this function shows that the resistant cross section can be modelled by a lacunar fractal set (i.e. with dimension lower than 2). Therefore fractality is proven and consequent size effects on strength and fracture energy can be pointed out.

1. INTRODUCTION

One of the most important topics in solid mechanics is the study of the so-called *size effect*, whose importance has been widely recognized during the last decades. By size effect we mean the dependence of one or more material parameters on the size of the structure made by that material. Particularly, on concrete-like materials tensile strength decreases with the size, whereas fracture energy increases.

The increase of brittleness along with the structural size is mainly due to the localization of the damage in the failure process of the structures composed by quasi-brittle materials. Nowadays, the most widely used model to describe damage localization in materials with disordered microstructure (also called quasi-brittle or concrete-like materials) is the *cohesive crack model*, introduced by Hillerborg *et al.* /1/. According to Hillerborg's model, the material is characterized by a stress-strain relationship (σ-ε), valid for the unbroken zones (Figure 1a), and by a stress-crack opening displacement relationship (σ-w, the cohesive law), describing how the stress decreases from its maximum value σ_u to zero as the distance between the crack lips increases from zero to the critical displacement w_c (Figure 1b). The area below the cohesive law represents the energy G_F spent to create the unit crack surface. The cohesive crack model is able to simulate tests where high stress gradients are present, e.g. tests on pre-notched specimens. On the other hand, relevant scale effects are encountered also in uniaxial tensile tests on dog-bone shaped specimens /2,3/ (see Figure 2a), where smaller stress gradients are present. Figure 2b clearly shows that the cohesive law and its parameters G_F and σ_u are size-dependent. For this kind of tests, size effects should be ascribed to the material behavior rather than to the stress-intensification. Anyway, they cannot be predicted by the cohesive crack model.

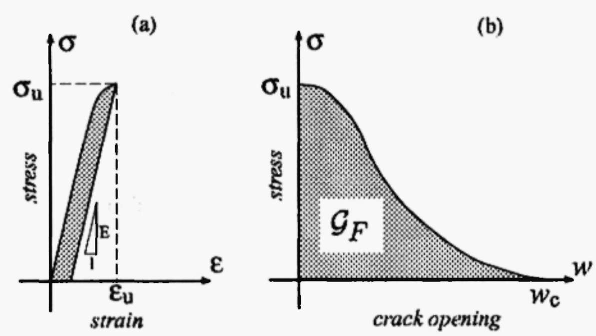


Fig. 1: The cohesive crack model by Hillerborg *et al.* /1/.

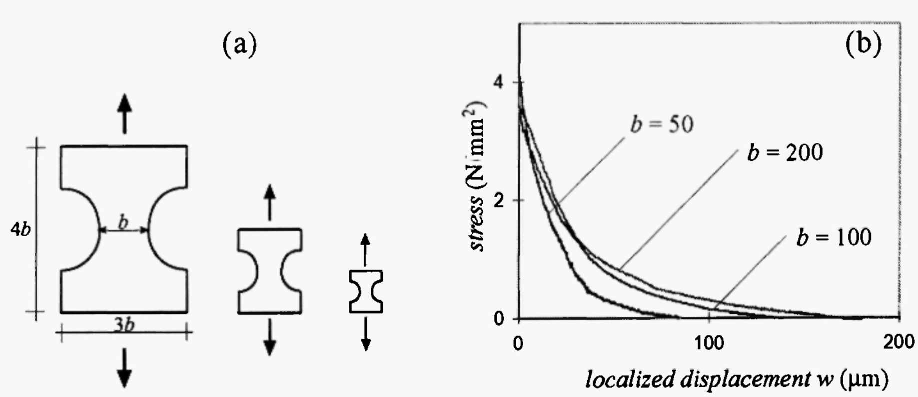


Fig. 2: Uniaxial tensile tests /2/ over dog-bone shaped specimens (a): cohesive laws (b) for different specimen sizes.

