Materials and Structures/Matériaux et Constructions, Vol. **31**, June 1998, pp 303-309

SCIENTIFIC REPORTS

Scaling behaviour and dual renormalization of experimental tensile softening responses

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Paper received: March 17, 1997; Paper accepted: April 22, 1997

ABSTRACT

The aim of the present paper is to investigate the scaling behaviour of the experimental tensile softening (or catastrophic) curves and it concludes a series of papers from the same research group published in *Materials and Structures* [1-4]. Experimentally, it is evident how these curves become steeper by increasing the size of the specimens (ductile-to-brittle transition). To study this phenomenon, a completely new testing apparatus, made up of three orthogonally placed actuators, was built at the Politecnico di Torino by the authors [1, 5-7]. This set-up makes it possible to apply a purely tensile force, so that the secondary flexural stresses, if kept under control, constitute a degree of error comparable to the values allowed for normal testing apparatus. The method enables a stress vs. strain curve to be plotted with the descending (softening) branch up to the point where the cross section of the tensile specimen breaks away. The results of a new experimental investigation, performed over a very large scale-range of unnotched concrete specimens (16:1), will be presented. Particular attention will be paid to the scale effects both on nominal tensile strength and on fracture energy. The renormalized experimental curves will be presented in a load vs. displacement plane characterized by anomalous physical dimensions. The renormalization of experimental curves enables obtaining the same response for all specimen dimensions, and then defining the universal (scale-independent) behaviour of the material.

RÉSUMÉ

Cet article étudie les effets d'échelle sur le comportement des courbes expérimentales de radoucissement à la traction (ou catastrophiques) ; il conclut une série d'articles publiés dans Matériaux et Constructions *par la même équipe de recherche [1-4]. Les expériences démontrent de façon évidente que ces courbes deviennent plus raides en fonction de l'augmentation de la taille des éprouvettes, montrant la transition de la ductilité à la fragilité. Afin d'étudier ce phénomène, un appareil d'essai innovant, composé de trois dispositifs de commande placés orthogonalement, a été construit par les auteurs à la Politecnico di Torino [1, 5-7]. Cet appareil permet d'appliquer une force de traction pure, afin que les contraintes de flexion secondaires, à condition d'être sous contrôle, constituent un degré d'erreur comparable aux valeurs permises pour un appareil d'essai classique. Cette méthode permet de déterminer une courbe contrainte/déformation ayant une branche décroissante (radoucissement) jusqu'au point ou les parties de l'éprouvette en traction se séparent. On présente les résultats d'une nouvelle étude expérimentale, menée sur une gamme d'éprouvettes de béton non-entaillé de dimensions variées (16:1). Une attention particulière est accordée aux effets d'échelle sur la résistance à la traction nominale et à l'énergie de rupture. Les courbes expérimentales normalisées sont présentées sur un plan charge/déplacement caractérisé par des dimensions physiques anormales. La normalisation des courbes expérimentales permet d'obtenir la même réponse pour toutes les dimensions des éprouvettes, et de définir ensuite le comportement universel (indépendant de l'échelle) du matériau.*

1. INTRODUCTION

Interest in the tensile properties of heterogeneous materials, such as concrete and rocks, has increased during the last decades. In particular, scale effects on nominal tensile strength and on fracture energy have been determined. On the other hand, direct tensile tests were performed experimentally only on a fixed specimen size [8-15], while only two series of tests were performed over a limited scale range (1:4) and on notched specimen geometries [16-17]. Slowik *et al.* [18] emphasized again in a recent paper the importance of the specimen size on the experimental determination of the mechanical parameters in direct tensile testing.

Editorial Note:
Prof. Alberto Carpinteri is a RILEM Senior Member. He is a member of the Editorial Group of RILEM Technical Committee 090-FMA (Fracture Mechanics of Concrete) and is involved in the work of TCs 147-FMB (Fracture Mechanics Applications to Anchorage and Bond), 148-SSC (Test Methods for the Strain Softening Response in Concrete) and QFS (Size Effect and Scaling of Quasibrittle Fracture). Prof. Carpinteri was awarded the Robert l'Hermite Medal in 1982.

When an experimental investigation is performed on a set of geometrically similar unnotched concrete specimens over a scale range greater than 10:1, the nominal tensile strength shows a decreasing trend in the bilogarithmic strength vs. size plane, as characterized by a curve with an upwards concavity, $\frac{\partial^2 \ln\sigma_N}{\partial \ln d^2} > 0$. In other words, for size scales tending to infinity, *i.e.* for very large specimens, the nominal tensile strength of concrete structures appears constant, whereas for size scales where random self-similarity holds, *i.e.* at the microscale, the nominal tensile strength increases by decreasing the structural size. This trend is accurately described by the Multifractal Scaling Law, recently proposed by Carpinteri [19] and Carpinteri *et al.* [2, 20-21]. In the physical reality, material ligaments at the peak load can be considered as multifractals, with a dimension of 1.5 at small scales and a dimension of 2 at large scales. This means that, for small scales, a self-similar distribution of Griffith cracks is prevalent, whereas for large scales, the disorder is not visible, with the size of the defects and heterogeneities being limited. A transition from extreme disorder (slope -1/2) to extreme order (zero slope) may therefore be detected in the bilogarithmic strength versus size diagram. A slope equal to - 1/2, therefore, may not be considered as a universal behaviour throughout the whole size range of unnotched structures.

