Magazine of Concrete Research, 1997, **49**, No. 178, Mar., 45–53

A new explanation for size effects on the flexural strength of concrete

A. Carpinteri*, B. Chiaia* and G. Ferro*

Politecnico di Torino

The size dependence of the flexural strength of concrete beams is discussed. It is shown that existing approaches fail to predict the strength of real-sized structures. The scaling of the modulus of rupture σ_u can be consistently modelled by means of a multifractal scaling law, the influence of microstructural disorder being predominant for the shallowest beams. At larger scales, homogenization comes into play, leading to the definition of an asymptotic constant strength f_t. This transition occurs more rapidly in the case of high-strength concrete, where a more brittle behaviour is observed, accompanied by the rapid vanishing of size effects. Validation of the law is pursued by means of best-fitting of relevant experimental data, which allows for determination of the asymptotic value of σ_u, valid for real-sized members.

Introduction: size-dependence of the modulus of rupture

It is widely believed that the true fracture properties of concrete structures can be unequivocally determined only by means of uniaxial tensile tests.[1] Unfortunately, tensile tests are difficult to carry out in standard laboratories, either with fixed or rotating boundary conditions. Therefore, the *modulus of rupture* σ_u, measured for beams in either three or four-point bending, turns out to be an experimentally convenient measure of strength owing to the relative simplicity of these tests. On the other hand, the strong size dependence of the bending properties (not only of the nominal strength, but also of the rotational capacity and ductility) has been detected earlier in several experimental investigations carried out on plain and reinforced concrete members.[2]

When dealing with the size dependence of the bending strength σ_u, it is customary to relate σ_u to f_t, this last parameter being the so-called *tensile strength*. It is usually assumed that f_t is an (ideal) material constant, which should be measured by means of tensile tests. Unfortunately, this is not the case, since the size dependence of f_t has been verified in direct[1] and indirect[3] tensile tests, as shown in Fig. 1. Therefore, the size dependence of (nominal) strength seems to be a peculiar property of concrete structures, regardless of the specimen geometry and loading conditions. It follows that a consistent explanation of these effects has to be sought based on more general considerations, that is, inserting the phenomenon of concrete fracture into the framework of critical phenomena.

The heterogeneous and disordered microstructure of concrete is mainly responsible for the scaling properties of strength and toughness, together with the competition between energy release due to macrocracking and stress redistribution due to progressive damage. A multifractal scaling law has been proposed,[4] which describes the whole range of scaling, capturing the peculiar transition from a disordered regime characterized by strong size effect, to the homogeneous regime, holding for the larger structures, where the size dependence vanishes and an asymptotic value of nominal strength can be determined.

Indeed, correct determination of bending strength σ_u is crucial from engineering viewpoints, due to the large number of concrete members subjected to flexure, and to the wide diffusion of bending tests throughout the various laboratories. This explains the theoretical and experimental efforts maintained by the scientific and engineering community in order to interpret the phenomenon of size dependence of the

* Politecnico di Torino, Department of Structural Engineering, Corso Duca degli Abruzzi 24, 10129 Torino, Italy.

(MCR 540) Paper received 2 January 1996; last revised 17 March 1996; accepted 22 April 1996

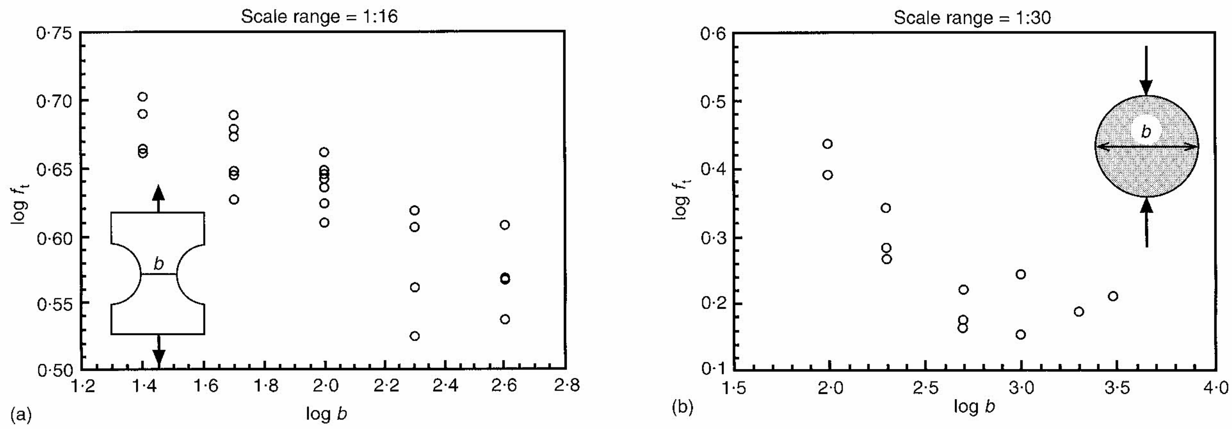

Fig. 1. Size effects on uniaxial tensile strength. (a) Direct[1]*, and (b) indirect*[3] *tensile tests*

modulus of rupture σ_u, whereas only in the last ten years has attention been drawn also to the other structural geometries.