In order to overcome the original cohesive crack model drawbacks, a *scale-independent* (or *fractal*) *cohesive crack model* has been proposed recently /4/. This model is based on the assumption of a fractal-like damage localization. The fractal nature of the damage process allows a coherent explanation of the size effect on the cohesive crack model parameters. In fact, if the damage zone is fractal, both the geometrical quantities (i.e. the area of the resistant cross section, the area of the crack surface and the thickness of the damaged band) and the physical quantities (i.e. the tensile strength, the fracture energy and the critical displacement) become nominal quantities and are, therefore, size-dependent. On the other hand, fractal quantities should be used: they are the true scale-invariant material parameters. The fractal strength and fractal fracture energy were introduced by Carpinteri in 1994 /5/, while the fractal critical strain has been introduced recently /4/.

In concrete structures, the hypothesis of damage domains showing fractal patterns received several experimental confirmations (see, for instance, /6,7/). The aim of the present paper is to provide a theoretical explanation of the fractality of the concrete damage domains based on the aggregate size distribution. As is well-known, the weakest link in normal strength concrete is represented by the interface between the cementitious matrix and the aggregates. Therefore it seems reasonable to look for a link between the aggregate grading and the fractal features (e.g. the fractal dimension) of the damage zones. Attention will be focused on the computation of the fractal dimensions (i) of the region where energy is dissipated during the failure process and (ii) of the region where stress is transferred at the peak load. Eventually, the good agreement between the results of the present analysis and the ones predicted by the multifractal scaling laws (MFSL) proposed by Carpinteri *et al.* /8/ will be shown.

The main tool we will use to perform the fractal analysis is *Stereology*, which encompasses the geometrical probability aspects of the problem. Regarding the stereology applied to concrete, we refer to the works by Stroeven /9,10,11/, while useful fractal concepts can be found in the book by Turcotte /12/, whose fragmentation analysis is close to the stereological one we are going to develop.

2. THE AGGREGATE SIEVE CURVE AND THE RELATED GRAIN SIZE DISTRIBUTION FUNCTION

The basis for the dimensional characterization of the concrete aggregate is the sieve analysis. The sieve curve describes the weight fraction $W(d)$ of the aggregate passing through a sieve with d-wide mesh. Thanks to its good packing property, the most common sieve curve used to prepare concrete is the so-called Füller curve:

$$W(d) = \sqrt{d / d_{\max}} \tag{1}$$

Henceforth we will always refer to the Füller aggregate size distribution. Furthermore, we will assume the aggregates to be spherical with diameter d, with maximum diameter d_{max} and minimum d_{min}, equal respectively to 20 mm and 0.2 mm (hence d_{min}/d_{max} = 1/100). Usually d_{min} = 0.2 mm and d_{max} = 30 mm, even if, in large buildings, the largest size can be proportional to the size of the structure: for instance, in dams d_{max} can be set equal to 120 mm.

It can be easily shown /10/ that the Füller sieve curve (1) can be expressed in terms of grain size distribution function as follows:

$$f(d) = 2.5\,\frac{d_{min}^{2.5}}{d^{3.5}} \tag{2}$$

where $f(d)$ is a probability density function (PDF), i.e., $f(d)\,\mathrm{d}d$ is the fraction of grains with diameter belonging to the interval $[d, d + \mathrm{d}d]$ and $\int_{d_{min}}^{d_{max}} f(d)\,\mathrm{d}d = 1$. Eqn (2) shows clearly that the number of the small particles is higher than that of the large particles, since the former ones must fill the gaps between the latter ones.

Our analysis needs the first three moments of the PDF (2). The first moment represents the average diameter in the concrete volume, while the second and third are proportional to the average grain area and volume:

$$\overline{d_V} = \int_{d_{min}}^{d_{max}} d\, f(d)\,\mathrm{d}d = \tfrac{5}{3} d_{min}\left[1-\left(\frac{d_{min}}{d_{max}}\right)^{1.5}\right] \simeq 1.7\, d_{min} \tag{3}$$

$$\overline{d_V^2} = \int_{d_{min}}^{d_{max}} d^2\, f(d)\,\mathrm{d}d = 5 d_{min}^2\left[1-\left(\frac{d_{min}}{d_{max}}\right)^{0.5}\right] \tag{4}$$

$$\overline{d_V^3} = \int_{d_{min}}^{d_{max}} d^3\, f(d)\,\mathrm{d}d = 5 d_{min}^{2.5}\, d_{max}^{0.5}\left[1-\left(\frac{d_{min}}{d_{max}}\right)^{0.5}\right] \tag{5}$$

The last factors in the previous expressions are close to the unity; nevertheless, differently than in other approaches /9/, we prefer not to neglect them.