The aim of the present paper is to investigate the scaling behaviour of the experimental tensile softening (or catastrophic) curves. Experiments show an increasingly steeper slope when increasing the size of the specimens, along with a dimensional ductile-to-brittle transition. Moreover, the shape of the softening curves is strictly dependent on the boundary conditions, as shown by van Mier *et al.* [22-23]. This implies that for the time being, the softening curve cannot be regarded and used as a true property of the material.

To study this phenomenon, a completely new testing apparatus made up of three orthogonally placed actuators was proposed at Politecnico di Torino by the authors [1, 5-7]. This set-up makes it possible to apply a purely tensile force so that the secondary flexural stresses, if kept under control, constitute a degree of error comparable to the values allowed for normal testing apparatus. The method enables a stress vs. strain curve to be plotted with the descending (softening) branch up to the point where the cross section of the tensile specimen breaks away. The results of the second experimental investigation, performed on a very large scale-range of concrete unnotched specimens (16:1), will be shown. Particular attention will be paid to the scale effects both on nominal tensile strength and on fracture energy. Van Mier *et al.* [22-23] performed numerical simulations of the first series of the tests presented in [1] by using the lattice model. The simulations indicate that a larger crack density must occur in specimens loaded between fixed platens; this is confirmed by the higher fracture energy measured in the experimental tests. The boundary conditions in the present experiments are the same for all specimen sizes. Therefore, the variation in the fracture energy, as well as in the tensile strength, is not influenced by the boundary conditions.

The renormalized constant values for the nominal tensile strength and for the fracture energy will be obtained, according to the monofractal hypothesis for the ligament at the peak load and for the fracture surface, respectively. Obviously, these constant values can be used only when the property of self-similarity is valid [24]. In general, the values of these two material parameters could be obtained by using the MFSL [2, 4]. The renormalized experimental curves will be shown in a load vs. displacement plane characterized by anomalous physical dimensions. Analogous procedures have been applied by de Arcangelis [25] to the scaling behaviour of the breakdown voltage. The renormalization of experimental curves enables obtaining the same response for all specimen dimensions, and then defining the universal (scale-independent) behaviour of the material.

2. EXPERIMENTAL DETAILS

In 1992, a new testing procedure was proposed at the Politecnico di Torino by the authors to study the phenomenon of the size effect on nominal tensile strength as well as on fictitious fracture energy [1, 5-7]. This set-up makes it possible to control the eccentricities of the tensile load during the tests. To perform this control, three orthogonally placed actuators were used: the central one and the other two placed on the two principal inertia planes of the specimen. The analogic command signal was supplied by the difference in the strains of the extensometers placed on the opposite sides of the specimen. In this way, during the test, the central restrained zone of the specimen was subjected to a uniform strain. In other words, a test machine was obtained which was characterized by an infinite rotational stiffness in the central zone. Unless a snap-back phenomenon occurs, this system allows obtaining complete curves, which are very stable and exhibit no bumps. A complete description of the experimental procedure is reported by Carpinteri and Ferro [1, 5], Carpinteri and Maradei [6], and Ferro [7], in which it is also possible to find a complete review on the experimental set-up designed for testing concrete specimens. The second testing series reported in this paper was selected and planned in order to confirm the MultiFractal Scaling Law over a scale range larger than 10:1 (namely 16:1). In fact, nearly 60 experimental investigations reported in the literature were considered [26], and the conclusion was that, in order to evaluate the trend of the second derivative $\frac{\partial^2 \ln\sigma_N}{\partial \ln d^2}$, it is necessary to have experimental results over a significant large scale-range.

For this reason, five specimen sizes were tested (one more than in the first series), while the testing geometry was kept the same. The thickness of the dog-bone specimens was equal to 10 cm for all of the sizes while the width of the central zone, with a characteristic size of d, was 0.25-0.5-1-2-4 times that of the thickness. The dog-bone specimens were glued to steel supporting plates through an epoxy resin-based bicomponent, and the plates were bolted to the machine. Grip problems