It is important to point out that uncertainties in the determination of the bending strength affect both *allowable stresses* design and *limit state* design, in the former case by lowering the reliability of the safety factor, and in the latter case by providing unacceptable stress–strain conditions under service loads. In both cases, excessive prudence due to uncertainty may lead to coarse oversizing or to the overestimation of the reinforcement percentage. Problems arise not only in the case of plain concrete structures (large foundation beams or massive walls) but also in reinforced members, where cracking of the most stressed concrete layers leads to corrosion of the steel bars and dramatically reduces durability. Moreover, the minimum reinforcement requirements for beams subjected to bending are strongly related to the modulus of rupture σ_u, since it has become clear that, especially for deep beams, the correct minimum reinforcement ratio can be computed only by means of a fracture mechanics approach.[5]

Classical interpretations of the bending strength size-dependence

In the following, we shall refer to f_t as *the asymptotic value of tensile strength*, obtained from uniaxial tests on large-sized specimens, and we shall relate the (nominal) bending strength σ_u to this constant quantity. According to the elastic bending theory, if M_u is the ultimate bending moment in the central cross-section of the beam (subjected to either three or four-point bending), and t and b are, respectively, the thickness and depth of the beam, the nominal bending strength (*modulus of rupture*) is given by:

$$\sigma_u = \frac{6M_u}{b^2 t} \tag{1}$$

If we assume that f_t is the limit stress that the material can *locally* undergo, it follows that the bending strength σ_u coincides with f_t in the case of an elastic, perfectly brittle material. No size effect is therefore provided: in this case, in fact, catastrophic failure is supposed to occur as soon as f_t is reached in any point of the beam (Fig. 2). Nevertheless, this would be true in the (ideal) case of pure bending while, in the case of three-point bending, the modulus of rupture σ_u exceeds f_t by almost 5% due to the asymmetrical stress field.

A trivial relation between the modulus of rupture σ_u and f_t is provided by the ACI Building Code,[6] which merely assumes, on the average:

$$\sigma_u = 1{\cdot}25 f_t \tag{2}$$

The size dependence of the bending strength is therefore not taken into account by equation (2), which only states that σ_u is larger than f_t, as had been reported much earlier.[7] An analogous approach is maintained by the Eurocode No. 2,[8] where the concrete tensile strength, if measured by means of flexural tests, has to be drastically reduced by 50%:

$$\sigma_u = 2{\cdot}0 f_t \tag{3}$$

If plasticity is supposed to occur in the most stressed layers of the beam, the nominal flexural strength, computed according to equation (1), is found to depend upon the strain gradient (Fig. 3(a)).

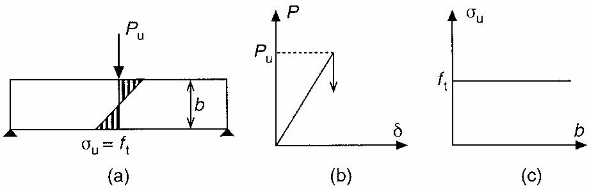

Fig. 2. (a) Elastic perfectly brittle model; (b) catastrophic failure; and (c) absence of size-effects

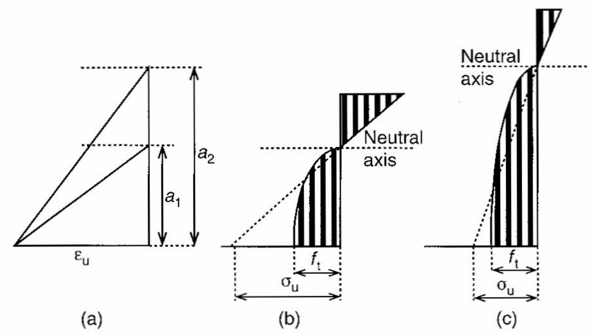

Fig. 3. Effect of the variable strain gradient (a) on the determination of nominal bending strength. Shallow (b) and deep (c) beam

Shallower beams (Fig. 3(b)) will therefore yield higher moduli of rupture, according to the larger strain decrease with respect to the depth (steep gradient). On the basis of this theory and of early experimental results, an empirical expression was proposed by the Deutscher Ausschuss für Stahlbeton,[9] relating the nominal flexural strength σ_u to the depth a of the tensile zone (which can be considered approximately proportional to the total beam depth b):

$$\sigma_u = (0{\cdot}8 + 0{\cdot}26a^{-0{\cdot}6})f_t \tag{4}$$

where a has to be measured in metres. Note that, according to equation (4), the bending strength of very deep beams ($a > 1500$ mm) becomes smaller than f_t (Fig. 4). Note also that these arguments would imply that the nominal flexural strength increases if applied compressive loads act upon the beam, thus reducing the size of the tensile zone.