In order to study the weakening effect of the aggregate in the stress transfer mechanism inside concrete, the size distribution of the particles on a plane is requested. To fix the ideas, let us refer to a concrete cube of side b cut by a plane α as shown in Figure 3a where, for the sake of clarity, only a few particles have been drawn. We denote by N_V and N_P respectively the number of grains inside the volume and the number of grains intercepted by the plane α. Considering now the grains of size d, it can be observed that only the fraction d/b is cut by α (see Figure 3b). Therefore, the link between N_P and N_V is straightforward:

$$N_P = \int_{d_{min}}^{d_{max}} \frac{d}{b} N_V\, f(d)\,\mathrm{d}d = \frac{N_V}{b}\overline{d_V} \tag{6}$$

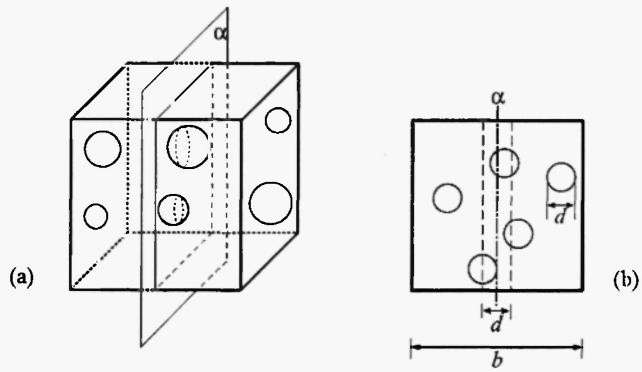


Fig. 3: A concrete specimen: grain distribution in the bulk (a) and on a plane α (b).

Wishing to find the relationship between the concrete volume and the number of grains inside it, we need one more parameter, i.e. the volume fraction f_a of the aggregates. This fraction must be high even if not too high, since the aim is a good particle packing but also a sufficient fluidity of the mixture when the concrete is cast. In normal strength concrete the aggregate occupies about three fourths of the total volume; hence $f_a \simeq 0.75$. The total number of particles inside the cube of side b is, therefore:

$$N_V = \frac{f_a b^3}{\frac{\pi}{6} \bar{d}_V^3} \tag{7}$$

This means that, when the above data are considered, the particles inside 1 m^3 of concrete are about 4 billions!

As stated before about the grains of size d, only the fraction d/b is cut by a generic plane α (see Figure (3b)): denoting by $g(d)$ the PDF of the grain diameters intercepted by the plane α, we get the fraction of the intercepted particles with diameter between d and $d + \mathrm{d}d$:

$$g(d)\,\mathrm{d}d = \frac{\frac{d}{b} N_V\, f(d)\,\mathrm{d}d}{N_P} \tag{8}$$

By applying Eqn. (6), we get the relation among the PDFs in the bulk and in the plane:

$$g(d) = \frac{d}{\bar{d}_V} f(d) \tag{9}$$

The comparison between functions (9) and (2) shows that the probability density of the large particles is higher over the plane than in the volume. This is because the probability of a plane to cut a particle is proportional to its size. Consequently, it is reasonable to expect that the average diameter $\overline{d_P}$ of the distribution (9) will be higher than the average diameter $\overline{d_V}$ in the bulk (Eqn. (3)). In fact:

$$\overline{d_P} = \int_{d_{\min}}^{d_{\max}} d\, g(d)\, \mathrm{d}d = \frac{\overline{d_V^2}}{\overline{d_V}} \simeq 2.7 d_{\min} \tag{10}$$

Let us emphasize that the fraction of the area occupied by the aggregates with respect to the total area of the cross-section is always equal to the volume fraction f_a. What differs is the size distribution and therefore the average values. Wishing to study the influence of the aggregates on a surface (e.g. a crack surface), one must refer to distribution (9) and not to (2). Since the average value over the plane (Eqn. (10)) is larger than the average value in the bulk (Eqn. (3)), in the literature this phenomenon is called *coarsening*.