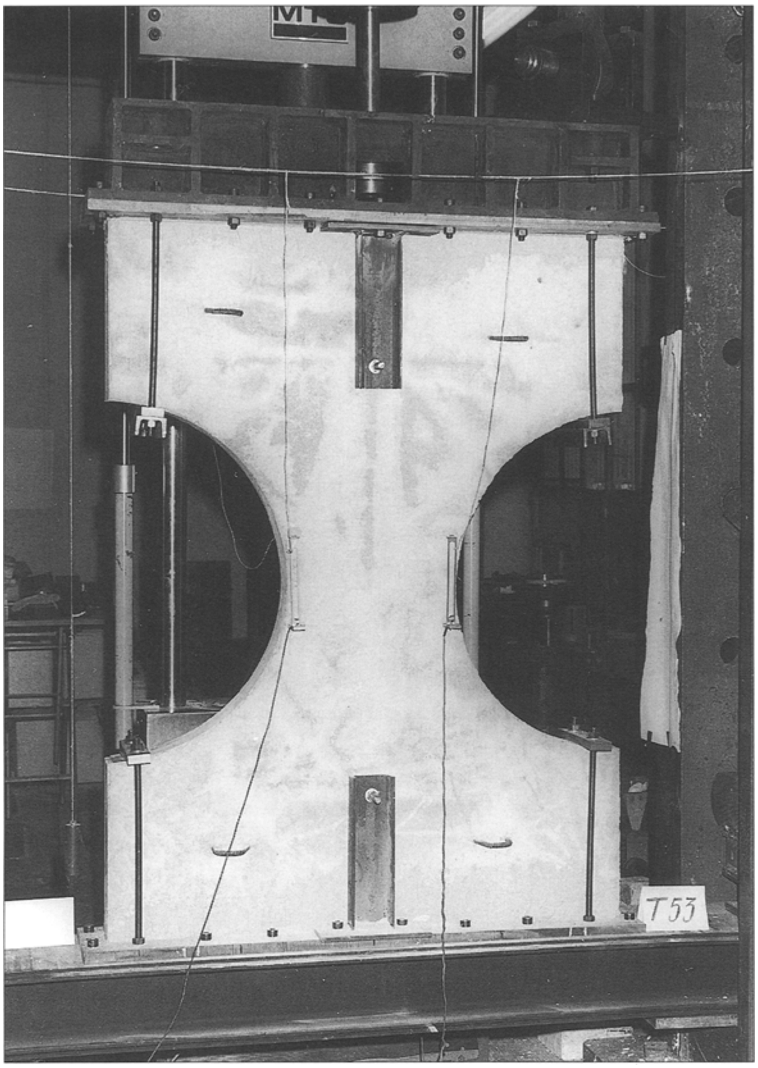

Fig. 1 – Grip system for the largest specimens.

occurred with the largest specimens, so that four lateral stirrups coupled to the wider specimen zone were inserted (Fig. 1). Moreover, two rests were riveted to the steel supporting plates in the central part. In this way, it was possible to obtain the complete force-displacement curves for the largest specimens.

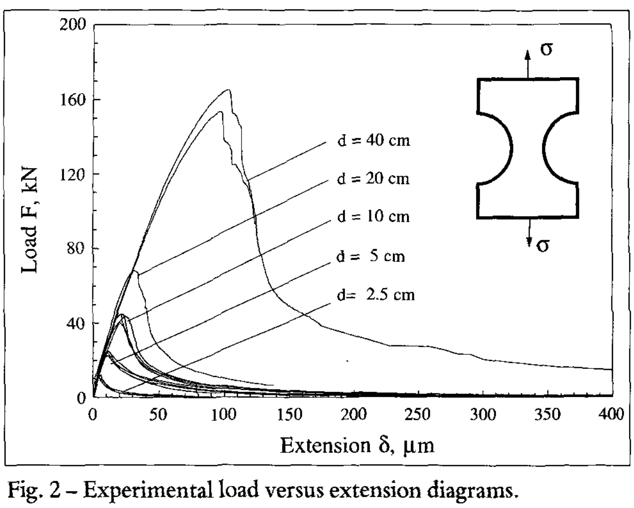

Fig. 2 – Experimental load versus extension diagrams.

3. EXPERIMENTAL RESULTS

In the second series, five specimen sizes were tested, with a transversal cross-section size of 2.5 × 10, 5 × 10, 10 × 10, 20 × 10 and 40 × 10 cm, respectively. Eight specimens for each size were cast. Furthermore, for this series the concrete mixture had a water/cement ratio of 0.5, while the maximum gravel size was d_{max} = 16 mm. The compression strength was obtained with cubic specimens of side length 160 mm; the average value was f_c = 42.7 MPa. For this series, five complete diagrams for each dimension were obtained, except for the dimension d = 20 cm. A diagram with all of the available curves is shown in Fig. 2. For the following diagrams, only one representative curve for the three most significant sizes has been selected, due to graphical reasons. A summary of the experimental results for nominal tensile strength and fracture energy is reported in Table 1.

For the smallest size, specimen B13 was chosen as representative, since it presents values which are closest to the mean values. The extensometers for this size had lengths equal to $2d$, it very being difficult to glue extensometers of length 2.5 cm. For the median size, specimen B38 was chosen. The specimens of height 80 cm, as mentioned previously, presented some problems. A few specimens were tested a greater number of times, since separation of the steel supporting plates occurred. This may justify the low values of both the nominal strength and the nominal fracture energy for this size. In addition, only the diagram of specimen B48 is complete. The values related to this specimen size were not considered in the analysis. For the largest specimens, the grip system has been modified, as described in the previous section. The length of the extensometers was equal to $d/2$, or 20 cm, in order to avoid snap-back problems, which occurred during the first series. However, with a shorter extensometer length, specimen B54 broke outside of the control zone. Specimen B52 was thus chosen as representative for the largest size.