Based on a similar strain-gradient approach, a size dependent empirical relationship between the modulus of rupture and the tensile strength has been specified in the CEB-FIP Model Code 1990,[10] and is given by:

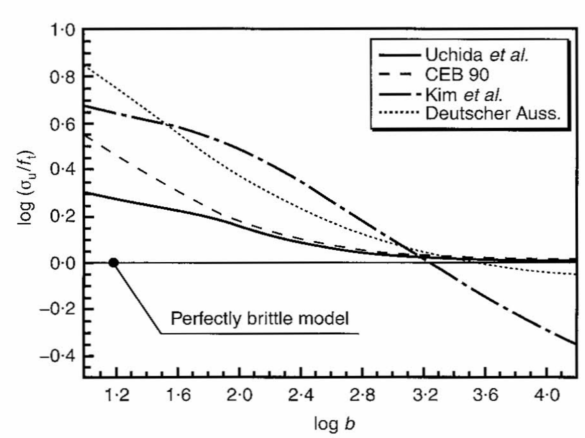

Fig. 4. Comparison of different empirical models for the size dependence of bending strength with some existing Code formulations

$$\sigma_u = f_t\left[\frac{1 + 2{\cdot}0(b/b_0)^{0{\cdot}7}}{2{\cdot}0(b/b_0)^{0{\cdot}7}}\right] \tag{5}$$

where b is the beam depth and b_0 is a reference size equal to 100 mm. Equation (5) is applicable to unnotched beams with $b \geqslant 50$ mm. Note that equation (5) comes from experimental observations and no consideration is given to the role of the microstructure, whereas aggregate interlocking is explicitly taken into account in the Codes when dealing with the ultimate shear strength.[8,10]

The earliest attempts to explain the bending strength decrease with size were related to a perfectly brittle *weakest-link* concept (statistical size effect[11]). The Weibull approach, however, turns out to be poor in the case of concrete, owing to the progressive chaotic damage occurring before the peak load, and to the stable crack growth that takes place before failure.

If the elastic, perfectly brittle constitutive model is abandoned, and the cohesive crack model[12] is assumed to represent the mesoscopic mechanical behaviour of concrete, the size dependence of the modulus of rupture can be more adequately described. Indeed, a unique relationship cannot be deduced, since solutions may appreciably differ from one another, depending on the shape of the softening curve.[2] Different failure mechanisms may take place, depending on the rate of consumption of the fracture energy $\mathcal{G}_F$. Nevertheless, the asymptotic behaviour predicted by the cohesive approach is $\mathcal{G}_F$-independent, and yields the following relations:

$$\sigma_u \to 3f_t \quad \text{for } b \to 0 \tag{6a}$$

$$\sigma_u \to f_t \quad \text{for } b \to \infty \tag{6b}$$

where the limit (6a) for structural sizes tending to zero represents a plastic limit solution, as has been shown.[13]

The interplay between plastic collapse, governed by limit analysis, and brittle failure, governed by linear elastic fracture mechanics (LEFM) represents the basis of the size effect law (SEL) by Bazant.[14] The original formulation of the law applies only to notched beams, with the size of the notch scaling proportionally to the structural size (Fig. 5(b)). On the

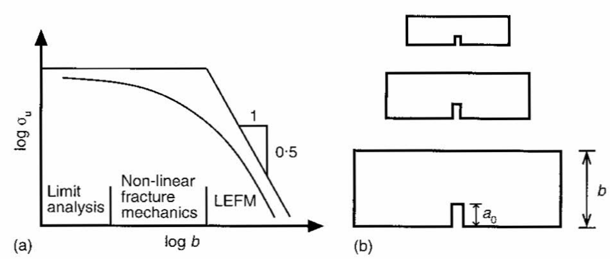

Fig. 5. Bazant's size effect law[14]: (a) bilogarithmic diagram; and (b) proportionally increasing notch size

basis of an energy-release failure criterion, the following expression is derived:

$$\sigma_u = \frac{Bf_t}{\left[1 + \frac{b}{b_0}\right]^{1/2}} \tag{7}$$

where B and b_0 are two empirical constants to be determined by best-fitting of the experimental data. In the bilogarithmic diagram, a continuous transition is provided from limit analysis to LEFM as the depth b of the beam increases (Fig. 5(a)), yielding the unrealistic complete vanishing of strength for very large structures.

Various attempts to apply equation (7) to different loading conditions and specimen geometries have been reported in the literature: nevertheless, it is now clear that the range of applicability of equation (7) is strictly limited by the two unrealistic assumptions of proportionally scaling notch size and purely energy-release controlled failure. By considering a constant maximum flaw size, as is likely to occur in real materials, a modified size effect law has been deduced by Kim *et al.*,[15] where a constant term αf_t is added to equation (7):

$$\sigma_u = \frac{Bf_t}{\left[1 + \frac{b}{b_0}\right]^{1/2}} + \alpha f_t \tag{8}$$

Note that equation (8) is a three-parameter equation (Fig. 4). Moreover, the value obtained for α, by best-fitting of the experimental data, is close to 0·15, which implies that the asymptotic value of the bending strength predicted by equation (8), as $\sigma_u = 0{\cdot}15 f_t$, is too low.[2]

Primary importance has been given by the Japanese scientific community to the proper understanding of the size effect on bending strength. Among the various proposed solutions, it is worth mentioning the empirical equation put forward by Uchida *et al.*:[16]

$$\sigma_u = f_t\left[1 + \frac{1}{0{\cdot}85 + 4{\cdot}5(b/l_H)}\right] \tag{9}$$

where $l_H = E\mathcal{G}_F/f_t^2$ is Hillerborg's characteristic length[12] and $b/l_H \geqslant 0{\cdot}1$. For a typical concrete $l_H \cong 300$ mm, whereas, in the case of high-strength mixtures, this value can eventually be halved.