3. FRACTAL ANALYSIS OF THE AGGREGATE SURFACE

As will be shown in Section 5, according to the *fractal approach*, the size effect on the fracture energy can be explained only if the domain where energy is dissipated is fractal. If it is not, no size effect should be expected.

We will make some hypotheses about the region where dissipation takes place and we will perform a fractal analysis of this region. By fractal analysis we mean essentially the computation of the fractal dimension of a given set. The easiest way to do it is probably to measure the area of the set at different resolution levels and to study how the measure varies on increasing the resolution. If it diverges, the set is an *invasive* fractal (i.e. dimension larger than 2); if it tends to zero, it is a *lacunar* fractal (i.e. dimension lower than 2); if it converges to a finite value different from zero, it is simply an Euclidean surface. The exponent of the power law describing the graph of the measure versus the resolution represents the difference from 2 of the fractal dimension of the given set. For the sake of simplicity, the resolution level will be identified by the grain diameter itself, even if, formally, the resolution should be the inverse of the diameter. For instance, a resolution of level d means to be able to capture the contribution of particles whose diameter is larger than or equal to d.

Now, let us consider the simple scheme of the uniaxial tensile test of a normal strength concrete specimen drawn in Figure 4. As experimentally detected, we assume the crack to be intergranular; i.e. no grain breaks. The grains remain attached to the side of the specimen containing their larger side. We further assume the portion of the crack that lies in the cementitious matrix to be flat.

About the energy dissipation region, we will make two different hypotheses: the former assumes this region to be the *crack surface* (Figure 4b), the latter assumes as dissipation zone the surface of all the grains included in a *damaged band* of thickness a (Figure 4a).

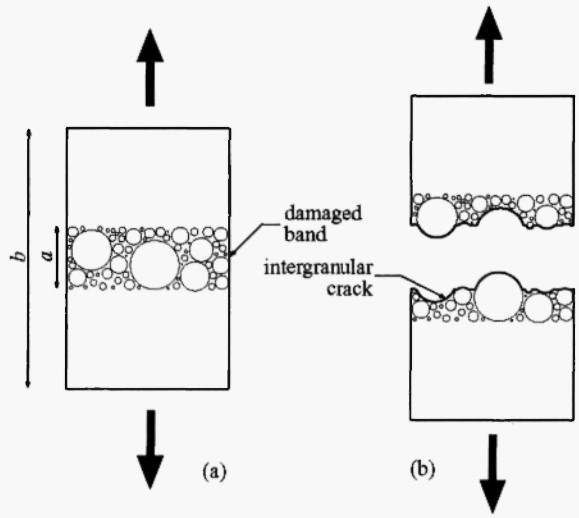


Fig. 4: Concrete specimen tensile test: damaged band (a) and intergranular crack (b).

Regarding the former hypothesis, we see from Figure 4b that the crack surface is composed of the surface of the cementitious matrix plus the surface of the grains. The area occupied by the matrix is simply equal to the fraction $(1 - f_a)$ of the cross-section while, increasing the measure resolution, more and more grains can be detected; hence the measure of the crack surface increases with the resolution. An upper bound for the portion of the crack area occupied by the grains is the sum S of the areas of the surfaces of all the grains cut by the plane of the crack, whose distribution is given by (9). S is a function of the resolution d according to:

$$S(d) = \int_{d}^{d_{max}} N_P\, g(d)\, \pi d^2\, \mathrm{d}d = \frac{5\pi N_P\, d_{min}^{2.5}\, d_{max}^{0.5}}{d_V}\left(1 - \sqrt{\frac{d}{d_{max}}}\right) \qquad (11)$$