The extensions of the 2.5 cm specimens were divided by two, while those related to the largest size were doubled, in order both to consider the extensometer's length and to obtain the same slope of the elastic portions. It is to highlight the distinct repetitiveness of the softening branches. The pattern of these curves follows a power law with an exponent of between - 0.85 and - 1.25 and a correlation coefficient larger than 0.95.

The average values of the apparent tensile strength are reported in a ln σ_N versus ln d plane (Fig. 4.a). The scale effect is represented by the slope of the linear regression, equal to 0.091. For the first series, a slightly larger value, 0.140, was obtained [1]. The dimensionless load versus extension diagrams for the (more) three representative

Table 1 – Summary of experimental results for tensile strength and fracture energy

Specimen no	Cross Sectional Area cm^2	Tensile Strength σ_N (Mpa)	Fracture Energy $\mathcal{G}_F$ (N/mm)
B13	25	4.91	0.138
B14	25	5.05	0.137
B15	25	4.59	0.180
B16	25	4.62	0.130
B22	50	4.41	0.244
B23	50	4.24	0.194
B25	50	4.73	0.318
B26	50	4.78	0.338
B27	50	4.45	0.197
B28	50	4.90	0.281
B29	50	4.41	0.231
B31	100	4.21	-
B32	100	4.46	-
B34	100	4.59	0.228
B35	100	4.33	-
B36	100	4.42	0.208
B37	100	4.07	0.270
B38	100	4.40	0.239
B45	200	4.16	-
B46	200	3.65	-
B47	200	4.04	-
B48	200	3.35	0.158
B52	400	4.05	0.286
B53	400	3.45	-
B54	400	3.70	-
B55	400	3.69	0.142

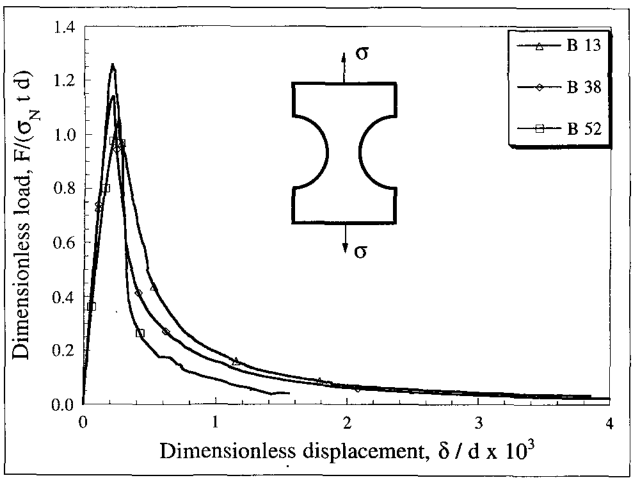

Fig. 3 – Dimensionless load versus extension diagrams, 2nd series.

sizes (smallest, medium and largest size, respectively) are shown in Fig. 3. The variation in the structural behaviour with the size is evident. The larger specimens show a more brittle response in comparison with the smaller ones. The slope of the softening curve for the 40-cm specimen is nearly vertical. If the extensometers had presented a length of 40 cm, the snap-back phenomenon would have occurred.

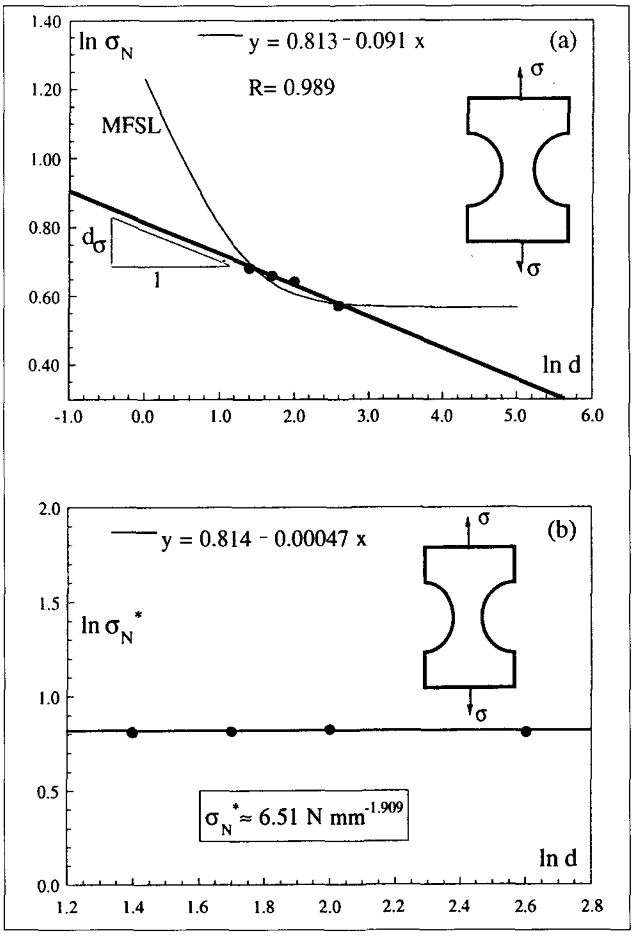

Fig. 4 – (a) Experimental scale effect on nominal tensile strength; (b) renormalized value of tensile strength, 2nd series.