The aforementioned empirical models fit the experimental results only in a narrow size range, due to the lack of theoretical bases. Moreover, appreciable differences arise between the curves, either in the case of the smaller or, which is more important, in the case of the larger beam sizes (Fig. 4). Other non-linear fracture mechanics models have recently been developed to explain the size dependence of the modulus of rupture, but they seem to work only in a limited range. This is the case of the boundary layer model by Bazant and Li[17] and of the two-parameter model by Jenq and Shah.[18] The former model gives an interpretation of size effect which is alternative to SEL, and is supposed to hold when macrocracking does not occur and energy release is not primarily involved in the definition of the peak load. In the case of Jenq and Shah's model,[18] astonishingly, the modulus of rupture seems to increase with the depth of the beam in the smaller sizes range, which represents an evident absurdity.[2]

Multifractal scaling law for the size effect on bending strength

It is now well established that scaling phenomena in disordered media are mainly due to the heterogeneity of the considered domains.[19] Moreover, the failure behaviour of concrete structures incontrovertibly possesses most of the features of critical phenomena, ranging from the microscopic scale, where self-organization of microcracks occurs prior to their coalescence and percolation, to the macroscopic one, where cusp instabilities are detected in the load–displacement characteristics.[19]

These self-similar and hierarchical damage processes occurring in the resisting section prior to the peak load have been extensively detected.[20] The multiscale accumulation of damage reflects the hierarchical character of the concrete microstructure, which ranges from the microscopic level of the cement clinker up to the macroscopic level of the coarse aggregates embedded in the paste.

Fractal geometry[19] permits one to abandon the integer topological dimensions of Euclidean sets, and to move to the anomalous non-integer dimensions of fractal domains. In this way, it is possible to *quantify the degree of disorder* possessed by the material microstructure, and to investigate the structural effects of the microscopic complexity. Due to the chaotic damage accumulation and to the heterogeneous distribution of internal stresses, it is consistent to model the resisting section of a concrete beam by means of a *lacunar fractal* set (Fig. 6). Thus, we have to abandon

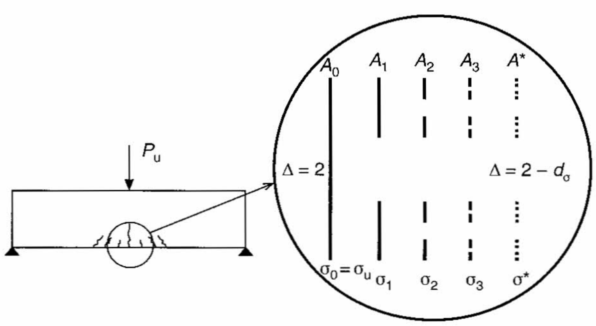

Fig. 6. Topological hypothesis[4] of rarefied fractal ligament at the peak load

the classical dimension ($[L]^2$) of a resisting area and to consider a dimension smaller than 2·0 ($[L]^{2-d_\sigma}$), where d_σ is the topological decrement due to disorder. An elegant and synthetic description of the microstress field in the reacting section can be obtained,[19] where the fractal stress σ^* ($[F][L]^{-(2-d_\sigma)}$) turns out to be the only scale-independent parameter. From this invariant quantity, the size dependence of the nominal stress σ_u (defined on the ideal ligament according to equation (1)) is obtained[19] as a function of the fractal dimensional decrement d_σ.

Extrapolating to the structural level, the nominal bending strength decreases as the beam depth increases, with a rate controlled by the fractal dimension of the rarefied ligament. On the other hand, as the depth of the beam increases, the progressive homogenization of the random fields comes into play. In the limit of the microscopic scales, fractal dimension cannot be smaller than 1·5, which represents the maximum (Brownian) degree of disorder in dissipative processes.[19] In the limit of the largest sizes,[4] fractality disappears *at the structural level* and the classical Euclidean description of the material can be adopted. According to this hypothesis, the size dependence turns out to be very strong for small beams, the influence of disorder being predominant, while size effect progressively vanishes as the depth of the beams increases.[4] This transition is controlled by the interplay between microstructural heterogeneity (quantified by the internal length l_{ch}) and external size.

Note that the multifractal approach is in agreement with the cohesive model, where the ratio between the characteristic size of the fracture process zone (where all the non-linear phenomena take place) and the beam depth can be considered as an indicator of the variable influence of disorder on strength. On the other hand, it represents a drastic conceptual revolution with respect to Bazant's size effect law, since the size-dependence rate follows an opposite trend.