As d tends to zero (infinite resolution), $S(d)$ converges to a finite value (see Figure 5). As stated above, this means that the crack surface is not a fractal set: it is simply a rough surface. Therefore, no size effect over the fracture energy should appear. Since this is not the case, we reject the former hypothesis and consider the latter one, according to which energy is dissipated over the interface between the matrix and the grains included in an a-wide damaged band (Figure 4a). In this case the area $A_{dis}(d)$ of all the interfaces is:

$$A_{dis}(d) = \int_{d}^{d_{max}} \frac{a}{b} N_V\, \pi d^2 f(d)\, \mathrm{d}d = 5\pi \frac{a}{b} N_V\, d_{min}^2 \left[\left(\frac{d_{min}}{d}\right)^{0.5} - \left(\frac{d_{min}}{d_{max}}\right)^{0.5}\right] \qquad (12)$$

where, for the sake of simplicity, we have assumed a specimen height equal to b. As the resolution increases (d tends to zero), $A_{dis}(d)$ diverges. This means that in a certain range of scale, the set represented by the

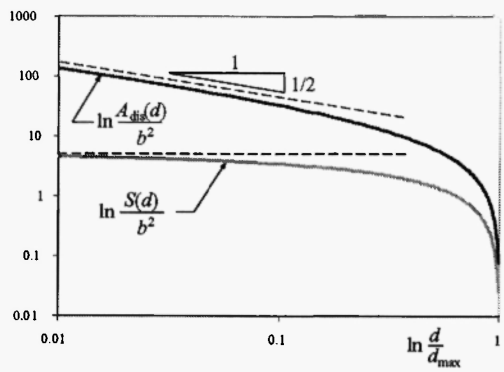


Fig. 5: Bilogarithmic and dimensionless plot of the area S of the grains belonging to the fracture surface (graay line) and of the area A_{dis} of the aggregate-matrix interface inside the damaged band (black line) vs. measure resolution.

interfaces shows a fractal behavior. Further, as shown in Figure 5 where S and A_{dis} vs. measure resolution are plotted in a bilogarithmic, dimensionless diagram (assuming a equal to $3d_{max}$), Eqn. (12) is very well approximated, for not too low resolutions, with a power law with an exponent equal to –0.5: this fact allows us to state that the aggregate surface inside the damaged band can be modelled by an *invasive fractal set* of dimension 2.5. The model is therefore able to predict the size effect over the fracture energy in tensile tests, as will be pointed out in Section 5.

Finally, observe that, even if less intuitive in comparison with the former hypothesis about energy dissipation, there are experiments that lead one to believe the latter hypothesis to be more realistic than the former one. For instance, acoustic emission analyses during tensile testing performed by Shah over concrete specimens /13/ showed that several damage phenomena take place inside the bulk all along the failure process. While at an early stage of loading they are spread all over the volume, near and after the peak load damage phenomena concentrate in a narrow band where, at the very last stage of the softening regime, the main crack appears. For these reasons, Bazant and Oh /14/ developed their crack band model. Our model is however more complex, since we consider the damage to be fractal-like inside the damage band.

4. FRACTAL ANALYSIS OF THE CEMENTITIOUS MATRIX SURFACE

Now we turn our attention to the stress transfer inside the damaged band of the concrete specimen subjected to a tensile load (Figure 4). As stated in the previous section, we assume that inside the damaged zone, microcracks develop all around the aggregate near by the peak load. Therefore the size distribution of the two-dimensional circles interception of the grains with the plane of the section (Figure 6a) could be considered as representing the spatial size distribution of the initial bond cracks that have weakened the

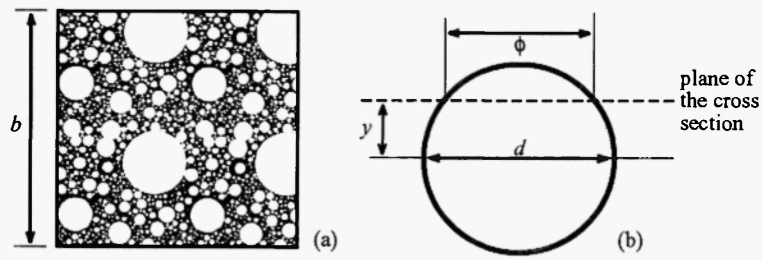


Fig. 6: Cross-section of a concrete specimen: 2D distribution of the grain circles intercepted by the section plane (a) and detail of a particle (b).