The fracture energy shows the opposite trend, *i.e.* it increases with the specimen size. The average values obtained are reported in Fig. 5.a in the ln $\mathcal{G}_F$ versus ln d plane. The scale effect on $\mathcal{G}_F$, as represented by the slope of the linear regression, is equal to 0.197. In Figs. 4.a and 5.a, the Multifractal Scaling Laws for both the tensile strength and the fracture energy [3] are also reported.

4. RENORMALIZED MATERIAL PROPERTIES

In this section, the procedure presented in Carpinteri [24] and in Carpinteri and Ferro [1] to obtain constant values over a limited range for both the tensile strength and the fracture energy will be presented. The price to pay to achieve this is the loss of the classical physical dimensions for these two material properties. The slope of the linear regression of the nominal tensile strength experimental data is equal to $d_\sigma = 0.091$. According to the monofractal hypothesis, it is possible to affirm that the dimension of the ligament at the peak load is equal to $\alpha = 2 - d_\sigma = 1.909$, a fractal set which is very similar to a two-dimensional sur-

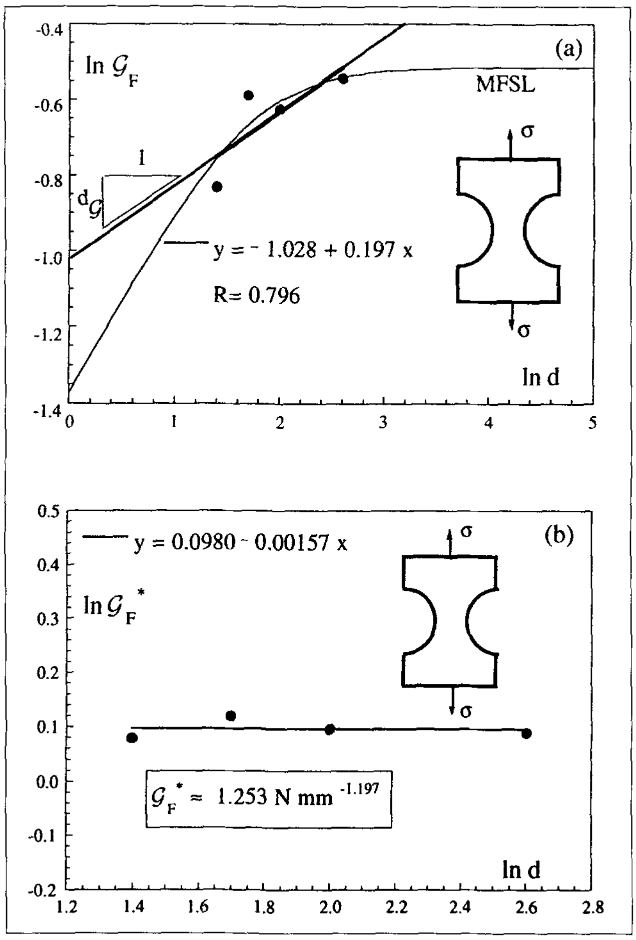

Fig. 5 – (a) Experimental scale effect on nominal fracture energy; (b) renormalized value of fracture energy, 2nd series.