Based on these arguments, a simple equation can be derived to model this scaling behaviour. By applying the renormalization procedure (which is an extension of dimensional analysis to chaotic processes) to the disordered stress field, the analytical expression of the multifractal scaling law can be obtained:

$$\sigma_u = f_t\left[1 + \frac{l_{ch}}{b}\right]^{1/2} \qquad (10)$$

where l_{ch} represents the characteristic length acting as a threshold between the *fractal regime*, where disorder plays a pre-eminent role and a strong size effect is present, and the *homogeneous regime*, where the influence of disorder vanishes and an asymptotic constant bending strength is reached, equal to f_t. Note that the rate of size dependence is governed by the interplay between l_{ch} and the beam depth b, and that the Brownian hypothesis provides the exponent of the scaling law, yielding the strongest size-effect rate, in the limit of the smallest beams, equal to 1/2. In the bilogarithmic diagram (Fig. 7(b)), the MFSL is therefore characterized by an upward concavity, whereas a downward concavity is shown by the SEL (Fig. 5(a)).

From an engineering point of view, the MFSL allows for the determination of the flexural strength of *very large* concrete beams, whereas the SEL predicts zero strength in that case. Thus, it is possible to compute the reliable minimum reinforcement ratio also for very deep beams or massive sustaining elements. In addition, the internal length l_{ch} marks the transition between a range of sizes (usually coincident with the laboratory-testing sizes) where the scaling effect is pronounced, and the larger sizes where scaling can be neglected and a constant value of bending strength can be defined.

A wide number of experimental tests confirm the MFSL hypotheses: the size dependence is found to be stronger in the smaller sizes range, as well as the scatter of the strength values corresponding to a certain size, thus clearly indicating that heterogeneity plays a fundamental role. Recent experimental results by Adachi *et al.*[21] demonstrate that the same transition is present also in the scaling of the *deformability* characteristics of concrete beams subjected to bending. The negative size effect on the maximum rotational angle turns out to be, in fact, more pronounced as the depth of the beams becomes shallower.

Application of the MFSL to relevant experimental results

The multifractal scaling law described in the previous section can be applied to relevant experimental data reported in the literature. This statistical analysis allows for the extrapolation of a reliable value of the bending strength, valid for real-sized structures, starting from laboratory-sized specimens (subjected to three or four-point bending).

An extensive application of the MFSL to various loading geometries (bending, direct and indirect tension, shear, torsion and their combination) has been

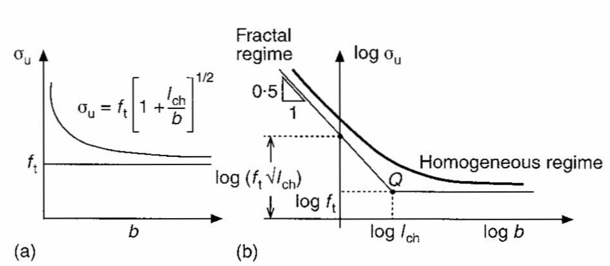

Fig. 7. (a) Multifractal scaling law for the bending strength. (b) Bilogarithmic diagram and transition from order to disorder

reported by Carpinteri *et al.*,[22] where the non-linear best-fitting Levenberg–Marquardt algorithm has been used throughout. Comparison between the correlation coefficients (R) provided by the MFSL and the SEL[14] reveals that the multifractal approach prevails in most cases when unnotched beams are considered, whereas the two approaches are almost equivalent in the case of notched beams. In any case, fitting of the data appears to be statistically significant only if (at least) one order of magnitude is considered in the size range.

In the following, data are presented in bilogarithmic scale to highlight the fundamental difference (concavity and asymptotics) between the energy-release based approach (SEL) and the multifractal theory. This should be clearly in mind when evaluating the amount of the strength reduction in real-sized beams. The actual decrement of the asymptotic bending strength with respect to the mean value and to the maximum value obtained in the tests is reported in Table 1. Note the strong size effect arising in all the considered situations.

Three-point bending tests on notched normal strength concrete beams were performed by Alexander.[23] Beams of sizes 100, 200, 300, 500 and 800 mm were used, thus resulting in a size range 1:8. The nominal stress at failure can be set, applying the elastic bending theory to the initially uncracked ligament, equal to:

$$\sigma_u = \frac{6M_u}{t(b - a_0)^2} \tag{11}$$

where a_0 is the initial notch length. The experimental data are reported in the bilogarithmic diagram, where fitting by the MFSL and by the SEL is also shown (Fig. 8). Note that, even if notched specimens are considered, an upward concavity of the data seems to come into play, indicating that the disorder → order transition prevails over the energy-release effects. Extrapolating to very large beams, an intrinsic flexural strength (f_t) would be present, while SEL would predict the absolute vanishing of the load-carrying capacity. Best-fitting of the data yields the two MFSL parameters as $f_t = 1{\cdot}67$ MPa and $l_{ch} = 288{\cdot}6$ mm, and the correlation coefficient R equal to 0·944, whereas fitting by SEL provides $R = 0{\cdot}860$.

