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A fractal model for simulating the formation of microcracks in the fracture process zone and a theoretical explanation of the size effect of the fracture energy of concrete

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Contribution by A. Carpinteri and B. Chiaia

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The paper deals with a fractal-tree model for simulating the formation of microcracks in the fracture process zone of concrete. The fractal-tree model for crack branching and propagation is not original, and has been proposed by various scientists in the last few years.[1–3]

Some basic mistakes should be pointed out in the paper by Ji *et al*., for the authors' and readers' precision. First of all, if invasive fractal sets, like fracture surfaces, are considered, their Euclidean measure diverges to infinity and there is no intrinsic area of the set. Instead, the fractal measure of the domain should be adopted with anomalous dimension depending on the fractal dimension D of the set.[4,5] Thus, the concept of 'total area of the fracture surface', put forward by Ji *et al*., represents a trivial conceptual error.

By considering only a geometrical approach, the authors obtain a size–effect law for the fracture energy of cement-based materials which fits the test data satisfactorily. Quite surprisingly, the authors seem to be unaware of many previous papers on this subject which have been published by our team in several international journals.[4–14] The size–effect on the concrete fracture energy has been exhaustively explained by us, by applying a physically based renormalization group approach in the framework of fractal geometry. A lot of papers, dating from 1994 to 1997, report our theoretical and experimental research on this subject.[6–11] We have already shown, and confirmed experimentally, that the fracture energy G_F increases with the size of the specimen by following a power law with an exponent equal to the fractional dimension increment d_G of the fractal surface ($d_G = D_G - 2$).

Moreover, we developed the so-called multifractal scaling law (MFSL), to take into account the asymptotic value of the fracture energy, which is attained with very large specimens.[4,11] This more general scaling law, which encompasses the whole size range of concrete structures, fits data better than the monofractal one, and has profound physical significance. It interprets the transition from microscopic fractal disorder to macroscopic Euclidean order occurring at large scales. From a practical point of view, the MFSL allows the engineer to predict the correct value of the material toughness in real structures, by extrapolating the values measured on smaller-scaled laboratory specimens. Finally, it is worth mentioning also the other MFSL,[6,8,12] which describes the complete scaling behaviour of the nominal tensile strength.

Reply by the authors

We are grateful to Carpinteri and Chiaia for their interest in our paper.

We will try to answer Carpinteri and Chiaia's questions paragraph by paragraph to the best of our knowledge:

- *Paragraph 1*. We admit that the fractal-tree model for crack branching and propagation is not original. During our research of fracture of concrete, we found that fractal geometry was very appropriate to describe the fracturing process of concrete; the purpose of our presenting the fractal-tree model is to show the usefulness and powerfulness of fractal geometry. In our paper we presented only an example of the fractal-tree model; many variations of the tree model could also be formulated to show that the fractal dimension of the fracture process zone is between 1 and 2.
- *Paragraph 2*. We admit that if the fracture surface were mathematically fractal, it would have an infinitely large surface area because the fracture surface would need to be measured with an infinitely small step length. However, we believe that the fracture surface is one of the 'natural fractal' objects. A 'natural fractal' object is not truly fractal—it does not possess self-similarity over all scales. Thus, below a certain resolution the natural fractal curve becomes smooth and possesses a finite length. So, if we could use a sufficiently small (but finite) step length to measure the surface, it would have a finite surface area. We believe that the total fracture area (A) of concrete could not be infinitely large as Carpinteri and Chiaia claim, because if $A = \infty$, the work (W) to fracture the concrete would be $W = 2\gamma A = \infty$ (where γ is the surface energy), which is not true.
- *Paragraph 3*. Our theoretical deduction and explanation of size effect of fracture energy of concrete is a development of the work by Pfeifer *et al*.[15] The fractal dimension (D_c) in our equation (12) is a compressive parameter: it is a fractal dimension of the whole microcrack network as well as the main fracture surface. The forms of our equation (12) and Carpinteri and Chiaia's size effect equation are basically similar, and both could be experimentally confirmed. It is probable that there is some relationship between our D_c and their D_G.
- *Paragraph 4*. The variation of equations (12) and (13) in our paper could predict theoretically that when the height of the ligament of specimen H is large enough, there will be no size effect of the fracture energy, that is, for a sufficiently large specimen of concrete, we could measure its intrinsic fracture energy. Although up to now we have not confirmed equation (13) experimentally with a large specimen, according to Bazant[16] when the ligament is 500 times the maximum diameter of aggregate, it is possible to find the true fracture parameter for concrete.

Again the fractal dimension (D_c) in our equation (12) or (13) is a compressive parameter: it cannot discriminate the multifractal dimensions of the microcrack network or fracture surface. We also observed the multifractal phenomenon in the porous structure of cement-based materials,[17] and we agree with Carpinteri and Chiaia that it would be appropriate to use multifractal dimensions to describe the fracturing process of concrete.

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