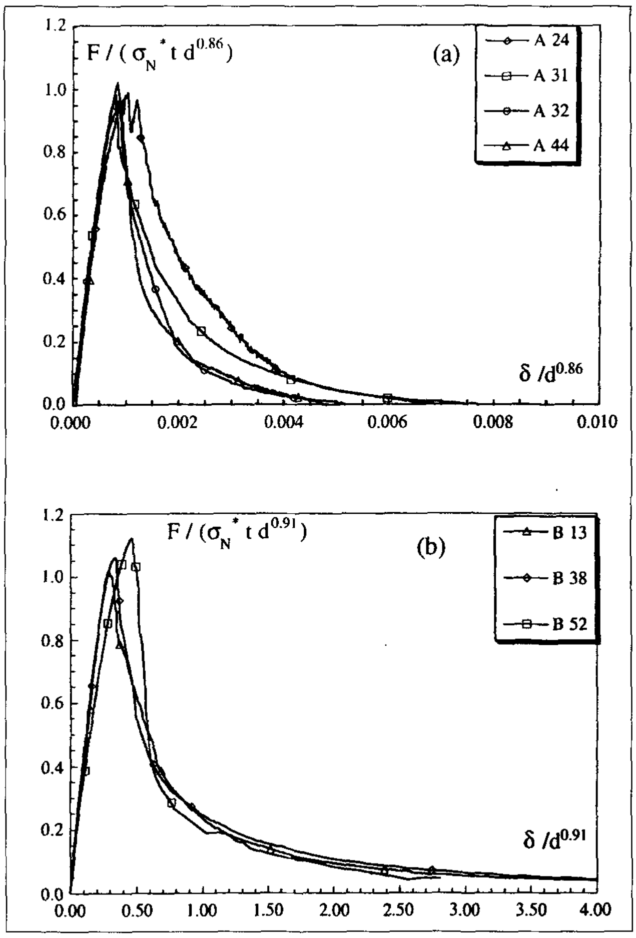

Fig. 6 – Renormalized experimental diagrams using the fractal dimension of the ligament at the peak load: (a) 1st series; (b) 2nd series.

face. The fractal nature of the material ligament emerges very clearly at the size scales of the specimens. On the other hand, the property of self-similarity is very likely to vanish or change at higher or lower scales, owing to the limited character of the particle size curve. From Fig. 4.a, it is clear that the monofractal hypothesis represents the mean tangent of the MFSL in the range of the test results. When the extrapolation of the tensile strength value for very large or very small structures is required, the MFSL should be used.

The relationship between nominal strengths related to the different sizes is:

$$\sigma_N^{(2)} = \sigma_N^{(1)} d^{-d_\sigma} \quad (1)$$

Using equation (1) it is possible to find the value of the renormalized strength σ_N^* of physical dimensions [*force*] × [*length*]$^{-1.909}$, with the ligament being a surface of dimension 1.909. The values of σ_N^* are almost constant with a varying d. The linear regression in the ln σ_N^* versus ln d plane gives a value equal to $\sigma_N^* = 6.51$ N mm$^{-1.909}$, while the slope of the straight line is nearly equal to zero (Fig. 4.b).

The same procedure can be performed in order to obtain a constant value of fracture energy with respect to the specimen dimension. The slope of the linear regression of the fracture energy experimental data is equal to $d_{\mathcal{G}} = 0.197$ (Fig. 5.a). In this case, the renormalized value of the fracture energy is obtained by supposing that the energy dissipation occurs in a fractal space of dimension $\alpha = 2.197$. The renormalized value results in being equal to $d_{\mathcal{G}}^* = 1.253$ Nmm$^{-1.197}$ (Fig. 5.b). Even in this case, from Fig. 5.a, it can be observed that the monofractal hypothesis is valid only over a limited range, where the property of self-similarity holds. The linear regression equals the mean tangent to the MFSL over the interval of the experimental results.

5. RENORMALIZED DIAGRAMS OF STRUCTURAL RESPONSE

The renormalization of the diagrams presented in Section 3 will be considered, so that the dispersion due to the scale effect is minimized with a tendency to superimpose the diagrams.

The experimental F vs. δ curves are characterized by two different regimes. The first regime corresponds to the pre-peak elastic stage, when microcracks form randomly in the specimen. In this stage, the external force increases linearly until it reaches the peak value and the statistical fluctuations are very small. In the second stage, which could be called the "catastrophic regime", the interactions between microcracks begin to dominate the

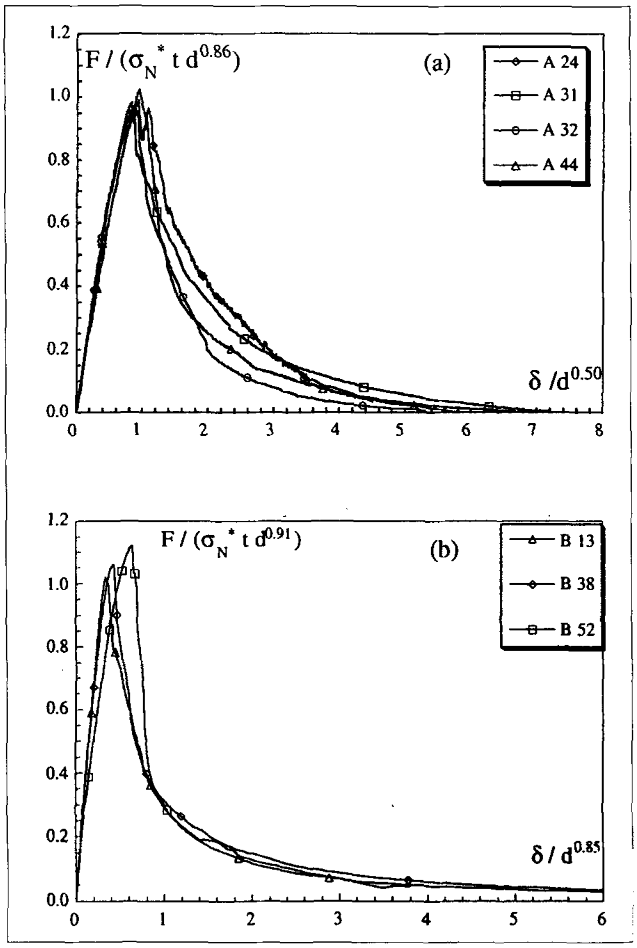

Fig. 7 – Renormalized experimental diagrams using two different best-fitting exponents: (a) 1st series; (b) 2nd series.

process, until a macrofracture forms and propagates through the whole specimen.