Four-point bending tests were carried out by Sabnis

Table 1. Decrement of the nominal bending strength predicted by the MFSL for large structures

	Ref. 23	Ref. 24	Ref. 25	Ref. 26
Max. experimental strength: MPa	3·90	8·90	2·90	10·11
Mean experimental strength: MPa	2·70	6·07	1·68	6·80
Infinite-size strength, f_t: MPa	1·67	3·83	0·57	4·10

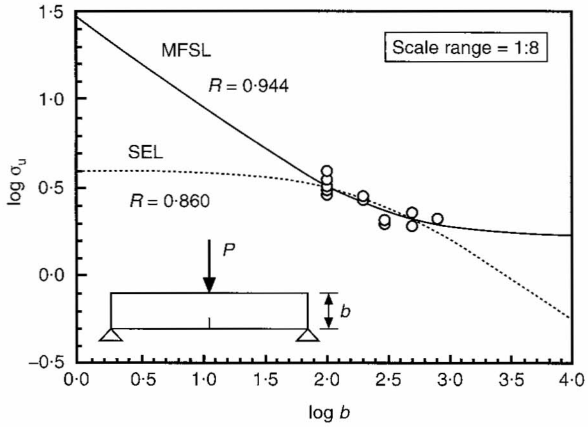

Fig. 8. Application of the MFSL to the TPB data by Alexander.[23] Comparison with the SEL

and Mirza[24] on unnotched concrete specimens in the size range 1:17. The span-to-depth ratio was set equal to 4 for all the beams. The nominal bending strength can be defined, according to equation (1), as the elastic stress acting upon the extreme fibre under the failure load. Best-fitting of the experimental data is shown in Fig. 9. The asymptotic bending strength f_t, valid for the largest beams, is found to be equal to 3·83 MPa, whereas the characteristic length is $l_{ch} = 42.02$ mm. The correlation coefficient R is equal to 0·999 in the case of MFSL and to 0·952 in the case of SEL, the concavity of the data being clearly upwards in the bilogarithmic diagram.

Another series of four-point bending tests was carried out by Bazant and Kazemi[25] on unnotched beams with longitudinal steel reinforcement. The size range examined was 1:16 ($b = 20{\cdot}64 \div 330.2$ mm), and a two-dimensional similitude was ensured, the thickness t being the same for all the beams ($t = 38{\cdot}1$ mm). A micro-concrete was used, with maximum aggregate size equal to 4·8 mm and average compressive strength $f'_c = 46{\cdot}2$ MPa. The span/depth ratio was

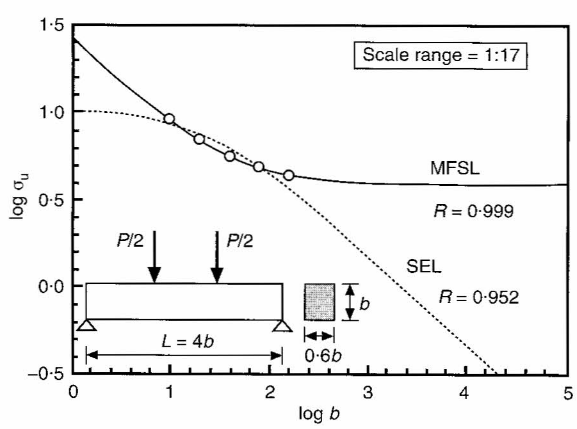

Fig. 9. Application of the MFSL to the FPB data by Sabnis and Mirza.[24] Comparison with the SEL

set equal to 7. Therefore, even if diagonal shear failure was forced by the presence of the bars, flexural effects cannot be excluded *a priori*. The steel bars, anchored with right-angled hooks at their ends to prevent bond slip and pull-out, provided a reinforcement ratio $\rho = A_{steel}/bt = 1{\cdot}62\%$.

The nominal strength, in this case, was set equal to the nominal shear strength, that is, $\tau_u = P_u/2bt$, where P_u is the failure load. Best-fitting of the data by means of MFSL yields the following parameters: $f_t = 0{\cdot}571$ MPa and $l_{ch} = 485{\cdot}9$ mm, with a correlation coefficient R equal to 0·986, while fitting by SEL, the value $R = 0{\cdot}980$ is obtained (Fig. 10). If the hypothesis is made that the internal length is directly related to d_{max}, a dimensionless quantity α can be deduced as $\alpha = l_{ch}/d_{max} = 101{\cdot}23$.

Three-point bending tests were performed by Gettu *et al.*[26] on notched *high-strength concrete* beams, with average compressive strength equal to 96 MPa. This kind of material is characterized by relatively small aggregates and by a strong interface bond between matrix and aggregates. The strength of the matrix is comparable with that of the aggregates, thus resulting in a more homogeneous cracking process with respect to ordinary concrete: the width of the fracture process zone, according to the cohesive model, decreases by 60%. Consequently, while the compressive strength increases by almost 160%, the material's fracture energy increases by only 25%, providing a definitely more brittle behaviour with respect to ordinary concrete. This implies, in the multifractal scaling law, a rapid transition towards the ordered regime, characterized by the absence (or, better, by the homogenization) of the beneficial contribution of microstructural disorder.