cross-section in an early stage of loading /9/. Denoting by ϕ the diameter of the circle intercepted by the section plane cutting a grain of diameter d (see Figure 6b), the following relation holds:

$$y = \frac{\sqrt{d^2 - \phi^2}}{2} \tag{13}$$

where y is the distance of the center of the spherical grain from the plane of the cross-section. Based on the PDF $g(d)$ of the diameters d of the particles intercepted by a plane (Eqn. (9)), we intend to obtain the PDF $r(\phi)$ of the diameters ϕ of the circles belonging to the cross-section. The total number of circles belonging to the section is obviously N_P (Eqn. (6)). The number of circles with diameter comprised between ϕ and $d\phi$ is given by the sum of the compound events to find a particle with diameter $d > \phi$ whose center shows a distance from the section plane comprised between y and $y + dy$. Since we are dealing with continuous distributions, the sum, in mathematical terms, is an integral between ϕ and d_{max}, if $\phi > d_{mim}$, and between d_{min} and d_{max}, if $\phi < d_{min}$. On the other hand, the probability that the center of a sphere lies at a distance y from the section plane is $2dy/d$ ($0 < y < d/2$). Therefore the number of circles with diameter comprised between ϕ and $\phi + d\phi$ is:

$$N_P r(\phi) d\phi = \int_{d_{min}}^{d_{max}} N_P g(d) \frac{2dy}{d} dd \tag{14}$$

The link between y and ϕ is given by Eqn. (13). Eqn. (9) as well as the differentiation of Eqn. (13) lead to the following result for the PDF $r(\phi)$:

$$r(\phi) = \begin{cases} \dfrac{\phi}{d_V} \displaystyle\int_{d_{min}}^{d_{max}} \frac{f(d)}{\sqrt{d^2 - \phi^2}} dd & \text{if } 0 \le \phi < d_{min} \\ \dfrac{\phi}{d_V} \displaystyle\int_{\phi}^{d_{max}} \frac{f(d)}{\sqrt{d^2 - \phi^2}} dd & \text{if } d_{min} \le \phi \le d_{max} \end{cases} \tag{15}$$

In the case of the Füller sieve curve, $f(d)$ is given by Eqn. (2) and no analytical solutions can be found for the function $r(\phi)$. In view of a numerical integration, it is convenient to change the variable of integration d with $x = \phi/d$. In this way Eqn. (15) becomes:

$$r(\phi) = \begin{cases} \dfrac{5d_{\min}^{2.5}}{2d_V\phi^{2.5}} \displaystyle\int_{\frac{\phi}{d_{\max}}}^{\frac{\phi}{d_{\min}}} \dfrac{x^{2.5}}{\sqrt{1-x^2}}\,dx & \text{if } 0 \le \phi < d_{\min} \\ \dfrac{5d_{\min}^{2.5}}{2d_V\phi^{2.5}} \displaystyle\int_{\frac{\phi}{d_{\max}}}^{1} \dfrac{x^{2.5}}{\sqrt{1-x^2}}\,dx & \text{if } d_{\min} \le \phi \le d_{\max} \end{cases} \tag{16}$$

First of all, note that the PDF $r(\phi)$ is different from zero in the interval $[0, d_{\max}]$ while $f(d)$ and $g(d)$ are different from zero only in the interval $[d_{\min}, d_{\max}]$. This is because there is no grain in the bulk with diameter lower than $d_{\min}$, while circles on the cross-section can also have a diameter comprising between $[0, d_{\min}]$. Moreover, since $r(\phi)$ is a PDF, its integral over the whole interval $[0, d_{\max}]$ is equal to 1. Then note that the integrand function of Eqn. (16) has a singular point in $x = 1$. In order to avoid numerical problems caused by this singularity, we can perform the numerical integration by the Gauss-Chebyshev formula:

$$\int_{-1}^{+1} \frac{f(x)}{\sqrt{(1-x^2)}}\,dx \approx \frac{\pi}{n}\sum_{i=1}^{n} f\left(\cos\frac{2i-1}{2n}\pi\right) \tag{17}$$

where n is the number of the nodes of the quadrature formula. The result is plotted in Figure 7. Note that function $r(\phi)$ is continuous over the whole domain of existence, even if it has a singularity for $\phi = d_{\min}$.