The renormalization of the experimental curves represents the determination of the scaling law for forces and/or for displacements. The experimental diagrams related to the various sizes can be rescaled by an appropriate power of d, so as to obtain the best superposition of the curves:

$$F = d^{\alpha}\Phi\left(\delta d^{-\beta}\right) \qquad (2)$$

where Φ is a scaling function.

The F vs. δ curves, in an initial attempt, have been plotted using a unique value for the exponent, equal to the fractal dimension obtained from the nominal strength renormalization. The diagrams of the first [1, 5] and second series are shown in Fig. 6. The loads have been divided by the renormalized strength σ_N^*, by the thickness t and by the characteristic size d, rescaled by the fractal dimension of the ligament at the peak load. Furthermore, the displacements have also been divided by the rescaled characteristic dimension d. According to this operation, the exponents in equation (2) have been assumed to be equal to $\alpha = \beta = 0.86$ for the first series and to $\alpha = \beta = 0.91$ for the second. This renormalization, according to the monofractal hypothesis presented in Carpinteri [24] and in Carpinteri and Ferro [1], works very well for the ascending portion of the curve, thereby producing a satisfactory accumulation of the different curves. In fact, up to reaching the peak load, the fractal dimension of the ligament is approximately that obtained with the renormalization of the nominal strength. On the other hand, in the softening regime, the curves still show considerable divergence. This is due to the decrease in the fractal dimension of the reacting section with the growth and percolation of damage. Such a fractal dimension decreases continuously down to zero, when the specimen breaks into two parts.

The renormalized experimental curves, with the exponent β superposing the softening curves of the two series as well as possible, are reported in Fig. 7. An improved superposition of the curves might be obtained with a renormalization that incorporates a continuous variation in the exponents α and β, in relation to the effective decrement of the fractal dimension of the ligament during the complete testing procedure. In mathematical terms, the exponents of equation (2) should be appropriate functions of δ, $\alpha(\delta)$ and $\beta(\delta)$, with values comprised between zero and one.

6. CONCLUSIONS

Three fundamental aspects are treated in the paper. First of all, the results of the second series of tensile experiments on dog-bone concrete specimens are reported. These tests were performed over a very large scale-range (16:1). The size effects on both nominal tensile strength and fracture energy are highlighted.

The second aspect consists in the renormalization of the experimental results, in order to obtain constant values for both tensile strength and fracture energy with respect to the structural dimension. The drawback to upholding this objective is the loss of the classical physical dimensions for these two material properties. It is obviously very high and, actually, it is impossible to use these results in a structural analysis, wherein noneuclidean (or fractal) mechanics are not available. For engineering applications, the MFSL, based on the self-affinity hypothesis of both the ligament for the tensile strength and the fracture surface for the fracture energy could be used.

Finally, an original idea concerning the fractal dimension of the ligament (or reacting section) in the tensile test is presented. The monofractal hypothesis proposed by Carpinteri [24], and used to obtain constant values for tensile strength, considers the fractal dimension of the ligament at the peak load. Beyond this stage, a softening (or catastrophic) regime starts in which a macrofracture forms and propagates through the whole specimen. The fractal dimension of the ligament decreases from the value at the peak load to zero when the specimen breaks into two parts. The dual renormalization, presented in this paper, involving both load and displacement, induces a scale-independent softening curve. The independence of the softening curve with respect to specimen size is fundamental, since the softening curve is a basic component of several computational models.

ACKNOWLEDGEMENTS

The present research was carried out with the financial support of the Ministry of Universities and Scientific Research (MURST) and the National Research Council (CNR). Financial support from the European Community, TMR contract no. ERBFMRXCT-960062, is also gratefully acknowledged.

REFERENCES

[1] Carpinteri, A. and Ferro, G., 'Size effect on tensile fracture properties: a unified explanation based on disorder and fractality of concrete microstructure', *Mater. Struct.* **27** (1994) 563-571.

[2] Carpinteri, A., Chiaia, B. and Ferro, G., 'Size effects on nominal tensile strength of concrete structures: multifractality of material ligaments and dimensional transition from order to disorder', *Ibid.* **28** (180) (1995) 311-317.

[3] Carpinteri, A. and Chiaia, B., 'Multifractal nature of concrete fracture and size effects on nominal fracture energy', *Ibid.* **28** (182) (1995) 435-443.

[4] Carpinteri, A. and Chiaia, B., 'Size effects on concrete fracture energy: dimensional transition from order to disorder', *Ibid.* **29** (1996) 259-266.