Four beam sizes have been tested: the reference size b was chosen equal to the total beam depth, in the size range 1:8 ($b = 38{\cdot}1 \div 304{\cdot}8$ mm). The maximum aggregate size was $d_{max} = 9{\cdot}5$ mm. The notch depth a_0, scaled in a proportional manner with b, was set

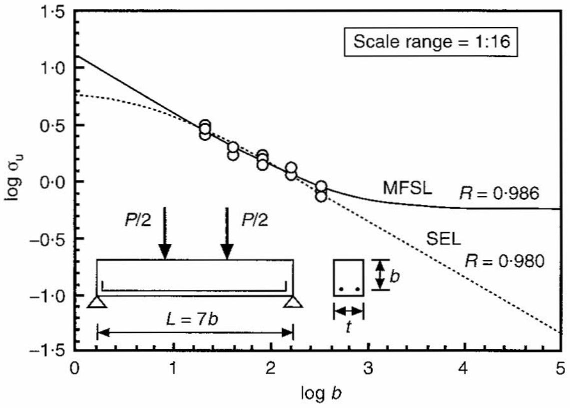

Fig. 10. Application of the MFSL to the FPB data by Bazant and Kazemi.[25] *Comparison with the SEL*

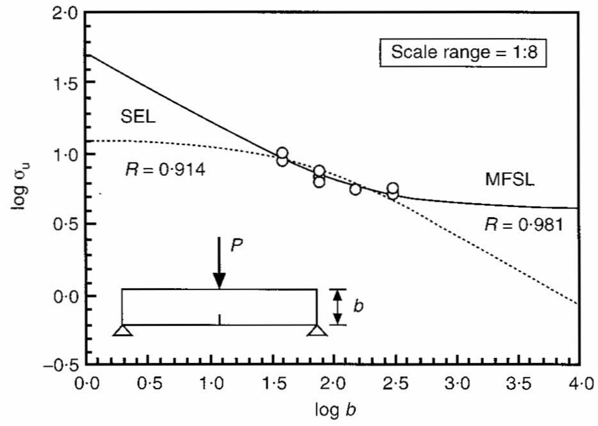

Fig. 11. Application of the MFSL to the TPB data by Gettu et al.[26] *Comparison with the SEL*

equal to $b/3$, while the net span between the supports was $L = 2{\cdot}5b$. Note that only a two-dimensional similitude is present, the thickness t of the beams being constant at 38·1 mm. The nominal bending strength can be computed according to equation (11).

Best-fitting of the data by means of the MFSL is directly plotted in the bilogarithmic diagram (Fig. 11), where fitting by SEL is also shown for comparison. The computed best-fit values are: $f_t = 4{\cdot}1$ MPa and $l_{ch} = 156{\cdot}9$ mm. From the internal length value, the non-dimensional parameter $\alpha = l_{ch}/d_{max}$ equals 16·51. The MFSL correlation coefficient turns out to be $R = 0{\cdot}981$, whereas the application of SEL yields $R = 0{\cdot}914$. The authors[26] affirm that the strength values obtained from the largest specimens are anomalously large, being in clear disagreement with the SEL predictions. In contrast, these values show perfect agreement with the MFSL, as they are placed in the asymptotic homogeneous scaling regime, which, in the case of high-strength concrete, comes into play much earlier. Recent results by Eo *et al.*[27] confirm that size effects on flexural strength attain their limiting values more rapidly for high-strength concrete than for normal-strength mixtures. Note also that the asymptotic strength f_t is equal to 60% of the average specimen strength (6·8 MPa), and to only 43% of the lowest specimen strength.

Conclusions

On the basis of theoretical and experimental evidence, the following conclusions may be drawn:

(1) The current building-code requirements need to be revised if consistent and reliable predictions are to be made for the bending strength of real-sized concrete beams. Poor extrapolations from laboratory-sized specimens are provided if the size-dependence of the modulus of rupture is not properly taken into

account (Table 1). Moreover, existing formulas (CEB, ACI, Eurocode 2) are based on an empirical approach which cannot be considered suitable for the huge variety of material properties and structural typologies that are encountered.

(2) The necessity for the correct prediction of the bending strength of concrete beams is neither restricted to unreinforced elements nor confined to the durability requirements of reinforced ones, but rather plays a fundamental role in the definition of the minimum reinforcement ratio and of the failure characteristics of the largest members. The determination of reliable values of modulus of rupture seems to be even more important in the case of high-strength concrete, where a more brittle behaviour is expected and bending failure may be catastrophic.

(3) The heterogeneity of the concrete microstructure is mainly responsible for the size dependence of the modulus of rupture. Therefore, mechanical arguments have to be supported by an adequate topological description of the failure process, for which fractal geometry seems to represent a powerful and successful tool. The interplay between a microstructural characteristic length and the external size of the beam implies the progressive vanishing of the disorder effects on strength characteristics, which can be adequately modelled by means of a multifractal scaling transition. The same transition also seems to affect the deformability properties, namely the maximum rotational angle.

(4) A multifractal scaling law has been put forward by the authors, which allows for the extrapolation of a reliable value of bending strength, holding for real-sized structural members. The validity of the MFSL has been confirmed by statistical investigation over a multitude of experimental data reported in the literature. On the other hand, wider ranges of sizes should be tested in order to get better statistical reliability.

(5) The very general physical arguments underlying the aforementioned approach make a claim for its applicability not only in the case of purely tensile and bending failures, but also in the case of shear failures, where aggregate interlock and mixed mode cracking are clearly affected by the microstructural heterogeneity.