We are now able to compute the area A_{res} occupied by the cementitious matrix on a generic cross-section varying the measure resolution. As stated above, this area represents the true resistant cross-section at the peak load, since the bond cracks fully develop around the grains inside the damaged band.

The area of the matrix (the black side of Figure 6a) can be found by subtracting from the nominal area b^2

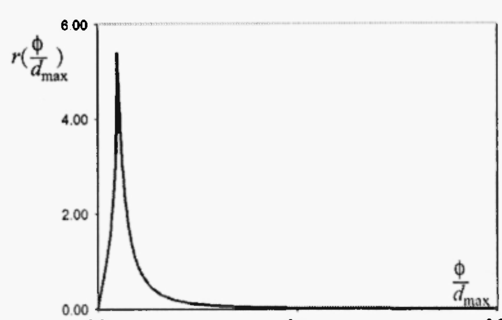


Fig. 7: Probability density function of the diameters of the grain circles intercepted by the plane of the cross-section.

of the cross-section the area occupied by the grains. On increasing the measure resolution (i.e. decreasing ϕ), more and more grains can be detected and subtracted from the nominal area. That is:

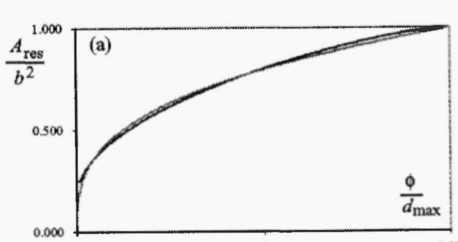
$$A_{\mathrm{res}}(\phi) = b^2 - \int_{\phi}^{d_{\max}} N_{\mathrm{P}} r(\phi) \frac{\pi\phi^2}{4} \mathrm{d}\phi \tag{18}$$

Upon substitution of Eqns. (6) and (7) and dividing by b^2, we get:

$$\frac{A_{\mathrm{res}}(\phi)}{b^2} = 1 - \frac{3 f_{\mathrm{a}} \overline{d_{\mathrm{V}}}}{2\overline{d_{\mathrm{V}}^3}} \int_{\phi}^{d_{\max}} f(\phi)\phi^2 \, \mathrm{d}\phi \tag{19}$$

Using the numerical evaluation of $r(\phi)$ previously performed, the dimensionless matrix area (19) can be plotted in Figure 8. In the same figure we also plotted the graph of the area of a deterministic fractal set of dimension 1.67. This graph is exactly a power law and a straight line respectively in Figures 8a and 8b. The two curves are very close. We can therefore conclude that the resistant cross-section inside the damaged band can be realistically modelled by a *lacunar fractal set* of dimension 1.67. Hence the model is able to predict the size effect over the tensile strength, as will be pointed out in Section 5.

Finally, observe that the fractal dimension of the resistant cross-section given by the analysis of Eqn. (19) depends on the volume fraction occupied by the aggregate f_{a}. More in detail, for very high values of the volume fraction $(f_{\mathrm{a}} \simeq 0.9)$ the fractal dimension tends to the value 1.5, while for low values of f_{a} it tends to 2, i.e. the dimension of an Euclidean surface. In other words, the aggregate is the cause of disorder inside the concrete microstructure. Eliminating the aggregates, both the fractal features and the related power laws describing the size effect of mechanical properties (see next section) disappear.



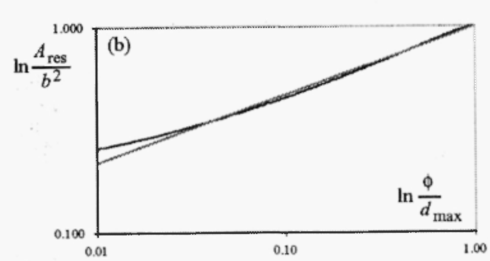


Fig. 8: Dimensionless area of the resistant cross-section (black line) and of a deterministic lacunar fractal set (gray line) vs. measure resolution (a); bilogarithmic plot (b).