[5] Carpinteri, A. and Ferro, G., 'Apparent tensile strength and fictitious fracture energy of concrete: a fractal geometry approach to related size effects', in 'Fracture and Damage of Concrete and Rock' (FDCR-2), Ed. H.P. Rossmanith, (Chapman & Hall/E&FN Spon, London, 1993) 86-94.

[6] Carpinteri, A. and Maradei, F., 'Three-jack solution to obtain a truly stable and symmetric tensile concrete test', *Experimental Mechanics* **35** (1995), 19-23.

[7] Ferro, G., 'Effetti di scala sulla resistenza a trazione dei materiali', Ph.D. Thesis, Politecnico di Torino, Torino, 1994.

[8] Rusch, H. and Hilsdorf, H.K., 'Deformation characteristics of concrete under axial tension', *Voruntersuchungen*, Bericht 44 (Münich 1963).

[9] Hughes, B.P. and Chapman, G.P., 'The complete stress-strain curves for concrete in direct tension', *Bulletin RILEM* (30) (1966) 95-97.

[10] Evans, R.H. and Marathe, M.S., 'Microcracking and stress-strain curves for concrete in tension', *Mater. Struct.* **1** (1968) 61-64.

[11] Heilmann, H.G., Hilsdorf, H.K. and Finsterwalder, K., 'Festigkeit und Verformung von Beton unter Zugspannungen', *Deutscher Ausschuss für Stahlbeton* Heft 203 (1969).

[12] Petersson, P.E., 'Crack growth and development of fracture zones in plain concrete and similar materials', Report TVBM-1006, Division of Building Materials, Lund Institute of Technology (1981).

[13] Gopalaratnam, V.S. and Shah, S.P., 'Softening response of plain concrete in direct tension', *J. ACI* **82** (1985) 310-323.

[14] Reinhardt, H. W., Cornelissen, A. W. and Hordijk, D. A., 'Tensile tests and failure analysis of concrete', *J. Struct. Eng.* **112** (1986) 2462-2477.

[15] Phillips, D. V. and Zhang, B., 'Fracture energy and brittleness of plain concrete specimens under direct tension', in 'Fracture Behaviour and Design of Materials and Structures', Ed. D. Firrao, Proceedings of the 8th European Conference on Fracture, Torino, Italia, October, 1990 (EMAS, Warley,1991) 646-652.

[16] Bažant, Z.P. and Pfeiffer, P.A., 'Determination of fracture energy from size effect and brittleness number', *ACI Material Journal* (November-December 1987) 463-480.

[17] Nooru-Mohamed, M.B. and van Mier, J.G.M., 'Size effect in mixed mode fracture of concrete', in 'Fracture Processes in Concrete, Rock and Ceramics', Eds. J.G.M. van Mier, J.G. Rots and A. Bakker, (E. & F.N. Spon, 1991) 461-471.

[18] Slowik, V., Saouma, V.E. and Thompson, A., ' Large scale direct tension test of concrete', *Cement and Concrete Research* **26** (6) (1996) 949-954.

[19] Carpinteri, A., 'Scaling laws and renormalization groups for strength and toughness of disordered materials', *International Journal of Solids and Structures* **31** (1994) 291-302.

[20] Carpinteri, A., Chiaia, B. and Ferro, G., 'Multifractal scaling law for nominal strength variation of concrete structures', in 'Size Effect in Concrete Structures', Eds. H. Mihashi, H. Okamura and Z.P. Bažant, (E. & F.N. Spon, London, 1994) 193-206.

[21] Carpinteri, A., Chiaia, B. and Ferro, G., 'Multifractal nature of material microstructure and size effect on nominal tensile strength', in 'Fracture of Brittle Disordered Materials: Concrete, Rocks, Ceramics', Eds. G. Baker and B.L. Karihaloo, (E. & F.N. Spon, London, 1995) 21-34.

[22] Vervuurt, A., Schlangen, E. and Van Mier, J.G.M., 'Tensile cracking in concrete and sandstone: part 1 - basic instruments', *Mater. Struct.* **29** (1996) 9-18.

[23] Van Mier, J.G.M., Schlangen, E. and Vervuurt, A., 'Tensile cracking in concrete and sandstone: part 2 - effect of boundary rotations', *Ibid.* **29** (1996) 87-96.

[24] Carpinteri, A., 'Fractal nature of material microstructure and size effects on apparent mechanical properties', *Mechanics of Materials* **18** (1994) 89-101.

[25] de Arcangelis, L., 'Randomness in breaking thresholds', in 'Statistical Models for the Fracture of Disordered Media', Ed. H.J. Herrmann and S. Roux, (Elsevier Science Publishers, B.V. North Holland, 1990) 229-260.

[26] Carpinteri, A., Chiaia, B. and Ferro, G., 'Multifractal Scaling Law: an extensive application to nominal strength size effect of concrete structures', *Atti del Dipartimento di Ingegneria Strutturale del Politecnico di Torino* **51**, Torino (1995).