Acknowledgements

The present research was carried out with the financial support of the Ministry of University and Scientific Research (MURST) and of the National Research Council (CNR).

References

1. Carpinteri A. and Ferro G. Size effects on tensile fracture properties: a unified explanation based on disorder and fractality of concrete microstructure. *Materials and Structures*, 1994, **27**, 563–571.
2. Planas J. *et al.* Rupture modulus and fracture properties of concrete. *Proc. 2nd Int. Conf. on Fracture Mechanics of Concrete Structures (FRAMCOS-2)*, Zurich, 1995, pp. 95–110. Aedificatio Publishers, Freiburg, 1995.
3. Hasegawa T. *et al.* Size effect on splitting tensile strength of concrete. *Proc. 7th Conference of the Japan Concrete Institute*, Tokyo, 1985, pp. 309–312.
4. Carpinteri A. *et al.* Size effects on nominal tensile strength of concrete structures: multifractality of material ligaments and dimensional transition from order to disorder. *Materials and Structures*, 1995, **28**, 311–317.
5. Bosco C. and Carpinteri A. Fracture mechanics evaluation of minimum reinforcement in concrete structures. In *Application of fracture mechanics to reinforced concrete*, Torino, 1992, pp. 347–377. Elsevier Applied Science, London, 1992.
6. American Concrete Institute *ACI-Building Code 318R-89*, Detroit, 1992.
7. Neville A. M. *Properties of concrete*. Pitman, London, 1981.
8. Commission Of The European Communities *Eurocode No. 2 – Design of Concrete Structures*. 1994.
9. Heilmann H. G. Zugspannung und Dehung in unbewehrten Betonquerschnitte bei exzentrischen Belastung. *Deutscher Ausschuss für Stahlbeton*, Heft 269, 1976.
10. Comite' Euro-International Du Beton *CEB-FIP Model Code-90 for Concrete*, Lausanne, 1991.
11. Weibull W. *A statistical theory for the strength of materials*. Swedish Royal Institute for Engineering Research, Stockholm, 1939.
12. Hillerborg A. *et al.* Analysis of crack formation and crack growth in concrete by means of fracture mechanics and finite elements. *Cem. Concr. Res.*, 1976, **6**, 773–782.
13. Carpinteri A. *Mechanical damage and crack growth in concrete: plastic collapse to brittle fracture*. Martinus Nijhoff, Dordrecht, 1986.
14. Bazant Z. P. Size effect in blunt fracture: concrete, rock, metal. *J. Engng Mech.*, ASCE, 1984, **110**, 518–535.
15. Kim J. K. *et al.* Size effect in concrete structures without initial crack. *Fracture mechanics: application to concrete* (ACI SP-118), Detroit, 1989, pp. 179–196.
16. Uchida Y. *et al.* Application of fracture mechanics to size effect on flexural strength of concrete. *Concrete Library of JSCE*, 1992, **20**.
17. Bazant Z. P. and Li Z. Modulus of rupture: size effect due to fracture initiation in boundary layer. *J. Struct. Engng*, ASCE, 1995, **121**, 739–746.
18. Jenq Y. and Shah S. P. Two parameter fracture model for concrete. *J. Engng Mech.*, ASCE, 1985, **111**, 1227–1241.
19. Carpinteri A. Scaling laws and renormalization groups for strength and toughness of disordered materials. *Int. J. Solids Struct.*, 1994, **31**, 291–302.
20. Schlangen E. and Van Mier J. G. M. Experimental and numerical analysis of micromechanisms of fracture of cement-based composites. *Cem. Concr Composites*, 1992, **14**, 105–118.
21. Adachi *et al.* Size effect on strength and deformation of RC beams failing in flexure. *Proc. 2nd Int. Conf. on Fracture Mechanics of Concrete Structures (FRAMCOS-2)*, Zurich, 1995, pp. 685–692. Aedificatio Publishers, Freiburg, 1995.
22. Carpinteri A. *et al.* *Multifractal scaling law: an extensive application to nominal strength size effect of concrete structures*. Report No. 51 of the Dept. of Structural Engineering, Politecnico di Torino, 1995.
23. Tang T. *et al.* Fracture mechanics and size effect of concrete in tension. *J. Struct. Engng*, ASCE, 1992, **118**, 3169–3185.
24. Sabnis G. M. and Mirza S. M. Size effect in model concretes? *J. Struct. Division*, ASCE, 1979, **105**, 1007–1020.
25. Bazant Z. P. and Kazemi M. T. Size effect on diagonal shear

failure of beams without stirrups. *ACI Struct. J.*, 1991, **88**, 268–276.

26. Gettu R. *et al.* Fracture properties and brittleness of high-strength concrete. *ACI Mater. J.*, 1990, **87**, 608–618.
27. Eo S. H. *et al.* Fracture characteristics and size effect in normal and high-strength concrete beam. *Proc. 2nd Int. Conf. on Fracture Mechanics of Concrete Structures (FRAMCOS-2)*, Zurich, 1995, pp. 685–692. Aedificatio Publishers, Freiburg, 1995.

Discussion contributions on this paper should reach the editor by 26 September 1997