5. SIZE EFFECTS ON TENSILE FRACTURE PROPERTIES

In the present section, we give a brief explanation of why fractal damage domains can justify size effect of concrete tensile properties. We obtain the size effect laws for the fracture energy and the tensile strength, whose derivation is a straightforward application of the fractal concepts developed in the previous sections. For a deeper insight into theoretical aspects of the fractal approach and its link with the Renormalization Group Theory, the reader is referred to the papers by Carpinteri /2,4,5/.

Consider the work W necessary to break a concrete specimen of cross section b^2. It is equal to the product of the fracture energy G_F times the nominal fracture area $A_0 = b^2$. On the other hand, we showed that the surface where energy is dissipated is not the flat cross-section: it is the surface of the aggregates inside the damaged band, whose area A_{dis} diverges as the measure resolution tends to infinity (Eqn. (12)). Therefore the fracture energy should be zero, which is meaningless. Finite values of the set where energy is dissipated can be achieved only via the fractal (Hausdorff) measure /15/ of the set considered, $A^*_{dis} \propto b^{2.5}$, to which the *fractal fracture energy* G^*_F corresponds:

$$W = G_F A_0 = G^*_F \; A^*_{dis} \Rightarrow G_F = G^*_F b^{0.5} \tag{20}$$

where G^*_F is the true scale invariant material parameter, whereas the nominal value G_F is subjected to a size effect described by a power law. Physically, it represents the increment of the fracture energy increasing the structural size.

A similar argument holds for the tensile strength. The maximum tensile load F is equal to the product of the strength σ_u times the nominal area $A_0 = b^2$. On the other hand we showed that the surface of the resistant cross-section is not the whole section: it is the portion of the surface occupied by the cementitious matrix, whose area A_{res} tends to zero as the measure resolution tends to infinite (Eqn. (19)). Therefore the strength should be infinite, which is meaningless. Finite values of the set where stress is transferred can be ahieved only via the fractal (Hausdorff) measure /15/ of the set considered, $A^*_{res} \propto b^{1.67}$, to which the *fractal strength* σ^*_L corresponds:

$$F = \sigma_u A_0 = \sigma^*_u A^*_{res} \Rightarrow \sigma_u = \sigma^*_u b^{-0.33} \tag{21}$$

where σ^*_L is the true scale invariant material parameter, whereas the nominal value σ_u is subjected to a scale effect described by a power law. Physically, it represents the decrement of the tensile strength as the structural size increases. Note that, on increasing f_a, the fractal dimension of the region carrying the stress can diminish till 1.5, i.e. the exponent of the power law (21) can reach the value –0.5. Anyway, the absolute value 0.5 is the upper bound of the exponents of the power laws (20) and (21), since they were derived in the hypothesis that all the bond interfaces fail inside the damage band. Therefore the Eqns. (20) and (21) confirm the scaling laws proposed by Carpinteri /5/:

$$G_{\mathrm{F}} = G_{\mathrm{F}}^{*} b^{+d_G}, \ \sigma_{\mathrm{u}} = \sigma_{\mathrm{u}}^{*} b^{-d_\sigma} \tag{22}$$

where he assumed $(d_G + d_\sigma) \leq 1$. Furthermore, there is agreement also with the Multifractal Scaling Laws proposed by Carpinteri *et al.* /8/, according to which the absolute value ½ for the scaling exponents d_G and d_σ is valid only at the smallest scales, where the influence of the microstructure disorder is greatest.

To conclude, we can say that we have shown how fractal patterns and related size effects, often detected in concrete tensile tests, can be seen as a consequence of the aggregate size distribution. Through stereological concepts and fractal analysis, we have tried to give a theoretical description of the role of the aggregate in the tensile failure of concrete specimens. This seems particularly interesting, since, as pointed out recently /16/, the aggregate is the cause of the tougher behavior of the concrete with respect to the mortar and to the hardened cement paste. In this direction, further experiments and analyses should be carried out in order to highlight the effects of the aggregate grading and of the aggregate volume fraction. Finally, in order to validate the fractal cohesive crack model /4/, a stereological analysis of the points where softening takes place along a concrete "fibre" inside the damage band /17/ should be performed in order to get the size effect on the critical displacement by the same arguments traced in this paper.

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